

# MISVALUATION OF INVESTMENT OPTIONS\*

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## Abstract

We study whether investment options are fairly priced by market participants. For this purpose, we build and estimate a real options model of optimal investment in the presence of uncertainty. We then classify stocks into undervalued and overvalued based on the difference between model-implied and observed firm values. A long-short strategy that buys stocks classified as the most undervalued by the model and shorts the most overvalued stocks generates annualized alphas from major asset pricing models that range between 10% and 17% for value-weighted portfolios. This relation between estimated misvaluation and future returns is only present within subsamples of firms with relatively high fractions of investment options to existing assets. We interpret these findings as evidence of investors having difficulties in valuing investment options, which leads to mispricing in equity markets that is gradually corrected over time.

**KEYWORDS:** Misvaluation, Investment options, Optimal investment, Demand uncertainty, Future returns, Structural estimation.

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*I really feel the valuation we've gotten is more than we have any right to deserve honestly.*  
*Elon Musk, CEO of Tesla Motors.*  
*December 2014.*

## 1 Introduction

A central theme in financial economics is whether stock market investors value financial assets correctly. It is often argued by behavioral economists that various assets are systematically mispriced by market participants.<sup>1</sup> These valuation errors subsequently get corrected, giving rise to the substantial volatility that we observe in financial markets. It has been famously argued by Shiller (1981) that the volatility of the stock market substantially exceeds the volatility of real economic activity, and in particular the volatility of firms' investment.

Our goal in this paper is to understand whether misvaluation in equity markets is driven by investors' inability to correctly price firms' investment (growth) options. Growth options are arguably the most difficult component of a firm to value. Their embedded optionality makes usual valuation techniques, such as discounted cash flows and valuation by multiples, which are most commonly used by equity analysts, less appropriate. First, it is more challenging to project cash flow of a growth firm as its cash flow is a function of its investment policy. Second, because the risk of such a firm changes as it exercises its investment options, the assumption of a constant discount rate embedded in a typical DCF valuation approach may also be inappropriate (see Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2004) for a formal analysis of the time-varying nature of risk for firms with real options). Investors' difficulties in adequately pricing growth options may lead to their persistent misvaluation and, as a result, to persistent misvaluation of firms' equity.

Because of the embedded optionality of firms with investment options, a real options framework may be more appropriate for valuing such firms. In this paper we build a real options model, estimate its key parameters, and use it to compute measures of equity misvaluation for publicly-traded U.S. firms. Our model is closely related to Pindyck (1988) and Abel and Eberly (1996), and features a firm with a Cobb-Douglas-type production function, a continuum of expansion options, and a stochastic demand shock that follows a geometric Brownian motion. The firm can purchase and install additional units of capital at a given price. The firm optimally exercises its expansion options when the shock to its profitability approaches the optimal investment boundary.

To the best of our knowledge, this paper represents the first effort in the literature to gauge mispricing of growth options at the firm level. We keep the model relatively simple primarily for the

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<sup>1</sup>See, for example, Baker and Wurgler (2007), Hirshleifer (2015), Hirshleifer and Jiang (2010), among many others.

sake of computational feasibility, as we need to estimate the model on a panel of firms. While the model abstracts from some aspects that have received considerable attention in the literature, such as managerial entrenchment and the agency costs of debt, it is specifically tailored to value investment options. A model with additional layers of complexity would make it more difficult to attribute any resulting mispricing to misvaluation of investment options.

We estimate the model at monthly frequencies at the industry level. First, we use firm-specific accounting inputs from Compustat, stock market variables from CRSP, and analyst estimates from IBES to calibrate the key parameters of the model: cash flow volatility, expected growth rates, and capital depreciation rate at the industry level, and the value of the capital stock and profitability at the firm level. We then estimate the curvature of the production function and the cost of installing new capital from observed market values of firm equity by minimizing valuation errors within each industry. In the next step, we use our model with estimated parameters to compute theoretical firm values. We then use the resulting firm-specific valuations to classify firms into undervalued and overvalued based on the difference between the value implied by the model and the actual market valuation. We study how misvaluation affects subsequent stock returns and whether this relation can be tied to the importance of investment options for overall firm values.

We start our empirical analysis by sorting all stocks every month into ten decile portfolios according to our measure of misvaluation: the ratio of the sum of market value of equity and book value of debt to the firm's model value. Ratios higher than one indicate overvaluation relative to the model; ratios lower than one indicate undervaluation. The relation between excess returns and firms' misvaluation has a pattern consistent with our valuation model. Mean value-weighted monthly excess return for stocks belonging to the most undervalued decile in the previous month is 1.05%, while for stocks belonging to the most overvalued decile it is 0.15% – an annualized difference of 11.4%. These differences persist at the 3-month horizon, but become economically weaker at the 12-month horizon, which is consistent with mispricing being corrected over time, and the returns to stocks in undervalued and overvalued portfolios converge.

These differences in excess returns are not due to differences to loadings on known risk factors. We estimate alphas from both established asset pricing models, such as CAPM, Fama and French (1993) 3-factor model, and Carhart (1997) 4-factor model, as well as more recent models, such as Fama and French (2015) 5-factor model and Hou, Xue and Zhang (2015) 4-factor  $q$  model, with or without the momentum factor. The differences between alphas of most undervalued and most overvalued portfolios are large – they range between 0.79% and 1.31% per month – and are highly statistically significant. Overall, these differences are highly statistically significant and are robust to varying the thresholds used for assigning

firms to most undervalued and overvalued portfolios and to changes in the model's estimation procedure.

We confirm our non-parametric results using parametric Fama-Macbeth cross-sectional regressions of excess returns on the logarithm of our measure of misvaluation, while controlling for the usual cross-sectional determinants of stock returns. The coefficients on log misvaluation are highly statistically significant and are also economically significant: increasing our misvaluation measure by one standard deviation is associated with a 25-30 basis points reduction in following-month excess returns, with a 80 basis points reduction in following-3-months returns, and with a 2.5 percentage points reduction in following-12-months returns, holding everything else constant.

To explore the role of growth options in the relation between misvaluation and future risk-adjusted returns, we decompose each firm's value into the value of growth options (GO) and value of assets in place (AP), with the help of our model. For each firm, our model generates both GO and AP values. We then construct the ratio of its GO relative to the value of its AP. Firms with higher GO/AP ratios are the firms whose values are derived to a larger extent from expansion options as opposed to existing assets. We sort all stocks into equally-sized terciles based on industry-average GO/AP ratios. Within each tercile we sort stocks into deciles based on our misvaluation measure. The idea is that if equity mispricing is driven to a large extent by misvaluation of investment options, and if our model helps value those options, then the strategy that buys stocks that are undervalued relative to model-implied value and shorts overvalued stocks is expected to perform better within subsamples of firms endowed with more growth options. The returns on these double-sorted portfolios exhibit patterns consistent with our conjecture that the power of the model comes from its ability to help value investment options. In particular, the differences between risk-adjusted returns of the portfolio with most undervalued stocks and those of the portfolio with most overvalued stocks range between 0.79% to 1.31% per month within the highest GO/AP tercile, which are highly statistically significant, and between 0% to 0.69% per month in the lowest GO/AP tercile, which are insignificant.

Since estimated GO/AP ratios are outputs from a structural model, they may potentially pick up some model misspecification as opposed to true value of firms' expansion options. Therefore, in addition to model-implied GO/AP sorts, we provide additional model-free evidence by sorting firms by market-to-book (M/B) ratio, which is an accepted measure of firms' investment opportunities (e.g., [Smith and Watts, 1992](#); [Barclay, Smith and Watts, 1997](#)). The idea is to examine whether our results are due primarily to mispricing of high market-to-book firms. Importantly, we cannot use a firm's own M/B ratio as a proxy for its investment opportunities, as M/B is potentially directly related to misvaluation (e.g., [Rhodes-Kropf, Robinson and Viswanathan, 2005](#)). Instead, we use industry-level M/B ratios, which are orthogonal to firm-level (relative to industry) misvaluation by construction. We sort firms

into terciles based on industry-median market-to-book (M/B) ratio and, within each tercile, we sort firms into misvaluation deciles. Within the tercile of firms belonging to high M/B industries, the difference between risk-adjusted returns of the portfolio with most undervalued stocks and those of the portfolio with most overvalued stocks ranges between statistically significant 0.83% to 1.29% per month, while in the low M/B tercile these differences range from 0.30% to 0.89%, and are generally statistically insignificant.

Finally, we provide one additional piece of evidence consistent with our results being driven by investors' inability to correctly price firms' growth options. We estimate our model, while shutting down growth options, effectively imposing that each firm's value is the value of its existing assets only. By construction, this version of the model is incapable of identifying any mispricing of investment options. We then repeat our procedure by defining the misvaluation measure as the ratio of the firm's market value to its modified model value. We then sort our stocks into deciles based on this misvaluation measure. The value-weighted return in the decile of the most undervalued firms is 1.15% per month while in the most overvalued firms it is 0.63%. The difference of 0.52% is not significant. The differences between risk-adjusted returns of undervalued and overvalued decile portfolios are not statistically different from zero. In general, the pattern of risk-adjusted returns across misvaluation deciles is highly non-monotonic when misvaluation is computed relative to the assets-in-place-only model. Overall, these pieces of evidence suggest that the differences in returns between undervalued and overvalued firms is at least partially due to the mispricing of investment options.

Our paper relates to several strands of literature. A large and growing body of literature examines implications of real option models for equity returns. For example, [Carlson, Fisher and Giammarino \(2004\)](#) model the role of growth option exercise in the dynamics of firm betas. [Aguerrevere \(2009\)](#) studies the effect of industry competition on equities' risk and return. [Hackbarth and Morellec \(2008\)](#) model the dynamics of betas around takeover transactions in a real option framework. [Babenko, Boguth and Tserlukevich \(2016\)](#) examine the role of idiosyncratic cash flow shocks. [Carlson, Fisher and Giammarino \(2004\)](#) study the dynamics of firm betas arising through operating leverage and past investments. [Sagi and Seasholes \(2007\)](#) focus on the convexity of firm value due to a mean-reverting price process. [Liu, Whited and Zhang \(2009\)](#) estimate a  $q$  theory investment model, which produces expected returns to a firm's investment as a function of firm observable characteristics and model parameters.

A common theme in these papers is explaining the relation between expected returns and growth options in a rational framework. Our paper, on the other hand, attempts to capture firm-level mispricing of growth options. For that purpose, a real options model that can be implemented at a firm level is

needed, using a unique set of inputs for every firm at various points in time. We propose such a model, which focuses on the valuation of expansion options. The goal of our paper, therefore, is not to study the effects of firm characteristics on risk and expected returns, but to examine whether values of investment options are adequately reflected in firms' market valuations.

Our paper is also related to the literature that structurally estimates parameters of dynamic corporate finance models. The paper that is most closely related to ours is [Warusawitharana and Whited \(2016\)](#). In their model, there is an exogenous misvaluation shock that follows a certain stochastic process and affects firms' financing and investment policies. [Warusawitharana and Whited \(2016\)](#) estimate the parameters of this process for an average sample/industry firm, and use these estimates to study the response of managers to misvaluation shocks and their effects on shareholder value for the average firm. In contrast, our model is designed to capture firm-level misvaluation. Thus, in every month and for each firm our model produces a unique firm-specific misvaluation measure.

The rest of the paper is organized as follows. Section 2 presents a simple dynamic model of a firm's optimal exercise of its expansion options. In Section 3, we identify the model through a combination of calibration and estimation of the model's parameters. Section 4 presents the empirical tests of our main hypothesis: that misvaluation is expected to be related to future risk-adjusted returns, predominantly in investment-options-rich firms. Section 5 concludes. All derivations required for the model's solution are found in Appendix A. Appendix B provides some details on the estimation procedure.

## 2 Model

We begin with a model of a firm that is characterized by a starting level of capital stock and faces stochastic demand for its products. The firm can purchase and add more capital to its capital stock at any given time. Below we discuss the details of our setup. Our approach follows earlier literature on investment with continuous capacity choice. [Pindyck \(1988\)](#) is one of the first models of incremental irreversible investments. [Abel and Eberly \(1994\)](#) and [Abel and Eberly \(1996\)](#) derive the optimal investment policy of a firm that can both purchase new capital as well as sell some of its existing capital. [Morellec \(2001\)](#) focuses on disinvestment options. In this paper we refrain from modeling disinvestment options and assume that any purchase of capital is completely irreversible. We do this for two primary reasons. First, our focus is on the misvaluation of investment options and its role in overall equity misvaluation. Those are expansion options that are optimally exercised at high demand states. On the contrary, disinvestment options are akin to put or default options that are exercised at low states

of demand.<sup>2</sup> Second, our goal is to estimate parameters of the model on a large panel of firms and, therefore, for computational feasibility it is necessary to have an analytical solution for the value of the firm. Furthermore, to the best of our knowledge, our study is the first to measure mispricing of growth options at the firm level, and we strive to use a model that is simple and transparent yet has the capacity to relate the values of growth options and assets in place to firm fundamentals.

We assume that firm  $i$  is infinitely lived, is endowed with capital stock  $K_{i,t}$ , and has a continuum of investment options: for a fixed price  $\eta$  per unit of capital it can purchase and install additional capital. Thus,  $\eta$  captures both the purchase price of one unit of new capital as well as any potential proportional installation costs. Capital depreciates at a rate  $\lambda$  per unit of time. Below, we omit subscript  $i$  for the sake of brevity. The firm's instantaneous operating profit is given by

$$\pi(K_t, x_t) = x_t K_t^\theta (1 - \tau), \quad (1)$$

where  $0 < \theta < 1$  is the curvature of the production function,  $\tau$  is the corporate tax rate, and the non-negative stochastic component  $x_t$  follows a geometric Brownian motion process:

$$dx_t = \mu x_t dt + \sigma x_t dB_t, \quad (2)$$

where  $\mu$  is the drift parameter,  $\sigma$  the volatility parameter, and  $B_t$  is a standard Brownian motion. This profit function is equivalent to a firm with a Cobb-Douglas production function and isoelastic demand.<sup>3</sup> We further assume that there exists a tradable asset whose value is perfectly correlated with  $x_t$  and hence the risk-neutral measure  $\mathbb{Q}$  exists.

The optimal investment policy of the firm is to purchase and install an infinitesimally small amount of additional capital as soon as  $x_t$  reaches the optimal investment boundary  $X(K_t)$  from below. This optimal investment boundary is increasing in  $K_t$  – a better capitalized firm optimally waits longer, until a higher realization of  $x_t$  is reached, before installing additional capital.  $X(K_t)$  divides the  $(x_t, K)$  plane into two regions. If  $x_t < X(K_t)$  then the firm is in the inaction region, as the marginal increase in firm value due to potential investment is lower than the cost of purchasing and installing new capital,  $\eta$ . If  $x_t > X(K_t)$ , then an immediate lumpy investment of  $\Delta K_t$  is optimal, such that  $x_t = X(K_t + \Delta K_t)$ . We prove in Appendix A.2 that  $X(K_t)$  has the following form:

$$X(K_t) = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu + \lambda \theta) \eta K_t^{1-\theta}}{(1 - \tau) \theta}, \quad (3)$$

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<sup>2</sup>See Eisdorfer, Goyal and Zhdanov (2016) for a study on the mispricing of default options.

<sup>3</sup>For details, see Appendix A.1 and also Morellec (2001).

where  $\beta_1$  is the positive root of quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + [\mu - \lambda(\theta-1)]\beta - (r + \lambda) = 0. \quad (4)$$

The value of the firm is then given by

$$V(K_t, x_t) = \underbrace{\frac{(1-\tau)x_t K_t^\theta}{r-\mu+\lambda\theta}}_{\text{Value of Assets-in-Place (AP)}} + \underbrace{\left[ \frac{(1-\tau)\theta}{\beta_1(r-\mu+\lambda\theta)} \right]^{\beta_1} \left( \frac{\beta_1-1}{\eta} \right)^{\beta_1-1} \frac{x_t^{\beta_1}}{(\beta_1(1-\theta)-1)K_t^{\beta_1(1-\theta)-1}}}_{\text{Value of Growth Options (GO)}}. \quad (5)$$

The value of assets in place in the first term of (5) equals the expectation (under  $\mathbb{Q}$ ) of all future cash flows generated by existing capital. The value of investment options in the second term of (5) is the sum of the present values of cash flows generated by additional capital that the firm optimally installs over time, net of the costs of acquiring capital. For the value of investment options to be finite, the production function has to be sufficiently concave. More specifically,

$$\theta < 1 - \frac{1}{\beta_1}. \quad (6)$$

Before proceeding to employ the model to value firms in our sample, it is useful to examine its comparative statics. Figure 1 provides the values of investment options and assets in place as functions of the curvature of production function  $\theta$ , depreciation rate  $\lambda$ , purchase price of capital  $\eta$ , and the volatility of cash flows  $\sigma$ . The remaining parameter values in Figure 1 are set as follows:  $\lambda = 0.1$ ,  $\sigma = 0.2$ ,  $\theta = 0.25$ ,  $\eta = 1$ ,  $\lambda = 0.1$ ,  $r = 0.05$ ,  $\mu = 0.01$ .<sup>4</sup>

Naturally, the value of assets in place decreases with the depreciation rate  $\lambda$ . Faster depreciating capital generates lower cash flows in the future. Furthermore, as equation (3) suggests, the effect of depreciation on the value of assets in place is amplified when the curvature of the production function,  $\theta$ , is higher: it is costlier to lose capital due to depreciation when its productivity is higher. The value of growth options, on the other hand, is increasing in  $\lambda$  because loss of capital due to depreciation makes it optimal to install additional units of capital at a faster pace, and a firm with a higher depreciation rate is forced to invest in new capital more aggressively. However, as intuitively expected, depreciation has a negative effect on the total value of the firm (the sum of the values of the assets in place and growth options). Thus, overall firm value is decreasing in  $\lambda$ .

The curvature of production function,  $\theta$ , has a positive effect on the values of both assets in place and growth options. More productive capital is reflected in higher values of existing production assets

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<sup>4</sup>The qualitative comparative statics are insensitive to the choice of parameter values.



as well as options to expand capital stock in the future. Yet, as Figure 1 demonstrates, the value of growth options is more sensitive to  $\theta$  than the value of assets in place. In particular, as  $\theta$  approaches its theoretical limit of  $1 - \frac{1}{\beta_1}$  (see equation (6)), the value of growth options becomes infinitely high. In this case, capital becomes so productive that installing additional capital would immediately make adding more capital even more optimal. When estimating the model, we make sure that this constraint on  $\theta$  is always satisfied.

As expected, volatility,  $\sigma$ , has a positive effect on the value of expansion options, due to the fundamental positive relation between the value of an option and the volatility of the underlying process, while an increase in the purchase price of capital  $\eta$  has a negative effect on their value. More expensive capital forces the firm to slow down its investment and reduces the present value of its investment options.

### 3 Estimation

This section explains our estimation and identification approach. Our dataset covers U.S. publicly traded firms over the period 1980-2014. Since the primary goal of our paper is to understand whether investors systematically misprice firms' investment options, we compare firms' theoretical values, which include values of their investment options, to firms' market values. We estimate the model's parameters at monthly frequency at the industry level. To obtain firms' market values and corresponding theoretical values, we use data from annual Compustat files and monthly CRSP files. We use Standard & Poor's Global Industry Classification Standard (GICS) to define industries. GICS is the most common classification used in the financial industry. Furthermore, [Bhojraj, Lee and Oler \(2003\)](#) in a study that compares GICS, NAIC, and SIC industry classifications find that the GICS classifications are significantly better at explaining stock return comovements, as well as cross-sectional variations in valuation multiples, forecasted and realized growth rates, research and development expenditures, and various key financial ratios. We exclude financial firms (GICS Sector 40) and regulated utilities (GICS Sector 55). Regulated utilities may not invest optimally. In other words, the investment process of regulated utilities may be subject to significant frictions that are inconsistent with our model. In the case of financial firms, productive capital differs from that in other sectors of the economy. This leaves us with 8 GICS sectors and 56 industries.

The empirical counterpart of firm  $i$ 's estimated theoretical value  $V(K_{it}, x_{it})$  is the sum of the market value of equity and the book value of debt. Market value of equity is defined as end-of-the-month price per share times the number of shares outstanding,  $PRC \times SHROUT$ . The book value of debt is defined as

the sum of debt in current liabilities (DLC) and long-term debt (DLTT).

The two firm-level variables that determine the differences in theoretical values across firms in the same industry are a firm's capital stock,  $K_{it}$ , and the value of the stochastic component of firm's profits,  $x_{it}$ . Capital stock,  $K_{it}$ , is defined as the gross value of property, plant, and equipment (Compustat item PPEGT). The empirical equivalent of firm's pre-tax operating profits,  $\pi(x_{it}, K_{it}) = x_{it}K_{it}^\theta(1 - \tau)$  is earnings before interest, taxes, depreciation, and amortization (EBITDA), which equals to  $\text{SALE} - \text{COGS} - \text{XSGA}$ . For given  $\theta$ , we back out  $x_{i,t}$  from  $\pi(x_{i,t}, K_{i,t})$  and  $K_{i,t}$ .

In addition to observable firm-level variables discussed above, we need to obtain industry-level parameters. It is conceivable that there are fundamental economic drivers that determine industry growth rates, which, in turn, determine the growth rates of individual firms. If investors are overly optimistic or pessimistic about individual firms, we are more likely to identify such firms by benchmarking them against comparable firms in the same industry. We, therefore, define the drift and volatility of the demand process, which captures underlying economic drivers in our setup, at the industry level. In our model a firm's capital depreciates at the rate of  $\lambda$ . In practice, there is a substantial time-series volatility in firm-level depreciation rates. To smooth out idiosyncratic fluctuation in depreciation, we use an average industry depreciation rate in the last three years as a proxy for the future depreciation rate,  $\lambda$ . Finally, the assumption that the curvature of production function and the cost of installing new capital are constant across all firms in an industry is required for model identification.

Each industry is characterized by 5 industry-level parameters: the drift of the stochastic process,  $\mu$ , its volatility,  $\sigma$ , capital depreciation rate,  $\lambda$ , the curvature of production function,  $\theta$ , and the cost of installing new capital,  $\eta$ . Our approach combines calibration and estimation techniques. We calibrate the parameters of the demand process, i.e.,  $\mu$ ,  $\sigma$ , and capital depreciation rate,  $\lambda$ , and estimate the curvature of the production function,  $\theta$ , and the cost of installing new capital,  $\eta$ , directly from the data. The next two subsections discuss our calibration and estimation approach in detail.

### 3.1 Calibration

To estimate the drift,  $\mu$ , of the profit process, we use equity analysts' forecasts of firms' cash flow growth from the Institutional Brokers Estimate System (IBES). The demand process features time-invariant parameters, and the model is set up with an infinite horizon. Therefore, we would have ideally liked to have access to forecasts of terminal growth rates. Unfortunately, equity analysts do not systematically report terminal growth rates, and instead issue so-called long-term growth (LTG) forecasts. These are typically forecasts of growth rates of firms' cash flows over a five-year horizon. We base our proxy for terminal growth rates on LTG rates, and use them to estimate the drift of the demand process at monthly

frequency. We aggregate forecasts issued by all analysts to all firms within industry. To avoid look-ahead bias, we use all forecasts available within three months prior to the month in which we estimate the model.

Since our model is set under the risk-neutral measure ( $\mathbb{Q}$ ), we convert the estimated growth rates from physical ( $\mathbb{P}$ ) to risk-neutral measure. In this procedure we follow [Morellec, Nikolov and Schurhoff \(2012\)](#), who show that the growth rate under  $\mathbb{Q}$ ,  $\mu_{jt}$ , equals  $g_{jt} - \beta_{jt}\text{MRP}$ , where  $g_{jt}$  is the growth rate under  $\mathbb{P}$  and  $\beta_{jt}\text{MRP}$  is the risk premium. Similar to [Morellec, Nikolov, and Shuerhoff \(2012\)](#), we assume that the market risk premium is time-invariant and equals 6% per year.<sup>5</sup> To estimate equity beta for each firm,  $\beta_{it}$  we run a rolling 36-month regression of the firm’s excess returns on market excess returns. To measure expected returns on debt we sort stocks into five distress quintiles based on the “naive” distance-to-default measure of [Bharath and Shumway \(2008\)](#). We then use the the risk free rate plus the credit spread on AAA bonds, credit spread on BAA bonds, and credit spread on BAA bonds plus 2% for the firms in the least distressed, the next two, and the two most distressed quintiles, respectively. We infer debt beta using the CAPM, compute the firm’s weighted average beta as the average of debt and equity betas, and compute average  $\beta_{jt}\text{MRP}$  across all firms in industry  $j$ . The details of the estimation of the drift of the demand process are outlined in Appendix B.

To estimate the volatility of the demand process we use the volatility of firms’ quarterly sales. We estimate the volatility of quarterly sales for each firm using quarterly sales data in the last eight quarters. For most industries, sales data is highly seasonal. To take out the seasonal variation in sales, we regress quarterly sales on seasonal dummies for each industry and use residuals as our quarterly time series of sales. Since our demand process is defined at the industry level, we use the sales volatility of the median firm in the industry to approximate the volatility of the demand process.

To approximate annual depreciation rates, we use the ratio of depreciation charges (DP) to the value of property, plant, and equipment (PPEGT) for each firm in an industry and average it across across all industry firms. Finally, when measuring after-tax operating profits, we set corporate annual tax rate  $\tau$  at 35% for every firm.

Table 1 summarizes the data we use in this paper. Observations with missing values for the GICS code, the gross capital stock, market value of equity, or negative operating profits are excluded from the final sample. Table 2 reports basic characteristics for each industry. The number of firms varies from as little as 2 in the Marine industry in 1980 to 143 in the Oil, Gas & Consumable Fuels industry in 2014. The table also reports time-series averages of demand growth rates, volatility of demand, and capital

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<sup>5</sup>Our results are robust to assuming time-varying risk premium, computed as the difference between realized return on the value-weighted CRSP index and the rate of return on the riskless asset, where the latter is the average of the yields on the short-term Treasury Bill and 10-year Treasury Note.

depreciation rates.

### 3.2 Estimation technique

We estimate the model using observed firm market values. Equation (5) gives us the theoretical value of the firm. Once the parameters discussed in the previous section have been calibrated, the firm value is the function of the curvature of the production function and capital costs. For each firm and each month we denote the theoretical firm value as  $V_{it}(\theta, \eta)$ .

The difference between the actual firm value,  $\tilde{V}_{it}$ , and the model-implied value should equal zero under the true parameter values  $\theta$  and  $\eta$ , if our valuation model is correct and there is no mispricing in the market. Therefore, for each firm and each month we compute the firm's valuation error as the ratio of the observed firm value to model firm value:

$$\varepsilon_{it} = \tilde{V}_{it}/V_{it}(\theta, \eta).$$

We interpret the deviations of  $\varepsilon_{i,t}$  from one as the market's mispricing of the firm. Values above (below) one imply that the market value of the firm is greater (lower) than the model-implied value, and, therefore, we refer to such firms as overvalued (undervalued). It is important to note that valuation errors may capture either true mispricing or model misspecification. Our setup does not allow us to distinguish between the two empirically. In the next section we investigate whether misvaluation gets corrected over time, and therefore the possibility of valuation errors capturing model misspecification works against us finding results.

We estimate the curvature of the production function,  $\theta$ , and the cost of installing new capital,  $\eta$ , by minimizing aggregate valuation error within industries. For each month and each industry, we pull the industry data for the past 12 months and estimate the model on these data. By doing this, we assume that the production technology is constant within a one-year period.

We define an objective function that is symmetric and does not overweigh either undervalued or overvalued firms. In particular, for each industry  $j$  and each month  $t$  we minimize:

$$(\hat{\theta}_{j,t}, \hat{\eta}_{j,t}) = \arg \min \sum_{i=t}^{t-12} \sum_{k=1}^{N_{j,t}} |\log \varepsilon_{k,i}|, \quad (7)$$

where  $N_{jt}$  is the number of firms in industry  $j$  in month  $t$ . The minimization is done subject to the upper bound constraint on the curvature of the production function according to inequality (6):  $\theta < 1 - 1/\beta_1$ . Note that the constraint depends on the value of  $\beta_1$ , which in turn depends on the value of  $\theta$  according

to equation (4).

This estimation procedure assumes that firms invest optimally at all times. However, in reality it is conceivable that firms may deviate from theoretically optimal investment policy for various reasons. In theory, a firm cannot end up outside of the investment boundary. In practice, however, we can observe a firm outside of the boundary, which means that  $x_{it} > X(K_{it})$ . Economically, if a firm finds itself outside of the investment boundary, it means that it has not been following optimal investment policy in the past. It should then immediately make a lumpy investment to bring itself back to the investment boundary. Thus, if  $x_{it} > X(K_{it})$  then we assume that the firm immediately invests  $K_{it}^* - K_{it}$ , where

$$K_{it}^* = \left( \frac{\beta_1 - 1}{\beta_1} \frac{(1 - \tau)\theta x_{it}}{(r - \mu)\eta} \right)^{\frac{1}{1-\theta}}.$$

In this case, the firm's theoretical value net of additional investment costs becomes

$$V(K_{it}^*, x_{it}) - (K_{it}^* - K_{it})\theta_{jt}. \quad (8)$$

We then use this adjusted measure when computing the firm's theoretical value. In our sample, only about 6% of all firm-months are located outside of the optimal investment boundary and require this adjustment.

At the conclusion of the estimation procedure, we obtain estimates of the curvature of the production function  $\hat{\theta}_{j,t}$  and the cost of capital  $\hat{\eta}_{j,t}$  for each industry  $j$  and each month  $t$ . We define a firm-level misvaluation measure that we use in the subsequent sections as a valuation error at the estimated value of the parameter, i.e.,  $\tilde{V}_{it}/V_{it}(\hat{\theta}_{jt}, \hat{\eta}_{jt})$ , where  $\tilde{V}_{it}$  is the sum of the market value of equity and book value of debt.

### 3.2.1 Separate identification of $\theta$ and $\eta$

In this subsection we discuss separate identification of the curvature of production function,  $\theta$ , and the cost of installing new capital,  $\eta$ . Recall that in equation (5) the firm value consists of two components. The first component is the value of assets in place,  $AP_{it}$ :

$$AP_{it} = \frac{(1 - \tau)x_{it}K_{it}^\theta}{r - \mu + \lambda\theta}. \quad (9)$$

Notice that the value of assets in place depends on  $\theta$  only and is not affected by  $\eta$ . The value of a firm's assets in place is increasing in  $\theta$ , as pictured in Figure 1. The curvature of the production function is primarily identified by this component of the firm value. The second part of value equation (5) captures

the value of a firm's growth options,  $GO_{it}$ :

$$GO_{i,t} = \left[ \frac{(1-\tau)\theta}{\beta_1(r-\mu+\lambda\theta)} \right]^{\beta_1} \left( \frac{\beta_1-1}{\eta} \right)^{\beta_1-1} \frac{x_t^{\beta_1}}{(\beta_1(1-\theta)-1)K_t^{\beta_1(1-\theta)-1}}, \quad (10)$$

which depends on both  $\theta$  and  $\eta$ . The value of  $GO_{it}$  is increasing in  $\theta$  and is decreasing in  $\eta$  as shown in Figure 1. The value of growth options identifies the cost of installing new capital,  $\eta$ .

Additional mechanism that helps to separately identify  $\theta$  from  $\eta$  works through the dynamic constraint in (6):  $\theta < 1 - 1/\beta_1$ . The curvature of production function has to be sufficiently low, otherwise the value of growth options become infinite. In this constraint, the value of  $\beta_1$  depends on  $\theta$ , according to the fundamental equation (4). Notice that this constraint is independent of the cost of installing new capital,  $\eta$ . If the value of  $\theta$  is sufficiently high, this constraint helps identify  $\theta$ . For low values of  $\theta$ , however, this constraint has no additional identifying power.

### 3.3 Estimation results

We report estimation results in Table 3. We aggregate estimates at the industry level (Panel A) and at the sector level (Panel B). The reported values are time-series averages of the curvature of production function and the cost of installing new capital at the industry and sector levels. In addition, the table reports the distribution of GO/AP ratios across firms in each industry and each sector.

Estimated curvature of production function,  $\theta$ , ranges between 0.26 and 0.46. On average, capital has the lowest average productivity in the Electrical Equipment industry (which belongs to the Industrials sector) and Household Durables and Textiles, Apparel & Luxury Goods industries (which belong to the Consumer Discretionary sector). Capital is most productive in the Information Technology sector, in particular in the Office Electronics industry. This result is in line with the vast literature in macroeconomics and finance that directly or indirectly estimates parameters of the production process. For example, [Olley and Pakes \(1996\)](#) estimate the productivity of capital in the Telecommunications Equipment industry at around 0.34 and [Levinsohn and Petrin \(2003\)](#) obtain estimates in the range of 0.2 to 0.29.<sup>6</sup>

Estimated cost of installing new capital,  $\eta$ , ranges between 1.00 and 1.36. In our model,  $\eta$  captures how much a firm has to pay to purchase \$1 worth of capital. In other words,  $\eta$  captures the wedge between the cost of buying capital and its value on a firm's balance sheet. The lower bound that we impose on this parameter in estimation is 1. This effectively assumes that firms in a given industry on

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<sup>6</sup>See [Akerberg, Caves and Frazer \(2015\)](#) for a review and criticism of various approaches to the estimation of the production function.

average do not pay for capital less than its book value. In addition to the wedge between the purchase price and the book value of capital, this parameter captures capital specificity, installation costs, and other overhead expenses associated with installing new capital in a given industry.

Our estimates show that capital is less firm-specific and is cheaper to install in the Information Technology sector. In particular, Software and Communications Equipment industries have the lowest costs of installing capital. On the other hand, firms in the Materials and Industrials sectors have the highest cost of installing new capital. In particular, the two industries with the highest cost of capital are Paper & Forest Products ( $\eta = 1.36$ ) and Transportation Infrastructure ( $\eta=1.34$ ).

To the best of our knowledge, our paper is the first to estimate the cost of installing new capital for a universe of all publicly traded firms in the U.S. [Cooper and Haltiwanger \(2006\)](#) estimate this cost on a panel of 7,000 large manufacturing firms in the U.S. that operated from 1972 until 1988. Despite differences in samples used in the two papers, our results are remarkably close to the estimates in [Cooper and Haltiwanger \(2006\)](#) – 1.32 in the Steel industry and 1.25 in the Transportation industry.<sup>7</sup>

The last six columns in Panels A and B of Table 3 show the distribution of estimated GO/AP ratios across industries across sectors. There are large differences in the value of growth options relative to the value of existing assets. On average, firms in Health Care, Information Technology, and Energy sectors tend to have large GO/AP ratios. For example, the average firm in the Information Technology sector has about one quarter of its value derived from investment options, and three quarters derived from existing assets. On the other hand, firms in Materials, Industrials, and Consumer Staples sectors have the lowest fractions of growth options in their values.

In addition to significant differences in GO/AP ratios across sectors, the variation in GO/AP across firms within industries and sectors is also very significant. For example, in the Software industry (Panel A, GICS 451030) the firm at the 25th percentile has only 6% of its value in investment options, while for the firm at the 75th percentile this number is 43%.

## 4 Misvaluation and expected stock returns

In this section we empirically test the hypothesis that firms that are overvalued (undervalued) relative to their model values earn lower (higher) subsequent risk-adjusted returns. This hypothesis relies on two assumptions. First, we assume that firm-level misvaluation produced by our model is non-random and is positively correlated with unobserved true misvaluation. Second, we assume that this potential

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<sup>7</sup>Note that in [Cooper and Haltiwanger \(2006\)](#) firms can sell capital, and therefore their model features both the buying and selling price of capital. When estimating the model, [Cooper and Haltiwanger \(2006\)](#) fix the buying price of capital at \$1 and estimate the selling price of capital.

misvaluation is gradually corrected by the market as new, tangible information, on firm’s performance arrives. We also hypothesize that the relation between misvaluation and future returns should be stronger for firms whose values are attributable to a larger degree to investment options that they possess. To summarize, our two hypotheses are:

**Hypothesis 1** *We expect to find a negative relation between firm misvaluation (relative to the model) and future risk-adjusted equity returns;*

**Hypothesis 2** *We expect to find that the negative relation between misvaluation and future risk-adjusted equity returns is stronger for firms with larger proportions of value represented by growth options.*

In our empirical tests, we use the same data as those described in the Model Estimation section – our data span years 1980-2014 and include all stocks available on CRSP monthly files that have relevant accounting information on Compustat.

#### 4.1 Portfolio sorts based on misvaluation measure

To test our first hypothesis, we first sort stocks each month  $t - 1$  by our misvaluation measure, estimated according to (7) and assign stocks into misvaluation deciles. Then, for each of the ten portfolios, we estimate the regression of value-weighted mean excess return in subsequent month ( $t$ ),  $R_{p,t}$ , on monthly returns of factors, suggested by various asset pricing models:

$$R_{pt} = \alpha_p + \beta_p \mathbf{R}_{Ft} + \varepsilon_{pt}, \quad (11)$$

where  $\alpha_p$  is the average abnormal return of portfolio  $p$ ,  $\mathbf{R}_{Ft}$  is a vector of factor returns,<sup>8</sup> and  $\beta_p$  is a vector of factor loadings. To save space, we do not report the results of estimating (11) using equally-weighted portfolio returns. These results are generally consistent with those using value-weighted returns.

We use eight benchmarks to estimate abnormal returns:

- 1) the “naive” benchmark in which the set of factors  $\mathbf{R}_{Ft}$  is empty. In this specification,  $\alpha_p$  is the mean portfolio return;
- 2) Capital Asset Pricing Model, in which  $\mathbf{R}_{Ft}$  includes MKT, defined as the difference between value-weighted market return and the risk-free rate;
- 3) Fama and French (1993) model, which includes HML and SMB factors in addition to MKT;

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<sup>8</sup>The majority of factor returns are obtained from Ken French’s data library [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Monthly returns on the  $q$ -factors were provided to us by Lu Zhang.



- 4) Carhart (1997) model, which includes, in addition to Fama and French (1993) three factors, a momentum factor, MOM;
- 5) Fama and French (2015) model, in which the set  $\mathbf{R}_{F,t}$  includes, in addition to MKT, HML, and SMB the following two factors: RMW (“robust minus weak”) – the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios, and CMA (“conservative minus aggressive”) – the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios;
- 6) Fama and French (2015) model augmented by the momentum factor, MOM;
- 7) Hou, Xue and Zhang (2015) model, which includes MKT, and three “ $q$ -factors”,  $r_{ME}$ ,  $r_{I/A}$ , and  $r_{ROE}$  from a triple 2-by-3-by-3 sort by size, investment-to-assets ratio, and return on equity (ROE);
- 8) Hou, Xue and Zhang (2015) model augmented by the MOM factor.

We report risk-adjusted returns from estimating the regression (11) in Table 4, which includes 8 specifications discussed above. In all 8 models, the standard errors are Newey-West adjusted with 6 lags. The mean value-weighted return of the portfolio of most undervalued stocks, 1.05% per month, is significantly larger than the mean return of the most overvalued portfolio, 0.15% per month. The annualized difference of 11.4% is significant at the 10 percent level. In general, mean portfolio returns tend to be monotonically decreasing in our misvaluation measure.

Controlling for the exposure to risk factors tends to strengthen the relation between our misvaluation measure and risk-adjusted returns. The differences in alphas from the CAPM, Fama and French (1993), and Carhart (1997) models between the most undervalued and most overvalued decile portfolios are highly statistically significant, with t-statistics of 3.70, 2.79, and 3.40, respectively. They are also highly economically significant: annualized differences between the alpha of the most undervalued and that of the most overvalued portfolio are between 9.9% and 13.6%. Annualized difference between the alphas of the two extreme portfolios is 10.3% in the case of the Fama and French (2015) model with a t-statistic of 2.89. Augmenting the Fama and French (2015) model by the momentum factor increases the gap between the alphas of two extreme misvaluation deciles to a highly significant annualized 12.4%. This is consistent with undervalued firms being past losers, while overvalued firms being past winners, on average. Even stronger results are obtained when we use the Hou, Xue and Zhang (2015) model, with or without the momentum factor, as a benchmark – the annualized difference between the two extreme misvaluation portfolios’ alphas exceeds 16% with a t-statistic exceeding 4.

Table 5 reports factor loadings for the most undervalued and overvalued deciles, and for the undervalued-minus-overvalued (UMO) strategy. There are a number of interesting observations from Table 5. First, the loadings of the UMO strategy on the MKT factor are negative and highly significant.

In the CAPM model, the most undervalued decile loads 1.07 on MKT while the most overvalued decile has a loading of 1.33. Therefore, the UMO strategy is short in stocks with higher exposure to the market risk, which explains improved performance of our UMO strategy after we control for MKT exposure.

Second, UMO is negatively exposed to SMB. The loading of UMO on SMB is -0.27 in the [Fama and French \(1993\)](#) model, with a t-statistic of -2.81. This implies that stocks in the undervalued portfolio tend to be larger than those in the overvalued portfolio. Third, UMO is a value strategy. The loadings on the HML factor range from 0.37 to 0.57, and are highly significant. Undervalued stocks have larger exposure to the HML factor relative to overvalued stocks. In other words, undervalued stocks tend to be value stocks while overvalued stocks tend to be growth stocks.

Fourth, the UMO strategy is counter-momentum. UMO loadings on MOM vary from -0.19 to -0.25 and are highly significant. Stocks in the undervalued portfolio tend to be past losers and stocks in the overvalued portfolio tend to be past winners. This explains better performance of our strategy once we control for the exposure to the momentum factor. In particular adding MOM to [Fama and French \(1993\)](#) model increases alpha from 0.79 to 0.97, while decreasing volatility of the strategy: the t-statistic increases from 2.79 to 3.40. Similarly, when we add MOM to [Fama and French \(2015\)](#) model, alpha goes from 0.82 to 0.98 and the t-statistic from 2.89 to 3.45. Fifth, the UMO strategy is not significantly exposed to the profitability factor RMW of [Novy-Marx \(2013\)](#). Undervalued stocks tend to have slightly higher gross profitability compared to overvalued stocks, but not significantly so.

Table 6 presents results of robustness tests of our first hypothesis, in which we alter the definition of extreme misvaluation groups, the method of computing portfolio returns as well as the horizon of returns following assignment to misvaluation groups, impose restrictions on the sample composition, and change the way in which we estimate the model's parameters and resulting misvaluation. Table 6 has 8 columns, similar to Table 11. To conserve space, in each robustness test, we only report risk-adjusted returns of the two extreme misvaluation portfolios and the differences between them.

In Panel A, instead of sorting stocks into deciles based on our misvaluation measure, we sort them into quintiles. The results are consistent with the baseline estimation. For example, when we use [Fama and French \(2015\)](#) model, the annualized risk-adjusted return on undervalued-minus-overvalued (UMO) strategy becomes lower, 9.7%, but the t-statistic increases to 3.56. This result alleviates a concern that our strategy is driven by a small number of stocks in undervalued and overvalued deciles, and suggests that our strategy is potentially tradable.

In Panel B, we sort stocks into two portfolios: the undervalued portfolio contains firms with misvaluation measure lower than one, while the overvalued portfolio contains firms with misvaluation

measure greater than one. The results become weaker compared to Table 4 and to Panel A of this table, but not significantly so. This is expected because instead of sorting firms into 10 or 5 portfolios, we sort them into 2 portfolios only. Despite this, all of the differences between risk-adjusted returns of undervalued and overvalued portfolios are still economically large and statistically significant. The lowest annualized alpha is from the Fama and French (2015) model and equals 6.4%, with a t-statistic of 1.91.

Panel C reports results for equally-weighted portfolio returns. The results are much stronger than the baseline results using value-weighted returns. The monthly mean return in the undervalued portfolio is 1.45% while in the overvalued portfolio it is 0.19%, an annualized difference of 15%, with a t-statistic of 2.66. Annualized risk-adjusted returns of the UMO strategy from the eight asset pricing models vary from 11.8% (Fama and French (2015) model) to 16.4% (Fama and French (1993) model augmented by the momentum factor). The lowest t-statistic is 4.42.

In Panel D, we structurally estimate only one parameter of the model – the curvature of production function,  $\theta$ , while fixing the cost of installing new capital,  $\eta$  to one, instead of two parameters –  $\theta$  and  $\eta$ . The reason is the potential concern that  $\theta$  and  $\eta$  are not separately identified. The difference between risk-adjusted returns of undervalued and overvalued portfolios declines, but only marginally. For example, annualized alpha of the UMO strategy within Fama and French (2015) model decreases from 9.8% to 9.5% with the associated decrease in the t-statistic from 2.89 to 2.81.

In Panel E, we estimate the model parameters on a subset of all firms that has relatively high analyst coverage. In particular, for each industry-month, we estimate the model using firms with above-industry-median analyst coverage. There are two motivations for performing this test. First, firms that are followed by more analysts are more likely to be correctly priced. Second, since one of our key model inputs – the drift of industry profit – is estimated using analyst projections, it is likely to be estimated more precisely within firms with relatively strong analyst coverage. Risk-adjusted returns of the UMO strategy decrease for all asset pricing models, but they remain economically large and statistically significant.

In Panels F and G we replace the one-month return on the left-hand side of (11) by 3-month and 12-month returns following the month of assignment into misvaluation deciles, respectively. The differences in 3-month risk-adjusted returns between the extreme misvaluation deciles are statistically significant in 7 out of 8 models. Annualized alphas of the UMO strategy range from 4.0% to 8.6%. Notably, the Fama and French (2015) alpha for the UMO strategy is not statistically significant with a t-statistic of 1.72. The annualized risk-adjusted returns of the UMO strategy decline even further in the case of 12-month returns. The the Fama and French (2015) alpha for the UMO strategy is 1.28. Annual Fama and French (2015) alpha is 5% down from the annualized alpha of 10.3% in the case of monthly returns. These results

imply that the performance of our strategy is strongest at the one-month horizon and starts deteriorating at the 3-month and 12-month horizons. We interpret this as evidence that investors gradually learn about firm mispricing and correct it over time.

Finally, Panel H shows results in the subsamples split by NBER expansions and recessions. Our results are economically strong in both recessions and expansions, with statistical significance being stronger in expansion periods. It is worth noting that our estimates are noisier in the subsample of recessions due to limited number of observations.

## 4.2 Cross-sectional firm-level tests

In this section we estimate monthly cross-sectional Fama and MacBeth (1973) regressions of individual firm excess returns  $R_{i,t}$  on a measure of misvaluation, and a vector of firm characteristics known at the beginning of month  $t$ :

$$R_{it} = \alpha_t + \beta_t MISVAL_{it} + \delta_t \mathbf{X}_{it} + \varepsilon_{it}, \quad (12)$$

where  $MISVAL_{it}$  is firm  $i$ 's measure of misvaluation in month  $t - 1$ . Because of skewness in our misvaluation measure, we use the natural logarithm of firm-level misvaluation, estimated as in (7).  $\mathbf{X}_{it}$  is a vector of firm characteristics that were identified in past literature to be related to future returns. Following common practice in the asset pricing literature (e.g., Fama and French (1993), Jegadeesh and Titman (1993), and Cooper, Gulen and Schill (2008) among many others), these characteristics are log market equity, log book-to-market, and past returns. Following Fama and French (1993), we measure the market value of equity as the share price at the end of June times the number of shares outstanding. Book equity is stockholders' equity, Compustat item TEQ, minus preferred stock plus balance sheet deferred taxes and investment tax credit, TXDITC, if available. If stockholders' equity is missing, we use common equity plus preferred stock par value, CEQ+PSTK. If these variables are missing, we use book assets less liabilities.  $AT-(DLTT+DLC)$ . Preferred stock is preferred stock liquidating value, PSTKRVL, preferred stock redemption value, PSTKL, or preferred stock par, PSTK, value in that order of availability. We match returns from January to June of year  $t$  with Compustat-based variables of year  $t-2$ , while the returns from July until December are matched with Compustat variables of year  $t-1$ . Past returns are defined as buy-and-hold returns for six months over months  $[t-7, t-2]$ . We then average the coefficients of cross-sectional regressions over time and compute the standard errors of the coefficients, while adjusting for potential autocorrelation in coefficient estimates.

We report the results in Table 7, which has 5 columns. The first one presents the baseline specification. In column 2, we compute misvaluation relative to model estimation in which we estimate

the curvature of production function,  $\theta$  only, while assuming that the cost of installing new capital,  $\eta$  equals one. In column 3, we estimate the model only for firms that have above-industry-median analyst coverage and compute misvaluation of all firms relative to model values corresponding to that estimation. In columns 4 and 5, we examine 3-month and 12-month returns respectively following the month in which misvaluation is estimated.

The main finding in Table 7 is that the coefficient estimates on the logarithm of our misvaluation measure is negative and highly statistically significant in all specifications. Moreover, the effect of misvaluation on returns is also economically large. The standard deviation of log misvaluation is 0.85. Multiplying it by the coefficient estimates on log misvaluation measure implies that a one standard deviation increase in misvaluation is associated with a 0.25%–0.30% reduction in monthly returns and around to 0.8% (2.5%) reduction in 3-month (12-month returns). These results of the cross-sectional tests, which demonstrate that firms that are undervalued relative to our model’s theoretical values outperform overvalued firms, are fully consistent with the results of time-series portfolio tests.

Overall, the results in Tables 11, 6, and 7 strongly support our first hypothesis – firms that our model considers undervalued significantly outperform overvalued firms following the assignment of firms to portfolios.

### **4.3 Misvaluation and growth options**

Our second hypothesis states that the differences between risk-adjusted returns of most undervalued stocks and those of most overvalued ones should be larger within subsamples of firms with abundant investment options than within subsamples of assets-in-place-based firms. Our hypothesized reason is that misvaluation is possibly driven by investors’ inability to correctly value firms’ growth options. To test this conjecture, we perform three sets of tests. First, we examine the performance of our undervalued-minus-overvalued strategy in the subsamples of stocks sorted based on our estimated measure of importance of firms’ growth options. Second, we document the performance of our strategy in the subsamples sorted based on industry market-to-book ratios. Lastly, we remove growth options from the valuation equation and estimate the model assuming that each firm consists of only existing assets and does not have expansion options and examine the performance of the undervalued-minus-overvalued strategy for an assets-in-place-based valuation model.

#### **4.3.1 GO/AP sorts**

If the performance of the investment strategy documented in Table 4 is driven by investors’ inability to correctly value firms’ investment options, we should expect stronger performance of our investment

strategy among more growth-oriented firms. To examine whether this is the case, for each firm we divide the theoretical value of growth options (GO) estimated from the model (first part of 5) by the theoretical value of assets-in-place (AP) (second part of 5). The resulting GO/AP ratio indicates the importance of growth options relative to existing assets in the overall value of the firm. We calculate median GO/AP ratio in each industry and sort firms into terciles based on the median GO/AP value in the industry to which each firm belongs. After assigning firms into GO/AP terciles, we sort them into misvaluation deciles, creating 3-by-10 portfolios. We then compute the difference between risk-adjusted returns of the two extreme misvaluation portfolios within each GO/AP tercile.

The results of estimating the regressions using double-sorted portfolios are presented in Table 9. The table has 3 panels, corresponding to 3 GO/AP terciles. Table 9 shows that the performance of the UMO strategy is driven by the tercile of highest GO/AP firms. In the lowest tercile, the mean annualized value-weighted return of the portfolio of most undervalued stocks is 12.4% and the mean annualized return of the most overvalued portfolio is 8.6%. The difference of 3.8% is not statistically significant. On the other hand, in the highest GO/AP tercile, the mean annualized value-weighted return of the portfolio of most undervalued stocks is 13.3% per month and the mean return of the most overvalued portfolio is 2.5% per month. The monthly difference of 10.8% is statistically significant at the 10 percent level.

Adjusting the returns to the exposure to risk factors strengthens the conclusion that the relation between misvaluation and returns is present only within growth-options-intensive terciles. Annualized differences between the alpha of the most undervalued and that of the most overvalued portfolio are between 9.4% and 16.9% in the high-GO tercile, all highly statistically significant with t-statistics ranging from 2.29 to 3.82. None of the differences in risk-adjusted returns between undervalued and overvalued portfolios are significant in the lowest GO/AP tercile.

#### **4.3.2 M/B sorts**

Since the GO/AP ratio used to sort firms into terciles of growth options in the previous section comes from a structural model, it may pick up some unobservable model misspecification that could correlate with GO/AP ratios but not with the true value of firms' expansion options. To alleviate this concern, we provide additional model-free evidence using an alternative, well accepted measure of the proportion of firm value represented by growth options – market-to-book (M/B) ratio (e.g., Smith and Watts (1992), Barclay, Smith and Watts (1997) among others). However, in addition to proxying for growth options, M/B is directly related to misvaluation (e.g., Rhodes-Kropf, Robinson, and Viswanathan (2005)). Thus, instead of firm-level M/B, we use industry-level M/B, which is orthogonal to firm-level M/B by construction, and sort firms into terciles based on the median market-to-book (M/B) ratio within a firm's



industry. We define firm's M/B as the ratio of the sum of market value of its equity,  $PRCC \cdot SHROUT$ , and book value of its debt,  $DLTT + DLC$ , to book value of its assets,  $AT$ . Within each industry M/B tercile, we sort firms into misvaluation deciles. We then estimate the difference between returns and alphas of the two extreme misvaluation portfolios within the terciles of industry M/B measure.

The results of double-sorting firms by industry M/B and estimated misvaluation are shown in Table 10. Going from the lowest to the highest M/B tercile, mean annualized excess return on the UMO portfolio increases from 6.7% to 11.9%. The t-statistic in the lowest industry M/B tercile is 0.91 while in the highest industry M/B tercile it is 1.93. Annualized risk-adjusted returns to the UMO strategy in the low industry M/B tercile range between 3.6% and 10.7% and are statistically insignificant in all specifications. In the highest industry M/B tercile, on the other hand, UMO strategy alphas range from 10% to 15.5% per month, and are significant at the 1% level in all specifications.

Overall, the evidence in Tables 8, 9, and 10 strongly supports our second hypothesis: The relation between misvaluation and future returns is stronger among firms with more growth options, potentially because of investors' difficulties in adequately assessing the value of investment options and gradual realization of this value by the market over time.

#### 4.3.3 Shutting down growth options: Counterfactual analysis

We shut down growth options in the valuation equation and estimate the model while assuming that each firm's value is derived only from its existing assets. If we disallow expansion (by assuming that the cost of adding new capital,  $\eta$ , is infinite), firm value becomes the present value of a perpetuity of after-tax EBIT:

$$V(K_t, x_t) = \frac{(1 - \tau)x_t K_t^\theta}{r - \mu + \lambda \theta} \quad (13)$$

Firm value in (13) is the function of the curvature of production function,  $\theta$ , only. Similar to the base case analysis, we define valuation errors as

$$\varepsilon_{it} = \tilde{V}_{it} / V_{it}(\theta),$$

and re-estimate the model.

The results are presented in Table 8. The structure of Table 8 is identical to that of Table 11. The mean value-weighted return of the portfolio of most undervalued stocks, 1.15% per month, is larger than the mean return of most overvalued portfolio, 0.63% per month, however the annualized difference of 6.4% is not statistically significant with a t-statistic of 0.67. The results remain insignificant after we control for the portfolio's exposure to risk factors. Importantly, the relation between risk-adjusted

returns and misvaluation is not monotonic at all in all 8 specifications, when misvaluation is computed relative to the model without growth options. Overall, the model that does not account for the value of growth options fails to produce significant differences in risk-adjusted returns between most undervalued and most overvalued firms. The results in this subsection present indirect evidence that misvaluation of firms' growth options is an important part of firms' misvaluation by investors.

## 5 Conclusions

Traditional valuation techniques (such as discounted cash flow or valuation by multiples) are not well suited for valuing real options in general and investment options in particular. In this paper we hypothesize that market participants' inability to correctly estimate values of firms with embedded investment options may lead to misvaluation of such firms, which is subsequently corrected as growth options are transformed into existing assets, which are easier to evaluate. To test our hypothesis, we first construct a real options model of firm's optimal investment, which we estimate at the industry level by matching firms' model-implied values to market values and searching for model parameters that provide the closest match between the model and market values within the industry on average. We then compute a firm-level measure of misvaluation for each firm-month and estimate empirically the relation between misvaluation and subsequent risk-adjusted returns.

Our empirical results are strongly consistent with our hypothesis. We find, using both time series portfolio regressions and cross-sectional regressions that our misvaluation measure is negatively related to future returns – a relation that is highly significant, both economically and statistically. This relation is robust to various adjustments for risk, sample splits, holding periods, and modifications in the estimation procedure. In addition, consistent with the hypothesis that misvaluation is driven by the difficulties market participants face when valuing investment options, the relation between misvaluation and future returns is much stronger among firms for which investment options constitute a relatively large proportion of value. Furthermore, this effect is strongly attenuated when the growth option component is “shut down” in the model. Overall, our findings suggest that investors tend to fail to appropriately incorporate the values of real options when evaluating firms, resulting in persistent misvaluation in equity markets.



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# Appendix

This Appendix contains 2 sections. Section A discusses issues related to the model and proofs. Section B discusses issues related to estimating the parameters of the demand process.

## A Model

### A.1 Production technology

Consider a firm with Cobb-Douglas production function

$$q = \kappa L^\phi K^{1-\phi},$$

where  $L$  is labor,  $\kappa$  is a constant, and  $0 < \phi < 1$  is the share of labor in the production process. The cost of labor is constant and given by  $w$ , the output price is given by the inverse demand function

$$p = y^{1-\gamma} q^\gamma,$$

where  $y$  is an uncertainty parameter and  $0 < \gamma < 1$ . The firm maximizes its instantaneous profit by optimally choosing the amount of labor:

$$L^* = \arg \max_L [y^{1-\gamma} (\kappa L^\phi K^{1-\phi})^\gamma - wL].$$

Solving this yields

$$L = \left[ \frac{w K^{(\phi-1)\gamma} y^{\gamma-1}}{\kappa^\gamma \phi \gamma} \right]^{\frac{1}{\phi\gamma-1}},$$

and

$$\pi = C y^{\frac{\gamma-1}{\phi\gamma-1}} K^{\frac{\gamma(\phi-1)}{\phi\gamma-1}},$$

where  $C > 0$  is a constant:

$$C = \kappa^{\frac{\gamma}{1-\phi\gamma}} w^{\frac{\phi\gamma}{\phi\gamma-1}} \left[ (\phi\gamma)^{\frac{\phi\gamma}{1-\phi\gamma}} - (\phi\gamma)^{\frac{1}{1-\phi\gamma}} \right].$$

This profit function is equivalent to the one in our model with  $x = C y^{\frac{\gamma-1}{\phi\gamma-1}}$  and  $\theta = \frac{\gamma(\phi-1)}{\phi\gamma-1}$ .

## A.2 Proofs

In this appendix we prove that the value of the firm is indeed given by (4). The value of assets in place equals the present value of all cash flows generated by the firm's (depreciating) capital:

$$AP(K, x) = (1 - \tau)K^\theta E \int_0^\infty x_t e^{-\lambda \theta t} e^{-rt} dt = \frac{(1 - \tau)xK^\theta}{r - \mu + \lambda \theta}.$$

Using standard arguments, one can show that the value of growth options  $F(K, x)$  follows the following PDE:

$$\frac{1}{2}\sigma^2 x^2 F_{xx} + \mu x F_x - \lambda K F_K - rF = 0.$$

The value of a marginal unit of capital to be installed is  $f(K, x)dK$ , where  $f(K, x) = F_K(K, x)$ .  $f(K, x)$  follows the following PDE:

$$\frac{1}{2}\sigma^2 x^2 f_{xx} + \mu x f_x - \lambda K f_K - (r + \lambda)f = 0.$$

The expected increase in the PV of one additional unit of capital is

$$v(K, x) = \frac{(1 - \tau)x\theta}{(r - \mu + \lambda \theta)K^{1-\theta}}.$$

Now, denote  $y = \frac{x}{K^{1-\theta}}$ . Assume that  $f(x, K)$  is a function of  $y$ :  $f(x, K) = g(y)$ . Substituting  $f_x(x, K) = K^{\theta-1}g'(y)$  yields:

$$\frac{1}{2}\sigma^2 y^2 g''(y) + \mu y g'(y) - \lambda(\theta - 1)y g'(y) - (r + \lambda)g(y) = 0,$$

or, equivalently

$$\frac{1}{2}\sigma^2 y^2 g''(y) + [\mu - \lambda(\theta - 1)]y g'(y) - (r + \lambda)g(y) = 0.$$

This ODE has the following solution:

$$g(y) = By^{\beta_1}$$

where  $\beta_1$  is the positive root of the equation

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + [\mu - \lambda(\theta - 1)]\beta - (r + \lambda) = 0.$$

The optimal investment threshold together with the constant  $B$  are determined by the following boundary conditions:

1) value matching condition:

$$B \left( \frac{X}{K^{1-\theta}} \right)^{\beta_1} = \frac{BX^{\beta_1}}{K^{\beta_1(1-\theta)}} = \frac{(1-\tau)X\theta}{(r-\mu+\lambda\theta)K^{1-\theta}} - \eta,$$

2) smooth pasting condition:

$$B\beta_1 X(K)^{\beta_1-1} K^{\beta_1(\theta-1)} = \frac{(1-\tau)X\theta}{(r-\mu+\lambda\theta)K^{1-\theta}}.$$

It follows that the optimal investment threshold

$$X(K) = \frac{\beta_1}{\beta_1-1} \frac{(r-\mu+\lambda\theta)\eta K^{1-\theta}}{(1-\tau)\theta},$$

and the constant  $B$  is

$$B = \frac{1}{\beta_1} \left[ \frac{(1-\tau)\theta}{r-\mu+\lambda\theta} \right]^{\beta_1} \eta^{1-\beta_1} \left( \frac{\beta_1}{\beta_1-1} \right)^{1-\beta_1} = \left[ \frac{(1-\tau)\theta}{\beta_1(r-\mu+\lambda\theta)} \right]^{\beta_1} \left( \frac{\beta_1-1}{\eta} \right)^{\beta_1-1}.$$

The value of growth options is

$$F(K,x) = Bx^{\beta_1} \int_K^{\infty} K^{(\theta-1)\beta_1} dK = \frac{Bx^{\beta_1}}{(\theta-1)\beta_1+1} K^{(\theta-1)\beta_1+1}.$$

Note that for the integral above to converge the following condition must hold:

$$\theta < 1 - \frac{1}{\beta_1}.$$

If this inequality is not satisfied, then the value of growth options is infinite. In that case, capital is so productive that installation of new capital makes it optimal to install even more capital. The total value of the firm is, then, given by

$$V(K,x) = \frac{(1-\tau)xK^\theta}{r-\mu+\lambda\theta} + \left[ \frac{(1-\tau)\theta}{\beta_1(r-\mu+\lambda\theta)} \right]^{\beta_1} \left( \frac{\beta_1-1}{\eta} \right)^{\beta_1-1} \frac{x^{\beta_1}}{(\beta_1(\theta-1)+1)K^{1-\beta_1(1-\theta)}}.$$

## B Estimation of the long-term growth rate of industry demand

We estimate the drift of the profit process from the equity analysts' forecasts of the long-term growth (LTG) rates. We use Institutional Brokers Estimate System (IBES) to obtain the LTG rates, which are typically coded by "0" in IBES.

We aggregate LTG at the industry level each month. In particular, for a given month for which we estimate the model, we take all LTG forecasts issued in the last 90 days by every analyst. Out of these forecasts, we take the latest forecast issued by each analyst for a given firm. This approach avoids biasing our results in favor of analysts who issue frequent updates to their forecasts during the quarter. This gives us a cross-section of unique analyst forecasts for each firm in the industry. We then average these forecasts in a two-step procedure. First, we take the median across all analysts for a given firm. Second, we take the median across all firms, which is our estimate of the industry long-term growth rate.

It is well known that the LTG forecasts are overly optimistic. We follow [Chan, Karceski and Lakonishok \(2003\)](#) and [Morellec, Nikolov and Schurhoff \(2012\)](#) approach and adjust these forecasts downwards using the following linear function: Adjusted LTG = 0.007264043 + 0.408605737 × median LTG, where the median LTG is our estimate of the industry growth rate forecast we discussed above. If, for example, a given industry has long-term estimate of the growth rate of 10%, the adjusted growth rate becomes 4.15%. The result in the original [Chan, Karceski and Lakonishok \(2003\)](#) is obtained by regressing LTG forecasts on the realized growth rates.

A typical LTG forecast issued by an analyst is a forecast of the growth rate over the next five years. In our model, however, firms are infinitely lived and demand process drifts with rate  $\mu$  into infinity. Therefore, we need to back out the terminal growth rate out of the five-year LTG forecasts. We employ the following procedure. We assume that a firm's cash flow will grow at the "Adjusted LTG" rate as specified above over the first five years, after which point the cash flows will grow at 2%.<sup>9</sup> From these two rates we back out single perpetual growth rate that corresponds to the same firm value as the combination of the two growth rates above, as we explain below.

Consider a firm that has a cash flow of \$1 today. This cash flow will grow at rate LTG over the next five years and from year 6 onwards it will grow at 2% in perpetuity. Assume that this firm's cost of capital is  $r_A$ . The value of this firm today is

$$\frac{1}{r_A - \text{LTG}} \left( 1 + \frac{(1 + \text{LTG})^5}{(1 + r_A)^5} \right) + \frac{1}{(1 + r_A)^5} \frac{1}{r_A - 0.02}.$$

Our goal is to find an equivalent perpetual growth rate of cash flows  $x$  that starts now and gives the same firm value. In other words,  $x$  is given as an implicit solution to the following equation:

$$\frac{1}{r_A - x} = \frac{1}{r_A - \text{LTG}} \left( 1 + \frac{(1 + \text{LTG})^5}{(1 + r_A)^5} \right) + \frac{1}{(1 + r_A)^5} \frac{1}{r_A - 0.02}$$

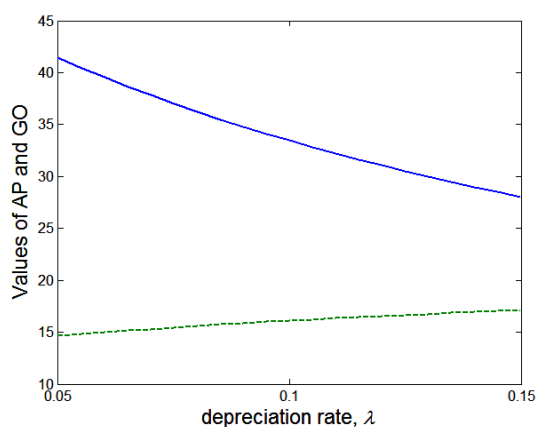
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<sup>9</sup>Although our choice of 2% terminal growth rate is ad-hoc, all our results hold if we assume terminal growth rate anywhere from 0.75% to 4.5%.

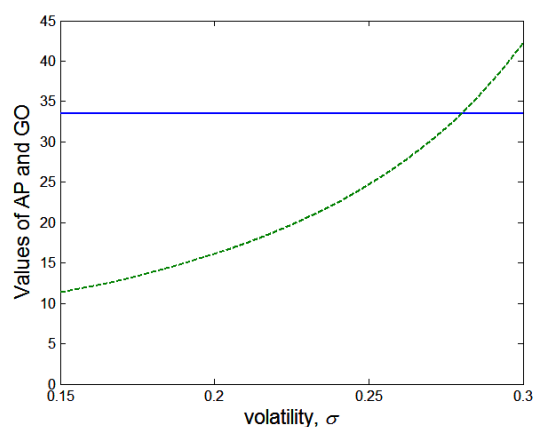
Note that the rate  $x$  is a function of discount rate  $r_A$ . To obtain the discount rate  $r_A$ , we estimate each firm's cost of capital every month. Since we do this adjustment at the industry level, we need to obtain industry-level cost of capital. We do this by taking the cost of capital of the median firm in the industry. Table 2 shows time-series averages of demand growth rates obtained by this procedure. Our final step is to risk-adjust the above growth rate. We discuss our risk-adjusting procedure in Section 3.1.

Figure 1: Values of growth options and assets in place.

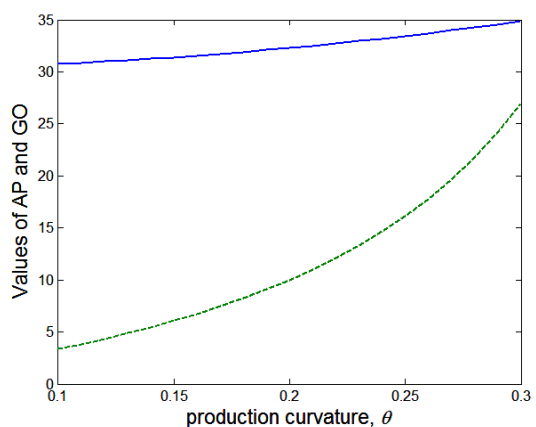
Figure 1 presents the values of assets in place and growth options as functions of model parameters  $\lambda$ ,  $\sigma$ ,  $\theta$ ,  $\eta$ .  $\lambda$  is the depreciation rate.  $\sigma$  is the volatility of the stochastic shock.  $\theta$  is the curvature of production function.  $\eta$  is the purchase price of capital (adjustment cost). Solid lines represent the values of assets in place. Dashed lines represent the values of growth options. The remaining parameter are set as follows:  $\lambda = 0.1$ ,  $\sigma = 0.2$ ,  $\theta = 0.25$ ,  $\eta = 1$ ,  $\lambda = 0.1$ ,  $r = 0.05$ ,  $\mu = 0.01$ .



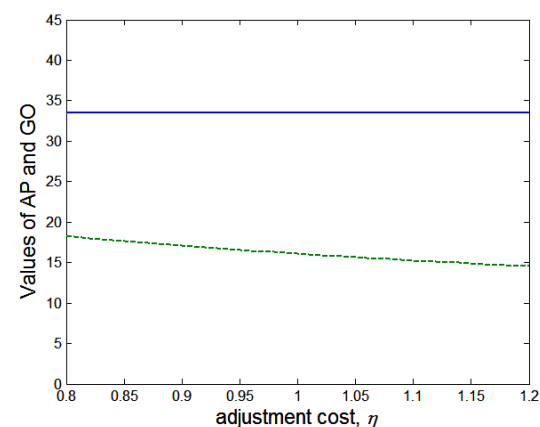
(a) Values of GO and AP as functions of  $\lambda$



(b) Values of GO and AP as functions of  $\sigma$



(c) Values of GO and AP as functions of  $\theta$



(d) Values of GO and AP as functions of  $\eta$



Table 1: DESCRIPTIVE STATISTICS FOR THE FINAL SAMPLE

This table provides summary statistics for the main variables used in estimation. The sample is based on 2014 Industrial Compustat Annual and Quarterly Fundamental Files, Center for Research in Security Prices (CRSP), and Institutional Brokers Estimate System (IBES). The sample covers the period from January 1980 to December 2014. Variables are either at the monthly or annual frequency. The last column in the table indicates the frequency with which a given variable is observed. Market value of equity, capital, revenue, COGS, SG&A, EBITDA, long-term and short-term debt are in million of dollars. Variables are defined in Section 3.

	Mean	S.D.	10%	25%	50%	75%	90%	Obs	Frequency
<b>Firm Characteristics</b>									
Market value of equity	1,890.88	11,272.75	8.49	27.13	127.04	649.82	2,650.45	1,506,719	Monthly
Capital	1,002.26	6,533.98	3.99	13.52	61.67	323.14	1,466.51	129,876	Annual
Revenue	1,602.59	8,610.16	15.25	45.41	169.01	706.34	2,620.35	129,876	Annual
Cost of Goods Sold	1,079.73	6,388.69	7.65	25.09	100.89	444.00	1,727.48	129,876	Annual
SG&A	280.79	1,457.47	2.96	8.56	31.15	127.03	463.01	129,876	Annual
EBITDA	242.07	1,409.62	1.22	4.32	18.69	91.10	360.87	129,876	Annual
Long-term debt	386.10	3,386.84	0.00	0.96	13.62	121.10	596.45	129,645	Annual
Short-term debt	127.97	2,315.13	0.00	0.20	2.23	13.04	69.60	129,742	Annual
Equity beta	1.13	0.75	0.25	0.66	1.08	1.56	2.11	1,323,780	Monthly
Firm beta	0.94	0.61	0.30	0.56	0.86	1.25	1.74	1,315,328	Monthly
<b>Industry Characteristics</b>									
Demand growth rate	0.03	0.01	0.02	0.03	0.03	0.04	0.04	25,260	Monthly
Demand volatility	0.19	0.27	0.08	0.11	0.15	0.20	0.28	27,447	Monthly
Capital depreciation rate	0.09	0.04	0.05	0.07	0.08	0.11	0.14	27,650	Monthly

Table 2: INDUSTRY CHARACTERISTICS

This table shows the composition of industries. Mean growth rate and growth rate volatility are time-series averages of the parameters of industry-specific processes. Depreciation rate is the time-series average of capital depreciation rate in a given industry.

Industry	GICS Code	Number of firms in 1980	Number of firms in 2014	Mean growth rate	Growth rate volatility	Depreciation rate
Energy Equipment & Services	101010	38	70	3.65%	26.45%	7.46%
Oil, Gas & Consumable Fuels	101020	121	143	2.83%	28.78%	6.17%
Chemicals	151010	77	71	2.85%	11.50%	6.36%
Construction Materials	151020	26	7	3.22%	14.78%	5.68%
Containers & Packaging	151030	28	20	2.77%	11.46%	6.93%
Metals & Mining	151040	93	62	2.99%	21.96%	5.41%
Paper & Forest Products	151050	35	15	2.81%	13.24%	4.93%
Aerospace & Defense	201010	67	53	2.95%	15.80%	7.95%
Building Products	201020	57	31	3.24%	14.31%	7.15%
Construction & Engineering	201030	32	25	3.26%	18.57%	9.43%
Electrical Equipment	201040	67	36	3.11%	14.89%	7.71%
Industrial Conglomerates	201050	9	6	2.89%	10.50%	7.96%
Machinery	201060	139	111	3.04%	15.02%	7.35%
Trading Companies & Distributors	201070	13	33	3.14%	11.66%	7.92%
Commercial Services & Supplies	202010	141	65	3.28%	13.68%	8.89%
Professional Services	202020	8	43	3.50%	14.62%	13.82%
Air Freight & Logistics	203010	9	11	3.21%	14.33%	10.08%
Airlines	203020	18	10	3.08%	15.11%	6.93%
Marine	203030	2	22	2.95%	20.41%	5.13%
Road & Rail	203040	21	10	3.00%	12.76%	8.17%
Transportation Infrastructure	203050	4	3	3.04%	22.47%	7.09%
Auto Components	251010	46	35	3.05%	13.93%	7.68%
Automobiles	251020	3	7	2.84%	17.26%	8.62%
Household Durables	252010	113	49	3.17%	15.14%	7.89%
Leisure Equipment & Products	252020	38	16	3.22%	16.09%	9.90%
Textiles, Apparel & Luxury Goods	252030	91	40	3.19%	12.22%	8.84%
Hotels, Restaurants & Leisure	253010	54	88	3.41%	12.06%	6.35%
Diversified Consumer Services	253020	4	27	3.25%	10.27%	9.80%
Media	254010	63	80	3.11%	14.20%	11.31%
Distributors	255010	60	8	3.24%	13.61%	10.30%
Internet & Catalog Retail	255020	3	19	4.43%	12.46%	13.07%
Multiline Retail	255030	37	14	3.05%	5.69%	6.73%
Specialty Retail	255040	73	97	3.53%	8.95%	9.06%
Food & Staples Retailing	301010	68	26	2.99%	5.83%	7.05%
Beverages	302010	12	21	2.78%	10.50%	7.03%
Food Products	302020	73	61	2.67%	9.76%	6.59%
Tobacco	302030	7	7	2.84%	13.68%	6.68%
Household Products	303010	17	12	2.68%	8.20%	6.57%
Personal Products	303020	22	21	2.98%	10.93%	8.72%
Health Care Equipment & Supplies	351010	51	76	3.45%	13.72%	10.63%
Health Care Providers & Services	351020	28	70	3.53%	15.42%	12.36%
Health Care Technology	351030	3	15	4.07%	29.63%	19.92%
Biotechnology	352010	9	21	5.33%	27.44%	9.74%
Pharmaceuticals	352020	31	36	3.16%	14.99%	9.69%
Life Sciences Tools & Services	352030	7	23	3.58%	12.32%	10.20%
Internet Software & Services	451010	6	68	4.70%	39.18%	19.11%
IT Services	451020	20	72	3.42%	14.72%	15.42%
Software	451030	20	88	4.56%	17.81%	17.85%
Communications Equipment	452010	50	61	4.25%	20.60%	12.47%
Computers & Peripherals	452020	45	28	3.93%	18.42%	13.12%
Electronic Equipment & Instruments	452030	80	92	3.79%	16.77%	9.71%
Office Electronics	452040	8	3	4.80%	16.91%	9.97%
Semiconductor Equipment & Products	452050	20	4	4.86%	21.06%	10.93%
Semiconductors & Semiconductor Equipment	453010	19	88	4.58%	23.83%	10.81%
Diversified Telecommunication Services	501010	5	26	2.50%	20.06%	9.75%
Wireless Telecommunication Services	501020	3	9	3.27%	16.06%	11.75%

Table 3: ESTIMATION RESULTS

This table reports estimation results aggregated at the industry level (Panel A) and sector level (Panel B). For each industry and each sector, we report average curvature of the production function,  $\theta$ , in Column 3, and average cost of installing capital,  $\eta$ , in Column 4. Columns 5 through 10 report the distribution of GO/AP ratios across firms in each industry/sector. To compute GO/AP ratios, we calculate the value of growth options (GO) and the value of assets-in-place (AP) for each firm at the estimated value of industry-level parameters ( $\theta, \eta$ ).

Panel A: Industry Level									
Industry	GICS Code	$\theta$	$\eta$	Value of Growth Options / Value of Assets in Place					
				Mean	10P	25P	Median	75P	90P
Energy Equipment & Services	101010	0.33	1.22	52.1%	0.4%	2.0%	7.9%	33.0%	88.9%
Oil, Gas & Consumable Fuels	101020	0.27	1.28	12.4%	0.1%	0.7%	3.0%	9.0%	20.5%
Chemicals	151010	0.29	1.19	2.1%	0.0%	0.1%	0.4%	1.5%	4.5%
Construction Materials	151020	0.35	1.31	4.5%	0.0%	0.1%	0.7%	3.6%	9.4%
Containers & Packaging	151030	0.41	1.34	2.8%	0.1%	0.2%	0.9%	2.9%	7.4%
Metals & Mining	151040	0.29	1.30	6.1%	0.0%	0.3%	1.2%	4.6%	13.2%
Paper & Forest Products	151050	0.33	1.36	1.5%	0.0%	0.0%	0.1%	0.6%	3.3%
Aerospace & Defense	201010	0.27	1.19	7.3%	0.1%	0.4%	2.1%	7.0%	17.2%
Building Products	201020	0.32	1.33	5.1%	0.1%	0.4%	1.6%	5.1%	13.5%
Construction & Engineering	201030	0.29	1.14	9.9%	0.2%	1.0%	4.0%	11.4%	26.1%
Electrical Equipment	201040	0.26	1.13	2.6%	0.1%	0.4%	1.3%	3.3%	6.6%
Industrial Conglomerates	201050	0.36	1.21	9.9%	0.1%	1.0%	4.9%	12.2%	24.8%
Machinery	201060	0.29	1.13	5.2%	0.1%	0.6%	1.9%	5.0%	13.3%
Trading Companies & Distributors	201070	0.31	1.26	9.5%	0.5%	1.7%	5.5%	14.1%	24.1%
Commercial Services & Supplies	202010	0.28	1.18	7.5%	0.2%	1.0%	3.5%	9.9%	20.4%
Professional Services	202020	0.33	1.15	25.5%	2.6%	8.0%	19.1%	32.6%	55.9%
Air Freight & Logistics	203010	0.36	1.13	12.6%	0.2%	1.0%	5.5%	13.3%	33.4%
Airlines	203020	0.29	1.26	3.5%	0.0%	0.0%	0.2%	1.3%	9.3%
Marine	203030	0.37	1.31	21.8%	0.1%	0.5%	1.9%	9.6%	38.2%
Road & Rail	203040	0.38	1.20	4.6%	0.0%	0.1%	1.0%	4.1%	12.2%
Transportation Infrastructure	203050	0.31	1.34	39.3%	0.1%	0.7%	3.6%	37.8%	107.7%
Auto Components	251010	0.34	1.21	6.3%	0.1%	0.4%	1.9%	7.4%	17.7%
Automobiles	251020	0.32	1.17	13.3%	0.1%	0.5%	2.1%	7.0%	17.8%
Household Durables	252010	0.26	1.26	6.4%	0.2%	0.8%	3.4%	8.8%	16.9%
Leisure Equipment & Products	252020	0.27	1.17	8.2%	0.3%	1.2%	4.4%	11.6%	21.9%
Textiles, Apparel & Luxury Goods	252030	0.26	1.32	6.8%	0.1%	0.8%	3.1%	9.1%	18.2%
Hotels, Restaurants & Leisure	253010	0.29	1.24	4.2%	0.0%	0.1%	0.8%	4.0%	12.8%
Diversified Consumer Services	253020	0.38	1.17	11.7%	0.4%	1.9%	6.7%	16.2%	30.1%
Media	254010	0.27	1.17	9.0%	0.4%	1.8%	5.6%	13.2%	23.0%
Distributors	255010	0.27	1.23	13.7%	0.2%	1.6%	7.0%	19.8%	35.4%
Internet & Catalog Retail	255020	0.31	1.09	16.5%	0.6%	3.2%	9.5%	22.2%	39.4%
Multiline Retail	255030	0.36	1.20	1.1%	0.0%	0.0%	0.1%	0.6%	2.6%
Specialty Retail	255040	0.28	1.10	2.7%	0.1%	0.4%	1.4%	3.7%	7.1%
Food & Staples Retailing	301010	0.38	1.21	3.6%	0.0%	0.2%	0.7%	2.8%	9.8%
Beverages	302010	0.29	1.33	10.3%	0.2%	1.0%	3.9%	12.6%	28.2%
Food Products	302020	0.31	1.21	6.1%	0.1%	0.4%	1.4%	5.2%	17.7%
Tobacco	302030	0.29	1.28	17.5%	0.8%	3.1%	9.9%	24.1%	39.9%
Household Products	303010	0.29	1.33	10.2%	0.0%	0.1%	1.9%	11.5%	34.7%
Personal Products	303020	0.27	1.21	7.8%	0.2%	0.9%	3.8%	10.3%	20.1%
Health Care Equipment & Supplies	351010	0.35	1.01	31.6%	0.4%	2.7%	9.5%	26.4%	83.7%
Health Care Providers & Services	351020	0.32	1.13	22.3%	1.6%	5.2%	14.8%	28.1%	50.6%
Health Care Technology	351030	0.34	1.06	103.4%	5.1%	14.0%	39.4%	80.7%	147.9%
Biotechnology	352010	0.34	1.14	209.0%	3.0%	16.3%	52.7%	147.6%	480.9%
Pharmaceuticals	352020	0.35	1.08	51.8%	0.0%	0.7%	13.4%	62.8%	125.8%
Life Sciences Tools & Services	352030	0.33	1.01	20.7%	0.3%	1.4%	5.5%	13.2%	33.5%
Internet Software & Services	451010	0.33	1.04	49.4%	6.7%	14.9%	30.1%	52.3%	77.4%
IT Services	451020	0.28	1.02	18.7%	1.3%	4.8%	12.3%	24.3%	37.7%
Software	451030	0.40	1.00	69.7%	1.7%	6.8%	27.6%	76.1%	173.4%
Communications Equipment	452010	0.39	1.00	76.5%	0.8%	2.7%	8.7%	31.7%	96.4%
Computers & Peripherals	452020	0.33	1.01	13.6%	0.3%	1.3%	4.7%	13.2%	40.7%
Electronic Equipment & Instruments	452030	0.28	1.05	6.9%	0.3%	1.3%	3.7%	8.9%	17.6%
Office Electronics	452040	0.46	1.18	100.1%	0.6%	2.2%	9.8%	47.5%	171.1%
Semiconductor Equipment & Products	452050	0.35	1.04	44.1%	0.5%	1.3%	3.9%	12.4%	45.2%
Semiconductors & Semiconductor Equipment	453010	0.39	1.02	103.8%	0.4%	1.7%	9.4%	54.2%	171.0%
Diversified Telecommunication Services	501010	0.35	1.23	21.1%	0.0%	0.1%	1.2%	6.4%	27.8%
Wireless Telecommunication Services	501020	0.34	1.12	31.0%	0.1%	0.7%	3.9%	10.4%	31.9%

Table 3: ESTIMATION RESULTS – CONTINUED

Panel B: Sector Level									
Sector	GICS Code	$\theta$	$\eta$	Value of Growth Options / Value of Assets in Place					
				Mean	10P	25P	Median	75P	90P
Energy	10	0.30	1.25	32.2%	0.2%	1.3%	5.4%	21.0%	54.7%
Materials	15	0.33	1.30	3.4%	0.0%	0.1%	0.7%	2.7%	7.5%
Industrials	20	0.32	1.21	11.7%	0.3%	1.2%	4.0%	11.9%	28.8%
Consumer Discretionary	25	0.30	1.19	8.3%	0.2%	1.1%	3.8%	10.3%	20.2%
Consumer Staples	30	0.31	1.26	9.3%	0.2%	0.9%	3.6%	11.1%	25.1%
Health Care	35	0.34	1.07	73.1%	1.7%	6.7%	22.5%	59.8%	153.7%
Information Technology	45	0.36	1.04	53.6%	1.4%	4.1%	12.2%	35.6%	92.3%
Telecommunication Services	50	0.34	1.17	26.1%	0.1%	0.4%	2.5%	8.4%	29.8%
Mean		0.32	1.18	24.5%	0.6%	2.1%	6.9%	19.8%	49.2%
Standard Deviation		0.04	0.10	35.9%	1.2%	3.5%	9.8%	25.3%	73.2%

Table 4: MISVALUATION DECILE PORTFOLIO REGRESSIONS

This table provides mean monthly raw and risk-adjusted returns of portfolios of firms belonging to 10 misvaluation deciles. The sample period is 1980-2014. Misvaluation is computed according to (7). Portfolio returns are computed as value-weighted mean returns of stocks belonging to a portfolio. Column 1 presents mean returns. Column 2 presents alphas relative to the MKT factor. Column 3 presents alphas relative to MKT, HML, and SMB factors, following Fama and French (1993). Column 4 presents alphas relative to MKT, HML, SMB, and MOM factors, following Carhart (1997). Column 5 presents alphas relative to MKT, HML, SMB, RMW, and CMA, following Fama and French (2015). Column 6 presents alphas relative to MKT, HML, SMB, RMW, CMA, and MOM. Column 7 presents alphas relative to MKT,  $r_{ME}$ ,  $r_{I/A}$ , and  $r_{ROE}$  factors, following Hou, Xue and Zhang (2015). Column 8 presents alphas relative to MKT,  $r_{ME}$ ,  $r_{I/A}$ , and  $r_{ROE}$ , and MOM factors. The description of factors is found in Section 4. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses. The difference row presents the difference in mean raw or risk-adjusted returns between two extreme misvaluation deciles with the t-statistics for the difference in parentheses.

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
1 (Most undervalued)	1.05 (3.45)	0.38 (1.96)	0.36 (1.85)	0.44 (2.25)	0.63 (3.24)	0.69 (3.54)	0.90 (4.67)	0.90 (4.66)
2	0.82 (2.92)	0.15 (0.98)	0.18 (1.20)	0.28 (1.91)	0.26 (1.75)	0.35 (2.35)	0.46 (3.02)	0.46 (3.03)
3	0.81 (3.50)	0.24 (2.10)	0.22 (1.95)	0.31 (2.76)	0.14 (1.19)	0.22 (1.93)	0.26 (2.15)	0.26 (2.20)
4	0.80 (3.48)	0.23 (2.05)	0.24 (2.08)	0.37 (3.45)	0.08 (0.68)	0.20 (1.93)	0.24 (2.05)	0.24 (2.20)
5	0.65 (2.65)	0.03 (0.24)	-0.01 (-0.06)	0.04 (0.35)	-0.15 (-1.38)	-0.10 (-0.96)	-0.08 (-0.66)	-0.08 (-0.70)
6	0.70 (2.87)	0.08 (0.76)	0.05 (0.49)	0.11 (0.96)	-0.13 (-1.23)	-0.08 (-0.73)	-0.08 (-0.64)	-0.08 (-0.69)
7	0.58 (2.37)	-0.04 (-0.32)	0.01 (0.09)	0.01 (0.05)	-0.19 (-1.78)	-0.19 (-1.73)	-0.24 (-2.03)	-0.24 (-2.07)
8	0.48 (1.82)	-0.19 (-1.69)	-0.13 (-1.12)	-0.13 (-1.06)	-0.32 (-2.87)	-0.31 (-2.74)	-0.37 (-3.03)	-0.37 (-3.07)
9	0.35 (1.21)	-0.35 (-2.27)	-0.22 (-1.42)	-0.37 (-2.51)	-0.30 (-1.90)	-0.43 (-2.86)	-0.53 (-3.35)	-0.53 (-3.40)
10 (Most overvalued)	0.15 (0.40)	-0.69 (-3.22)	-0.43 (-2.08)	-0.53 (-2.55)	-0.19 (-0.92)	-0.28 (-1.39)	-0.41 (-1.85)	-0.41 (-1.86)
Difference 1-10	0.90 (1.91)	1.07 (3.70)	0.79 (2.79)	0.97 (3.40)	0.82 (2.89)	0.98 (3.45)	1.31 (4.46)	1.31 (4.48)

Table 5: MISVALUATION DECILE PORTFOLIO REGRESSIONS: FACTOR LOADINGS

This table provides mean loadings on factors used in the 7 asset pricing models used in Table 4 for the most undervalued and most overvalued deciles of stocks. The description of factors is found in Section 4. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses. The difference row presents the difference in mean factor loadings between the two extreme misvaluation deciles with the t-statistics for the difference in parentheses.

		CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
MKT	1 (Most undervalued)	1.07 (25.34)	1.09 (24.19)	1.07 (23.85)	0.98 (20.95)	0.97 (20.78)	0.92 (21.00)	0.92 (20.95)
	10 (Most overvalued)	1.33 (28.34)	1.22 (25.95)	1.24 (26.37)	1.14 (23.07)	1.16 (23.58)	1.21 (23.92)	1.21 (24.40)
	Difference	-0.26	-0.14	-0.17	-0.16	-0.18	-0.29	-0.29
	1-10	(-4.11)	(-2.10)	(-2.60)	(-2.32)	(-2.69)	(-4.29)	(-4.44)
SMB	1 (Most undervalued)		-0.08 (-1.26)	-0.11 (-1.68)	-0.08 (-1.22)	-0.10 (-1.58)		
	10 (Most overvalued)		0.18 (2.67)	0.22 (3.14)	0.15 (2.13)	0.18 (2.67)		
	Difference		-0.27	-0.33	-0.23	-0.29		
	1-10		(-2.81)	(-3.43)	(-2.38)	(-3.02)		
HML	1 (Most undervalued)		0.00 (0.06)	-0.02 (-0.27)	0.20 (2.26)	0.17 (1.96)		
	10 (Most overvalued)		-0.41 (-5.71)	-0.39 (-5.35)	-0.37 (-4.07)	-0.33 (-3.65)		
	Difference		0.42	0.37	0.57	0.51		
	1-10		(4.18)	(3.68)	(4.51)	(3.99)		
MOM	1 (Most undervalued)			-0.11 (-2.64)		-0.10 (-2.34)		0.00 (0.03)
	10 (Most overvalued)			0.13 (3.03)		0.15 (3.42)		0.19 (3.56)
	Difference			-0.25		-0.24		-0.19
	1-10			(-4.01)		(-4.09)		(-2.65)

Table 5: MISVALUATION DECILE PORTFOLIO REGRESSIONS: FACTOR LOADINGS – CONTINUED

	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
RMW	1 (Most undervalued)			-0.36 (-4.30)	-0.35 (-4.22)		
	10 (Most overvalued)			-0.45 (-5.18)	-0.47 (-5.40)		
	Difference			0.10	0.12		
	1-10			(0.80)	(0.99)		
CMA	1 (Most undervalued)			-0.48 (-3.69)	-0.46 (-3.58)		
	10 (Most overvalued)			-0.13 (-0.97)	-0.16 (-1.17)		
	Difference			-0.35	-0.30		
	1-10			(-1.84)	(-1.63)		
$r_{ME}$	1 (Most undervalued)					0.10 (1.56)	0.10 (1.50)
	10 (Most overvalued)					0.25 (3.48)	0.18 (2.45)
	Difference					-0.15	-0.08
	1-10					(-1.60)	(-0.85)
$r_{I/A}$	1 (Most undervalued)					-0.50 (-4.96)	-0.50 (-4.95)
	10 (Most overvalued)					-0.51 (-4.43)	-0.50 (-4.39)
	Difference					0.01	0.00
	1-10					(0.09)	(0.02)
$r_{ROE}$	1 (Most undervalued)					-0.42 (-5.83)	-0.42 (-4.93)
	10 (Most overvalued)					-0.07 (-0.84)	-0.25 (-2.62)
	Difference					-0.35	-0.17
	1-10					(-3.19)	(-1.29)

Table 6: MISVALUATION DECILE PORTFOLIO REGRESSIONS: ROBUSTNESS

This table provides mean monthly raw and risk-adjusted returns of portfolios of firms belonging to extreme misvaluation groups using various asset pricing models. The sample period is 1980-2014. See Table 4 for descriptions of columns. The description of factors is found in Section 4. In Panel A, we report the mean raw and risk-adjusted returns for the extreme misvaluation quintiles instead of deciles. In Panel B, we report mean raw and risk-adjusted returns for subsamples of firms that have misvaluation below and above one (i.e. subsamples of undervalued and overvalued firms). In Panel C we use equally-weighted monthly portfolio returns when computing mean raw and risk-adjusted returns. In Panel D, we compute theoretical firm values and resulting misvaluation while estimating the model with only one free parameter,  $\theta$ , while fixing  $\eta$  at one. In Panel E, we compute theoretical firm values and resulting misvaluation while estimating  $\theta$  and  $\eta$  using only firms with above-median analyst coverage in their industries. In Panel F, the dependent variable is 3-month return following the formation of decile portfolios. In Panel G, the dependent variable is 12-month return following the formation of decile portfolios. In Panel H, we report mean returns and alphas separately for periods of NBER recessions and expansions. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses. The difference row in each Panel presents the difference in mean returns or alphas between two extreme misvaluation groups with the t-statistics for the difference reported in parentheses.

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Panel A: Quintiles of misvaluation								
Most undervalued	0.93 (3.36)	0.26 (1.79)	0.28 (1.90)	0.36 (2.44)	0.49 (3.27)	0.55 (3.71)	0.73 (5.10)	0.73 (5.09)
Most overvalued	0.23 (0.75)	-0.52 (-3.24)	-0.33 (-2.13)	-0.47 (-3.13)	-0.29 (-1.81)	-0.42 (-2.70)	-0.51 (-3.09)	-0.51 (-3.14)
Difference	0.70 (1.69)	0.78 (3.60)	0.61 (2.85)	0.83 (3.94)	0.78 (3.56)	0.97 (4.52)	1.24 (5.67)	1.24 (5.73)
Panel B: Undervalued vs overvalued								
Undervalued	0.97 (3.26)	0.30 (1.64)	0.30 (1.60)	0.35 (1.91)	0.47 (2.49)	0.51 (2.73)	0.64 (3.41)	0.64 (3.41)
Overvalued	0.26 (0.72)	-0.57 (-2.68)	-0.32 (-1.54)	-0.39 (-1.86)	-0.07 (-0.32)	-0.14 (-0.66)	-0.26 (-1.17)	-0.25 (-1.16)
Difference	0.71 (1.51)	0.87 (3.10)	0.61 (2.21)	0.74 (2.66)	0.53 (1.91)	0.65 (2.33)	0.89 (3.09)	0.89 (3.10)
Panel C: Equally-weighted returns								
Most undervalued	1.45 (4.42)	0.72 (3.49)	0.68 (3.92)	0.87 (5.18)	0.98 (5.78)	1.13 (6.99)	1.12 (7.20)	1.11 (7.77)
Most overvalued	0.19 (0.54)	-0.61 (-3.12)	-0.48 (-2.82)	-0.51 (-2.96)	0.00 (0.00)	-0.04 (-0.31)	-0.16 (-1.15)	-0.15 (-1.15)
Difference	1.26 (2.66)	1.34 (4.68)	1.17 (4.77)	1.38 (5.73)	0.98 (4.42)	1.18 (5.44)	1.27 (6.19)	1.27 (6.47)



Table 6: MISVALUATION DECILE PORTFOLIO REGRESSIONS: ROBUSTNESS – CONTINUED

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Panel D: Estimating one parameter								
Most undervalued	1.13 (3.77)	0.46 (2.47)	0.50 (2.60)	0.62 (3.24)	0.69 (3.57)	0.79 (4.11)	0.95 (4.97)	0.95 (4.98)
Most overvalued	0.21 (0.59)	-0.62 (-2.91)	-0.36 (-1.76)	-0.45 (-2.20)	-0.10 (-0.48)	-0.19 (-0.93)	-0.31 (-1.40)	-0.31 (-1.41)
Difference	0.91 (1.94)	1.08 (3.81)	0.86 (3.06)	1.07 (3.81)	0.79 (2.81)	0.98 (3.49)	1.26 (4.32)	1.25 (4.34)
Panel E: Estimation based on firms with above-median analyst coverage								
Most undervalued	0.97 (3.26)	0.30 (1.64)	0.30 (1.60)	0.35 (1.91)	0.47 (2.49)	0.51 (2.73)	0.64 (3.41)	0.64 (3.41)
Most overvalued	0.26 (0.72)	-0.57 (-2.68)	-0.32 (-1.54)	-0.39 (-1.86)	-0.07 (-0.32)	-0.14 (-0.66)	-0.26 (-1.17)	-0.25 (-1.16)
Difference	0.71 (1.51)	0.87 (3.10)	0.61 (2.21)	0.74 (2.66)	0.53 (1.91)	0.65 (2.33)	0.89 (3.09)	0.89 (3.10)
Panel F: 3-month returns								
Most undervalued	2.92 (5.52)	0.85 (2.67)	0.90 (2.71)	0.92 (2.66)	1.16 (3.29)	1.20 (3.31)	1.73 (4.84)	1.69 (4.71)
Most overvalued	1.34 (2.00)	-1.24 (-2.96)	-0.53 (-1.26)	-0.71 (-1.63)	0.18 (0.41)	-0.01 (-0.02)	-0.23 (-0.48)	-0.33 (-0.68)
Difference	1.58 (1.85)	2.09 (3.97)	1.43 (2.67)	1.64 (2.93)	0.97 (1.72)	1.21 (2.08)	1.97 (3.26)	2.02 (3.35)
Panel G: 12-month returns								
Most undervalued	10.88 (9.21)	2.03 (2.70)	2.45 (2.93)	4.91 (5.68)	2.11 (2.11)	4.63 (4.62)	5.64 (5.30)	5.89 (5.60)
Most overvalued	7.10 (2.41)	-5.61 (-1.96)	-4.72 (-1.49)	-5.62 (-1.62)	-2.93 (-0.77)	-3.68 (-0.91)	-5.52 (-1.27)	-5.63 (-1.29)
Difference	3.78 (1.19)	7.64 (2.58)	7.17 (2.19)	10.53 (2.94)	5.03 (1.28)	8.31 (1.99)	11.16 (2.50)	11.52 (2.57)

Table 6: MISVALUATION DECILE PORTFOLIO REGRESSIONS: ROBUSTNESS – CONTINUED

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Panel H: NBER recessions and expansions								
NBER recessions								
Most undervalued	-0.43 (-0.41)	0.44 (0.89)	0.49 (0.97)	0.47 (0.95)	0.42 (0.79)	0.32 (0.60)	0.62 (1.21)	0.50 (0.94)
Most overvalued	-2.28 (-1.60)	-1.21 (-1.39)	-1.20 (-1.51)	-1.17 (-1.49)	-1.02 (-1.21)	-0.89 (-1.06)	-1.15 (-1.33)	-0.88 (-0.98)
Difference	1.85 (1.04)	1.65 (1.65)	1.69 (1.79)	1.64 (1.76)	1.45 (1.44)	1.21 (1.22)	1.77 (1.76)	1.38 (1.33)
NBER expansions								
Most undervalued	1.29 (4.16)	0.39 (1.83)	0.38 (1.74)	0.45 (2.04)	0.68 (3.17)	0.76 (3.53)	0.99 (4.75)	0.99 (4.75)
Most overvalued	0.54 (1.54)	-0.58 (-2.78)	-0.30 (-1.49)	-0.40 (-1.97)	-0.06 (-0.32)	-0.17 (-0.85)	-0.27 (-1.27)	-0.29 (-1.39)
Difference	0.75 (1.61)	0.97 (3.26)	0.69 (2.29)	0.85 (2.84)	0.74 (2.54)	0.93 (3.16)	1.26 (4.23)	1.29 (4.33)

Table 7: FAMA-MACBETH REGRESSIONS OF RETURNS ON MISVALUATION MEASURE AND CHARACTERISTICS

This table presents results of estimating cross-sectional regressions of returns on the natural logarithm of misvaluation measure and the following characteristics: book-to-market ratio, the natural logarithm of market value, and 6-month past return. All independent variables are winsorized at the 1st and 99th percentiles. See Section 4 for variable definitions. The regressions are estimated monthly. The table reports means of coefficient estimates and their t-statistics in parentheses. In column 1, we report coefficient estimates for the baseline estimation procedure. In column 2, we compute theoretical firm values and resulting misvaluation while estimating the model with only one free parameter,  $\theta$ , while fixing  $\eta$  at one. In column 3, we compute theoretical firm values and resulting misvaluation while estimating  $\theta$  and  $\eta$  using only firms with above-median analyst coverage in their industries. In column 4, the dependent variable is 3-month return following the formation of decile portfolios. In column 5, the dependent variable is 12-month return following the formation of decile portfolios.

	Baseline	Estimating 1 param.	Above median coverage	3-month returns	12-month returns
Intercept	0.89 (2.75)	0.87 (2.68)	0.86 (2.65)	2.82 (2.36)	12.19 (2.29)
BM	0.25 (2.88)	0.27 (3.01)	0.26 (2.93)	0.70 (2.22)	5.07 (3.16)
Log(ME)	-0.07 (-2.14)	-0.07 (-2.13)	-0.07 (-2.09)	-0.22 (-1.96)	-1.13 (-2.14)
6-month return	0.01 (2.60)	0.01 (2.53)	0.01 (2.62)	0.03 (4.82)	0.09 (2.95)
Log(misvaluation)	-0.35 (-8.33)	-0.32 (-7.71)	-0.30 (-8.04)	-0.96 (-6.87)	-2.80 (-3.60)
# obs	419	419	419	417	408
R squared	0.03	0.02	0.02	0.03	0.04

Table 8: MISVALUATION DECILE PORTFOLIO REGRESSIONS – ONLY ASSETS IN PLACE

This table provides mean monthly raw and risk-adjusted returns of portfolios of firms belonging to 10 misvaluation deciles. The sample period is 1980-2014. Misvaluation is computed according to (13). Portfolio returns are computed as value-weighted mean returns of stocks belonging to a portfolio. Column 1 presents mean returns. Column 2 presents alphas relative to the MKT factor. Column 3 presents alphas relative to MKT, HML, and SMB factors, following Fama and French (1993). Column 4 presents alphas relative to MKT, HML, SMB, and MOM factors, following Carhart (1997). Column 5 presents alphas relative to MKT, HML, SMB, RMW, and CMA, following Fama and French (2015). Column 6 presents alphas relative to MKT, HML, SMB, RMW, CMA, and MOM. Column 7 presents alphas relative to MKT,  $r_{ME}$ ,  $r_{I/A}$ , and  $r_{ROE}$  factors, following Hou, Xue and Zhang (2015). Column 8 presents alphas relative to MKT,  $r_{ME}$ ,  $r_{I/A}$ , and  $r_{ROE}$ , and MOM factors. The description of factors is found in Section 4. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses. The difference row presents the difference in mean raw or risk-adjusted returns between two extreme misvaluation deciles with the t-statistics for the difference in parentheses.

Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
1 (Most undervalued)	1.15 (2.28)	0.52 (1.11)	0.23 (0.52)	0.43 (0.94)	0.13 (0.29)	0.31 (0.67)	0.27 (0.55)	0.30 (0.64)
2	-0.10 (-0.19)	-0.82 (-1.87)	-1.04 (-2.41)	-0.93 (-2.13)	-0.88 (-1.98)	-0.78 (-1.74)	-0.86 (-1.88)	-0.85 (-1.88)
3	0.47 (1.17)	0.01 (0.02)	-0.12 (-0.34)	-0.14 (-0.37)	-0.16 (-0.41)	-0.17 (-0.45)	-0.13 (-0.33)	-0.12 (-0.31)
4	1.06 (2.71)	0.50 (1.46)	0.27 (0.79)	0.36 (1.05)	0.19 (0.52)	0.27 (0.76)	0.33 (0.89)	0.33 (0.91)
5	1.33 (3.00)	0.59 (1.60)	0.43 (1.14)	0.51 (1.35)	0.32 (0.82)	0.40 (1.02)	0.36 (0.90)	0.36 (0.91)
6	1.04 (2.56)	0.38 (1.13)	0.26 (0.76)	0.36 (1.06)	0.06 (0.16)	0.16 (0.45)	0.07 (0.18)	0.07 (0.21)
7	0.65 (1.67)	-0.09 (-0.29)	-0.18 (-0.57)	-0.18 (-0.58)	-0.28 (-0.88)	-0.28 (-0.87)	-0.34 (-1.03)	-0.33 (-1.02)
8	1.04 (2.57)	0.34 (1.02)	0.36 (1.10)	0.33 (0.97)	0.31 (0.92)	0.28 (0.80)	0.13 (0.37)	0.14 (0.40)
9	0.44 (1.02)	-0.37 (-1.13)	-0.27 (-0.82)	-0.25 (-0.76)	-0.20 (-0.58)	-0.18 (-0.53)	-0.43 (-1.23)	-0.43 (-1.23)
10 (Most overvalued)	0.63 (1.06)	-0.28 (-0.55)	-0.16 (-0.32)	-0.21 (-0.39)	0.48 (0.93)	0.42 (0.80)	0.27 (0.50)	0.26 (0.49)
Difference 1-10	0.52 (0.67)	0.80 (1.16)	0.39 (0.58)	0.64 (0.92)	-0.35 (-0.50)	-0.10 (-0.15)	0.00 (-0.00)	0.04 (0.05)

Table 9: MISVALUATION AND INDUSTRY GO/AP DOUBLE-SORTED PORTFOLIO REGRESSIONS

This table provides mean monthly raw and risk-adjusted returns of portfolios of firms obtained by sorting firms into misvaluation deciles and independently into terciles of industry median GO/AP ratios estimated from the model. Misvaluation is computed according to (7). We only report mean raw and risk-adjusted returns for the most undervalued and most overvalued deciles and the differences between them for each misvaluation tercile. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses.

GO/AP tercile	Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Lowest	Most undervalued	1.03 (3.08)	0.36 (1.51)	0.16 (0.68)	0.34 (1.50)	0.17 (0.73)	0.33 (1.44)	0.50 (1.97)	0.49 (2.02)
	Most overvalued	0.72 (1.31)	-0.06 (-0.12)	0.12 (0.25)	-0.03 (-0.06)	0.17 (0.35)	0.04 (0.07)	-0.20 (-0.38)	-0.19 (-0.38)
	Difference	0.32 (0.49)	0.42 (0.79)	0.04 (0.07)	0.37 (0.69)	0.00 (-0.00)	0.30 (0.54)	0.69 (1.21)	0.69 (1.21)
Middle	Most undervalued	0.92 (3.13)	0.27 (1.36)	0.11 (0.61)	0.23 (1.23)	0.00 (0.02)	0.12 (0.59)	0.18 (0.88)	0.19 (0.93)
	Most overvalued	-0.02 (-0.05)	-0.90 (-3.35)	-0.74 (-2.73)	-0.77 (-2.83)	-0.37 (-1.40)	-0.42 (-1.55)	-0.42 (-1.53)	-0.42 (-1.53)
	Difference	0.93 (1.85)	1.17 (3.51)	0.85 (2.58)	1.01 (3.03)	0.38 (1.14)	0.54 (1.61)	0.60 (1.75)	0.61 (1.78)
Highest	Most undervalued	1.11 (3.29)	0.42 (1.78)	0.51 (2.16)	0.59 (2.46)	0.83 (3.48)	0.88 (3.69)	1.05 (4.48)	1.05 (4.48)
	Most overvalued	0.21 (0.54)	-0.62 (-2.51)	-0.27 (-1.18)	-0.34 (-1.44)	0.07 (0.32)	0.01 (0.02)	-0.26 (-1.04)	-0.26 (-1.04)
	Difference	0.91 (1.78)	1.04 (3.04)	0.79 (2.37)	0.93 (2.76)	0.75 (2.29)	0.88 (2.65)	1.31 (3.82)	1.31 (3.82)

Table 10: MISVALUATION AND INDUSTRY M/B DOUBLE-SORTED PORTFOLIO REGRESSIONS

This table provides mean monthly raw and risk-adjusted returns of portfolios of firms obtained by sorting firms into misvaluation deciles and independently into terciles of industry median M/B ratios. Misvaluation is computed according to (7). We only report mean raw and risk-adjusted returns for the most undervalued and most overvalued deciles and the differences between them for each misvaluation tercile. In all regressions, standard errors are Newey-West adjusted (with 6 lags). t-statistics are reported in parentheses.

M/B tercile	Misvaluation decile	Mean	CAPM	FF3	FF3 +Mom	FF5	FF5 +Mom	Q	Q +Mom
Lowest	Most undervalued	1.17 (3.99)	0.58 (2.78)	0.41 (2.12)	0.53 (2.75)	0.34 (1.73)	0.45 (2.29)	0.58 (2.65)	0.57 (2.70)
	Most overvalued	0.61 (1.12)	-0.16 (-0.33)	0.01 (0.03)	-0.18 (-0.37)	0.04 (0.08)	-0.13 (-0.27)	-0.31 (-0.62)	-0.31 (-0.61)
	Difference	0.56 (0.91)	0.74 (1.42)	0.40 (0.78)	0.70 (1.38)	0.30 (0.58)	0.58 (1.11)	0.89 (1.61)	0.88 (1.61)
Middle	Most undervalued	0.88 (2.75)	0.19 (0.87)	-0.02 (-0.09)	0.16 (0.77)	-0.04 (-0.18)	0.13 (0.59)	0.21 (0.90)	0.22 (0.98)
	Most overvalued	0.20 (0.56)	-0.52 (-2.02)	-0.48 (-1.85)	-0.48 (-1.83)	-0.40 (-1.48)	-0.40 (-1.48)	-0.47 (-1.71)	-0.47 (-1.71)
	Difference	0.67 (1.40)	0.71 (2.10)	0.46 (1.37)	0.65 (1.91)	0.36 (1.03)	0.53 (1.53)	0.68 (1.88)	0.69 (1.94)
Highest	Most undervalued	1.01 (3.00)	0.33 (1.38)	0.43 (1.81)	0.55 (2.30)	0.74 (3.12)	0.84 (3.51)	0.97 (4.11)	0.97 (4.10)
	Most overvalued	0.02 (0.05)	-0.85 (-3.53)	-0.44 (-1.98)	-0.48 (-2.17)	-0.09 (-0.40)	-0.14 (-0.64)	-0.32 (-1.33)	-0.32 (-1.33)
	Difference	0.99 (1.93)	1.17 (3.48)	0.87 (2.68)	1.03 (3.16)	0.83 (2.58)	0.98 (3.03)	1.29 (3.84)	1.29 (3.84)