Up-Cascaded Wisdom of the Crowd*

Lin William Cong† Yizhou Xiao§

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Abstract

Financial activities such as crowdfunding and IPO underwriting involve aggregating information from diverse investors, sequential sales, observational learning, and most interestingly, all-or-nothing (AoN) rules that contingent the financing upon achieving certain fundraising targets. We incorporate these features into a classical model of information cascade, and find that AoN leads to uni-directional cascades in which investors rationally ignore private signals and imitate preceding investors only if the preceding investors decide to *invest*. Consequently, an entrepreneur prices issuance more aggressively, and fundraising may succeed rapidly but never fails rapidly. Information production also becomes more efficient, especially with a large crowd of investors, yielding more probable financing of good projects, and the weeding-outs of bad projects that are absent in earlier models. More generally, endogenous pricing with AoN targets leads to greater financing feasibility and better harnessing of the wisdom of the crowd under informational frictions.

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†University of Chicago Booth School of Business. Email: will.cong@chicagobooth.edu
§The Chinese University of Hong Kong Business School. Email: yizhou@cuhk.edu.hk

1 Introduction

Since its inception in the arts and creativity-based industries (e.g., recorded music, film, video games), crowdfunding has quickly become a mainstream source of capital for entrepreneurs. In the span of a few years, its total annual volume has reached a whopping 34.4 billion USD globally at the dawn of 2017. It has surpassed the market size for angel funds in 2015, and the World Bank Report estimates that global investment through crowdfunding will reach $93 billion in 2025.[1] The US deregulation also passed the law to allow non-accredited investors to join equity-based crowdfunding, further fueling the development.[2] What is more, with the rise of initial coin offerings, alternative corporate crowdfunding emerges, with over two billion dollars raised in the US in the first half of 2017. In Appendix A, we provide two examples from well-known crowdfunding platforms.

While early news articles laud mitigation of financial constraints as the main reason for crowdfunding, recent empirical studies provide convincing evidence that entrepreneurs use crowdfunding as an information aggregation mechanism (Xu (2017) and Viotto da Cruz (2016)). For example, Mollick and Kuppuswamy (2014) find in a comprehensive survey of entrepreneurs on Kickstarter that learning about demand to be the single most important benefit or motive for crowdfunding. Reduction of search and matching online costs through the Internet, which in turn allows divisibility of funding and low communication costs, facilitates greater outreach, decentralized participation, timely disclosure and monitoring, and information aggregation. As such, it is generally recognized that the key advantage of crowdfunding platforms lies in aggregating information and harnessing the wisdom from the crowd,

[1] $http://www.infodev.org/infodev-files/wb_{c}rowdfundingreport-v12.pdf$

[2] On April 5, 2012, President Obama signed into law the Jumpstart Our Business Startups (JOBS) Act. Adding to then extant donation and reward based crowdfunding platforms, the JOBS Act Title III legalized crowdfunding for equity by relaxing various requirements concerning the sale of securities in May 2016.

in addition to financing.[3]

Importantly, crowdfunding exhibit two salient features in the process of fundraising and information aggregation. First, potential backers often randomly chance upon crowdfunding websites or products within the window of offering. Investors making decisions later can thus infer from earlier investors, or at least observe how well an offering has sold to date, or sold relative to offerings undertaken in the past. Second, the most common type of crowdfunding scheme involves an "all-or-nothing" (AoN) implementation where the entrepreneur sets a target threshold for fundraising and gets the capital if and only if the target is reached (Chemla and Tinn (2016)).[4] The Crowdfund Act also indicates that AoN feature will likely be mandated.[5] How do sequential sales and AoN target affect information aggregation and financing? Do they lead to underpricing and inefficient information aggregation as in classical information cascade models? Do they give crowdfunding an edge over traditional forms of financing?

To answer these questions, we incorporate information aggregation from diverse investors and the AoN feature into a standard model of sequential sales and dynamic learning, and characterize equilibrium pricing, optimal AoN targets, and information production. We find that the simple addition of AoN alters many important results from extant literature on information cascades. In particular, AoN leads to uni-directional cascades in which investors rationally ignore private signals and imitate preceding investors only if the preceding

[3] In fact, SEC also recognizes in its final rule of regulating crowdfunding that "individuals interested in the crowdfunding campaign members of the 'crowd'fund the campaign based on the collective 'wisdom of the crowd' " (Li (2017) and 17 CFR Parts 200, 227, 232, 239, 240, 249, 269, 274).

[4]Take Kickstarter, for example. The entrepreneur is typically asked to provide the following pieces of information: (1) a description of the reward to the consumer, typically the entrepreneur's final product; (2) a pledge level ; (3) a target level. The crowdfunding campaign lasts typically for a fixed period of time – usually 30 days. During the campaign, Kickstarter provides information on the aggregate level of pledges, therefore a supporter can condition his decision based on previous consumers actions.

[5]Because intermediaries need to ensure that all offering proceeds are only provided to the issuer when the aggregate capital raised from all investors is equal to or greater than a target offering amount, and allow all investors to cancel their commitments to invest, as the Commission shall, by rule, determine appropriate (Sec. 4A.a.7). See http: // beta.congress .gov / bill / 112th- congress / senate- bill / 2190 / text.

investors decide to *invest*. Consequently, an entrepreneur prices issuance more aggressively, and fundraising may succeed rapidly but never fails rapidly. Yet information production becomes more efficient, especially with a large crowd of investors, leading to more successes of good projects and failures of bad projects, and more generally a better harnessing of the wisdom of the crowd under informational frictions.

Specifically, we build on the framework of Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992): an entrepreneur approaches sequentially N investors who choose to support or reject the entrepreneur's startup. Supporters pay a fixed price pre-determined by the entrepreneur and gets a payoff normalized to one if the project is good. All agents are risk-neutral and have a common prior on the project's quality. Investors receive private, informative signals, and observe the decisions of preceding investors. Deviating from the standard setup, the entrepreneur also decides on AoN target—supporters only pay the price and enjoy the project payoff if the fundraising reaches a target number of supporters.

We show that in equilibrium the aggregation of private information only stops upon an UP cascade, in which the public Bayesian posterior belief is so positive that investors always support the project regardless of their private signals. The intuition is that with AoN, investors are encouraged to invest even when the aggregated information is not good. In particular, investors with positive private signals always find it optimal to support because they only pay the price when either there is an UP cascade later or the total number of good observations reaches the AoN target, both suggesting a high posterior on the project's quality. Therefore, DOWN cascades do not occur because they are all interrupted by investors with positive signals who do not care about DOWN cascades before AoN is reached. After AoN is reached, the situation returns to the standard cascade setting, with the caveat that the entrepreneur endogenously determines both price and AoN so that in equilibrium no DOWN cascade occurs, as opposed to setting price alone to avoid DOWN cascade as in Welch (1992). That said, investors with negative private signals are reluctant to support because in equilibrium their actions may be misinterpreted as positive signals, resulting in either a too-early UP cascade or reaching the AoN target without enough number of positive signals, both implying a not-high-enough posterior on the project's quality. Taking this concern of regretting supporting a project into consideration, the entrepreneur optimally sets AoN targets to completely exclude the possibility of DOWN cascades. Apparently, without DOWN cascades which stop private information aggregation, good projects are financed almost surely when the crowd base N is very large.

The exclusion of DOWN cascades has important implications on the availability of financing. In standard financial market models with information cascades, the feasible price range is limited because the price must be lower than the posterior of the first investor with a positive signal to prevent an early DOWN cascade. This limited price range makes it impossible to finance costly projects with potentially high qualities. With AoN target, entrepreneur can charge a sufficiently high price to cover the project implementation cost without worrying about DOWN cascades. Uni-directional cascades thus enlarge the feasible pricing range for fundraising. As a result, crowdfunding and the like can lead to financing of projects that would not have been funded by centralized experts, consistent with empirical findings in Mollick and Nanda (2015).[6]

The exclusion of DOWN cascades also affects the optimal pricing. In the standard information cascade setting, Welch (1992) shows that the entrepreneur endogenously charges a low price to induce an UP cascade from the very beginning, preventing the potential arrival

[6]Mollick and Nanda (2015) find that of the projects where there is no agreement, the crowd is much more likely to have funded a project that the judge did not like than the reverse. Around 75% of the projects where there is a disagreement are ones where the crowd funded a project but the expert would not have funded it. This is consistent with uni-directional cascades.

of DOWN cascades. This underpricing thus destroys information aggregation in financial market because information cascades start very early. Our model demonstrates that AoN provides the entrepreneur an additional tool to avoid DOWN cascades. On the one hand, a higher price increases the profit the entrepreneur collects from each supporting investor. On the other hand, high price sets a higher bar for implementation and associated UP cascades, resulting a smaller chances of project implementation and the delay of UP cascades. Since the delay of UP cascade is less costly given a large investor base, the entrepreneur facing a large base of potential investors will charge a higher price for issuance, and the information aggregation continues until an UP cascade arrives. Uni-directional cascades thus reduces underpricing, and partially restores information aggregation by avoiding information cascades from the very beginning.

By aggregating information before investment is sunk, crowdfunding platforms adds an option value to experimentation, which can facilitate entrepreneurial entry and innovation (Manso (2016)). In a sense, pre-selling through crowdfunding platforms can be viewed as credible surveys on consumer demand. Chemla and Tinn (2016) find that even for a failed crowdfunding, because the target is higher than the optimal investment threshold, the firm may still invest. Moreover, more successful at crowdfunding stage typically leads to greater success later for product implementation and future performance (Xu (2017)).

While the recent rise of crowdfunding certainly motivates our study, we note that aggregating dispersed information under frictions has been prevalent in finance and economics. The discussion of financial systems for aggregating information dates back to Hayek (1945). Bond, Edmans, and Goldstein (2012) survey recent contributions related to the informational role of market prices for real decisions. One important and oft-discussed example is IPO book-building that aggregates information from investors to price the shares (e.g., Ritter and Welch (2002)). With limited distribution channels by investment banks, it takes the underwriter times to approach interested investors, who are typically institutions that do not communicate amongst one another. Strong initial sales encourage subsequent support while slow initial sales discourage subsequent investing. During an IPO book building process, the issuer may decide to not continue with its proposed offering of securities if he observes a poor investor interest. IPO book-building is therefore also characterized by sequential arrival and AoN. In both Internet-based crowdfunding and IPO, there is no market for investors to trade, and prices are fixed by entrepreneurs or the underwriter with evolving quantities of financing in the process. We show that the introduction of explicit or implicit AoN target substantially changes equilibrium outcomes. In particular, issuance is less under-priced, and as we move from smaller investor base such as venture financing, to intermediate investor base such as IPO bookbuilding, to large investor base such as Internet-based crowdfunding, the issuance becomes more and more overpriced (less underpriced) relative to the prior average project quality.

Beyond Internet-based crowdfunding and IPO book building, our findings also shed light on other situations where decisions are made sequentially with AoN target. For example, in many elections a candidate is only voted into the office if the number of votes passes a threshold. In initial coin offerings, the upside speculation will only come to realize if there are enough ICOs or wide range users so that the government would not shut down such markets. Disclosure, accounting, and reporting practices may exhibit similar features. Scharfstein and Stein (1990) argue that managers imitate the investment decisions of other managers to appear to be informed. If new attempts have no cost upon failure, but can benefit the firms if there is a critical mass that triggers regulatory changes, then it is essentially an AoN implementation. Also defined by these features is the provision-point mechanism, also known as assurance contract or crowdaction, which solves a classic coordination and free-riding issue in the provision of public goods (e.g., Bagnoli and Lipman (1989)). Finally, to curb informational cascades in bank runs, an AoN measure could be explored in which no one can withdraw if the total withdrawal exceeds certain thresholds.

Literature

Our paper foremost relates to the large literature on information cascades, social learning, and rational herding. Bikhchandani, Hirshleifer, and Welch (1998) and Chamley (2004) provide comprehensive surveys. Our model is largely built on Bikhchandani, Hirshleifer, and Welch (1992) which discusses informational cascade as a general phenomenon. Welch (1992) relates information cascade to IPO underpricing, and serves as a natural benchmark for our model implications on pricing. Studies such as Anderson and Holt (1997), Çelen and Kariv (2004), and Hung and Plott (2001) provide experimental evidence for information cascades. We add to the literature by introducing AoN into sequential sales and learning, and show that the resulting directional cascades reduces underpricing, reduces the detriments of information cascades, and facilitate financing and harnessing the wisdom of the crowd.

Related are Guarino, Harmgart, and Huck (2011) and Herrera and Hörner (2013) that consider information cascades when only one of the binary actions is observable, and either the agents do not know their position or they have Poisson arrivals. While Herrera and Hörner (2013) find under certain signal distributions welfare could improve over that in Bikhchandani, Hirshleifer, and Welch (1992) and Guarino, Harmgart, and Huck (2011) show cascades only occur in one direction, they do not consider endogenous pricing. Moreover, they compare equilibrium outcomes across two exogenous environments, whereas we study the consequence of endogenous AoN under the standard cascade setting.

The paper also adds to an emerging literature on crowdfunding. Agrawal, Catalini, and Goldfarb (2014) comment on the basic patterns and economic tradeoffs of crowdfunding. Belleflamme, Lambert, and Schwienbacher (2014) provides an early theoretical comparison of reward-based and equity-based crowdfunding, and shows the former is better for and only for small initial capital requirements. Strausz (2017) and Chemla and Tinn (2016) analyze demand uncertainty and moral hazard, and find that AoN is crucial in mitigating moral hazard, and Pareto-dominates the alternative "keep-it-all" (KiA) mechanism. Chang (2016) shows under common-value assumptions AoN generates more profit, because AoN makes the expected payments positively correlated with values, reducing information rents the entrepreneur pays, reminiscent of the linkage principle (Milgrom and Weber (1982)). Moreover, Cumming, Leboeuf, and Schwienbacher (2014) and Lau (2013, 2015) find that AoN performs better than KiA based on comparison between the two largest crowdfunding platforms, Kickcstarter and Indiegogo, and by comparing projects within Indiegogo. Like Strausz (2017), Ellman and Hurkens (2015) discuss optimal crowdfunding design, in the absence of moral hazard, but with a focus on price discrimination and demand uncertainty. Finally, Li (2017) similarly examines contract designs that harness the wisdom of the crowd and find profit-sharing to be optimal. Instead of introducing moral hazard or financial constraint, or derives optimal designs in static settings, we focus on pricing and information production, especially under endogenous AoN arrangements and with dynamic learning.

Empirically evidence on harnessing the wisdom of the crowd and on information cascades abound. Mollick and Nanda (2015) find significant agreement between the funding decisions of crowds and experts, and find no qualitative or quantitative differences in the long-term outcomes of projects selected by the two groups. Agrawal, Catalini, and Goldfarb (2011) finds suggestive empirical evidence of funding propensity increasing with accumulated capital on Sellaband, an Amsterdam based music-only platform started in 2006. Zhang and Liu (2012) documents rational herding on P2P lending on Prosper.com. Burtch, Ghose, and Wattal (2013) examine social influence in a crowd-funded marketplace for online journalism projects, and demonstrate that the decisions of others provide an informative signal of quality. Xu (2017) and Viotto da Cruz (2016) demonstrate the wisdom of the crowd benefits entrepreneurs' ex post decisions and real option exercises. Our paper complements these studies by providing a formal framework to rationalize these phenomena.

Given our focus on financing efficiency, pricing efficiency, and informational efficiency, closely related is Brown and Davies (2017) which shows that when investors make decisions simultaneously, an AoN leads to loser's blessing, and scarce profits create a winner's curse, both adversely affecting financing efficiency for crowdfunding. We complement by endogenizing AoN target and discussing the resulting gains in informational efficiency as well as financing efficiency, all relative to the standard dynamic information-cascade benchmark. Also closely related is Hakenes and Schlegel (2014) which, along the same line, argues that endogenous loan rates and AoN targets encourage information acquisition by individual households in lending-based crowdfunding, and enable more good projects to receive financing. We focus on information aggregation and observational learning instead of investors' costly information acquisition. Moreover, we differ from these studies in our focus on dynamic learning and sequential investment instead of simultaneous investment games. Whereas those studies discuss the loss and gain in efficiency relative to the standard static auction benchmark, our setup allows us to uncover the benefits of setting AoN in a dynamic environment, in a spirit akin to how commitment helps improve informational efficiency in Bagnoli and Lipman (1989) and Bond and Goldstein (2015).

Our paper is also broadly related to innovation and entrepreneurial finance. Startup firms receive venture funding often to experiment and uncover more information about the project's viability and future profitability (Gompers and Lerner (2004) and Kerr, Nanda, and Rhodes-Kropf (2014)). To the extent that such information can be gleaned from consumer surveys or aggregated from crowds, the entrepreneur can potentially reduce experimentation or learning costs. Moreover, crowdfunding arguably reduces the barrier to entry for entrepreneurs. Yet it may not select or monitor projects as well as VC does (Gompers, Gornall, Kaplan, and Strebulaev (2016) show that VCs mainly add value through selection). It thus serves as a complement to the traditional venture capital (e.g., Chemla and Tinn (2016)). Abrams (2017) document initial empirical evidence on how the US securities crwodfunding market provides a new way to finance quality startups. We add to the literature by showing how AoN rules commonly observed in crowdfunding help mitigate inefficiencies typically associated with information cascades, therefore further demonstrating the benefits and costs of these innovations in entrepreneurial financing and information aggregation from dispersed investors and consumers.

2 A Model of Directional Cascades

2.1 Setup

Consider an entrepreneur deciding whether to press forward with a startup project. He visits a sequence of investors $i = 1, 2, \ldots, N$, each can potentially support or reject the project. The action of investor i is $A_i \in \{S, R\}$, where S denotes a support and R a rejection. If the project is funded eventually, then every supporting investor contributes a predetermined amount of capital m to the entrepreneur, and receives the benefit V, which can be either 0 or 1.[7]

All agents including the entrepreneur are rational, risk-neutral, and share the same prior that the project type can be either $V = 0$ and $V = 1$ with equal probability.[8] Each investor i observes one conditionally independent private signal $X_i \in \{H, L\}$. Signals are informative in the following sense:

$$Pr(X_i = H|V = 1) = Pr(X_i = L|V = 0) = p \in (\frac{1}{2}, 1); \tag{1}$$

$$Pr(X_i = L|V = 1) = Pr(X_i = H|V = 0) = q \equiv (1 - p) \in (0, \frac{1}{2}). \tag{2}$$

We depart from the literature by incorporating the observed "all-or-nothing" (AoN) scheme into this setup: the entrepreneur receives "all" if the campaign succeeds and "nothing" if it fails to meet a pre-specified target. In other words, before investors make investment decisions, the entrepreneur determines the amount of each contribution m and an AoN target T_N; the proposal is implemented if and only if more than T_N investors support. In the baseline, we assume the entrepreneur commits to abandoning the project if there are too few investors willing to contribute. m is essentially the price investors all pay—in the case of IPO issuance, the SEC bans variable-price sales; in the case of equity crowdfunding, equity prices are also uniform.

In addition to mitigating moral hazard and augmenting profit (Strausz (2017) and Chang (2016)), another common justification for AoN is that there is a minimum efficient scale for

[7]For crowdfunding, we are not distinguishing equity-based vs reward-based platforms. It is natural to interpret our model as equity-based crowdfunding, in line with IPO book building. However, for reward-based and donation-based crowdfunding, as long as investors are learning some common component of product quality, our results apply. Even though many prominent examples of crowdfunding such as Kickstarter are reward-based, Abrams (2017) documents that as of November 12th, 2016, the SEC has approved 21 platforms for security-based crowdfunding and there has been 146 security issues totaling over $13.6 million in funding through 17,000 distinct investments.

[8]In a typical crowdfunding project, each individuals contribution is small, at least relative to his or her wealth, thus there is little wealth effect and investors are locally risk-neutral; IPO book building involves institutional investors who can be treated as risk-neural.

the project, which is equivalent to requiring mT_N passing some threshold. This is nested in our interpretation as the entrepreneur sets m and T_N to maximize profit. The order of investors is exogenous and is known to all.[9] This is equivalent to observing both supporting and rejecting actions of previous investors, a standard assumption in the literature on information cascades. In other words, when investor i makes her decision, she observes her own private signal X_i and decisions made by all those ahead of her, that is, $\{A_1, A_2, \ldots, A_{i-1}\}$. In the application in crowdfunding, this information set is equivalent to observing fund raised to-date (and time) and knowing the starting time of fundraising and the investor arrival rate. Evidence that funders rely heavily on accumulated capital as a signal of quality is abundant (Agrawal, Catalini, and Goldfarb (2011); Zhang and Liu (2012), and Burtch, Ghose, and Wattal (2013)). Investors Bayesian update their beliefs using their private information and inferences from the observed actions of their predecessors in the sequence. Let $H_i \equiv \{A_1, A_2 \ldots, A_i\}$ be the action history till investor i, and N_S be the total number of supporting investors. Investor i's problem is:

$$\max_{A_i} \left[E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) - m\right] \mathbb{1}_{\{A_i=S\}}, \tag{3}$$

where $\mathbb{1}_{\{A_i=S\}}$ is the indicator function for supporting. If $E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) > m$, an investor chooses $A_i = S$. When $E\left(V|X_i, H_{i-1}, N_S \geq T_N\right) = m$, we assume that:

Assumption 1 (Tie-breaking)**.** *When indifferent between supporting and rejecting, an investor supports if the AoN target is possible to reach (positive probability), and rejects otherwise.*

[9] While crowdfunding in reality may involve endogenous orders of investors, our abstract and simplified setup allows us to relate and compare to the large literature on information cascades which typically has exogenous orders of investors. Louis (2011) similarly treats crowdfunding as involving exogenous priorities of investment opportunities, but instead of observing actions and learning dynamically, investors invest simultaneously under constraint of aggregate investment.

This assumption states that investors, whenever indifferent in terms of payoff consideration, supports the project if it is still possible to reach the target threshold $T_N(m)$. It is natural because the entrepreneur can always lower m by an arbitrarily small amount to induce the contribution.

Let ν be the per contribution cost for the entrepreneur. In the context of reward-based crowd-funding, this could be the production cost of each reward product. In the IPO book building process, ν can be interpreted as the issuer's share reservation value. The entrepreneur chooses price m and AoN target T_N to solve the following problem:

$$\max_{m,T_N} \pi(m, T_N, N) = E[(m-\nu)N_S \mathbb{1}_{\{N_S \geq T_N\}}], \tag{4}$$

where $\mathbb{1}_{\{N_S \geq T_N\}}$ is the indicator function of funding the project. The entrepreneur tries to maximize his expected profit from collecting contributions from investors. For simplicity we assume $\nu = 0$ in our baseline model. We revisit the case of $\nu > 0$ in section 4.

2.2 Equilibrium

We use the concept of perfect Bayesian Nash equilibrium (PBNE), which is defined as:

Definition 1. *An equilibrium consists of entrepreneur's choice of $\{m^*, T_N^*\}$, investment strategies for investors $\{A_i^*(X_i, H_{i-1}, m^*, T_N^*)\}_{i=1,2\ldots,N}$ such that:*

1. *For each investor i, given the required contribution m^* and T_N^*, associated T_N^* and other investors' investment strategies $\{A_j^*(X_j, H_{j-1}, m^*, T_N^*)\}_{j=1,2,\ldots,i-1,i+1,\ldots,N}$, investment strategy $A_i^*(X_i, H_{i-1}, m^*, T_N^*)$ solves her optimal investment problem:*

$$A_i^* \in \text{argmax } [E(V - m|X_i, H_{i-1}, N_S \geq T_N)] \mathbb{1}_{A_i=Y}; \tag{5}$$

2. *Given investment strategies* $\{A_i^*(X_i, H_{i-1}, m^*, T_N^*)\}_{i=1,2\ldots,N}$, m^* *and* T_N^* *solve entrepreneur's profit maximization problem:*

$$\{m^*, T_N^*\} \in \text{argmax} \quad \pi(m, T_N, N). \tag{6}$$

2.3 Solution

We start our analysis with the posterior dynamics. The following lemma characterizes the posterior belief given a series of signals.

Lemma 1. *Given a series of signals* $X \equiv \{X_1, X_2, \ldots, X_n\}$, *the ratio of the posterior probability of* $V = 1$ *to that of* $V = 0$ *is*

$$\frac{Pr(V=1|X)}{Pr(V=0|X)} = \frac{p^k}{q^k},$$

where $k = \#of\ H\ signals - \#of\ L\ signals.$

Proof. See the Appendix. □

Lemma 1 states that the posterior belief of project type only depends on the difference between numbers of H and L signals so far, but not on the total number of observations. This result suggests that observing one H and one L signals does not change the posterior belief. In other words, opposing H and L signals cancel each other and have no effect in forming posterior, a convenient feature also in Bikhchandani, Hirshleifer, and Welch (1992). Given Lemma 1, an investor's expected project cash-flow conditional on observing k more H signals is then,

$$E(V|k \text{ more } H \text{ signals}) = \frac{p^k}{p^k + q^k}. \tag{7}$$

It is apparent that the expected project payoff is strictly monotonically increasing in k.

When investors act regardless of their private signals, the market fails to aggregate dispersed information. Our notion of informational cascade follows the literature standard (e.g. Bikhchandani, Hirshleifer, and Welch (1992)).

Definition 2. *An information cascade occurs if a subsequent investor's action does not depend on her private information signal. An* UP *cascade occurs if a subsequent investor supports the project regardless of her private signal. A* DOWN *cascade occurs if she rejects the project regardless of her private signal.*

Notice that we have taken the convention of calling it a cascade as long as the NEXT investor ignores the private information, even though the current investor may still use private signal. This is immaterial for our theory but simplifies exposition in the proof. In standard models of informational cascades, both UP and DOWN cascades are possible. If a few early investors observe H signals, their contributions may push the posterior so high that the project remains attractive even with a private L signal. Similarly, a series of L signals may doom the offering. An early preponderance towards support or rejection causes all subsequent individuals to ignore their private signals, which thus are never reflected in the public pool of knowledge. The first main result in our paper is to show that with the AoN feature, there exists an equilibrium such that only UP cascades may exist.

Proposition 1. *There exists an equilibrium such that:*

1. *Given the investment contribution (price) $m^* \in (0,1)$, the corresponding AoN target $T_N^* \leq N$ satisfies:*

$$E(V|T_N^*, N) \leq m^* < E(V|T_N^* + 1, N), \tag{8}$$

where $E(V|x, N)$ is the posterior mean of V given there are x number of H signals out *of N observations;*

2. *Investors with signal H always support the project;*

3. *Investor i with signal L contributes if and only if:*

$$E(V|k-1 \text{ more } H \text{ signals}) \geq m^*, \tag{9}$$

where k is difference between the numbers of supporting and rejecting predecessors before investor i.

Proof. See the Appendix. □

Proposition 1 states that in the equilibrium the optimal target leaves investors no ex post regret. This result roots from the fact that any deviation from the optimal target creates friction in information aggregation and hence reduces the investment commitments. The entrepreneur also chooses the AoN target so as to leave no money on the table. If the target is set so high that investors would support even below the target, the the entrepreneur can increase the price to extract more rent.

Let $m_k \equiv E(V|k \text{ more } H \text{ signals})$. The proof for Proposition 1 suggests both the possibility and arrival time of cascades, as summarized in the following corollary.

Corollary 1. *In the equilibrium characterized in Proposition 1, there would be no DOWN cascades. If $m \in (m_{k-1}, m_k]$, an UP cascade starts whenever there are $k+1$ more investors supporting rather than rejecting.*

One can interpret UP cascades as the source of type I error in information aggregation since it may falsely accept the project when it is bad. On the other hand, DOWN cascades introduce type II error, rejecting the proposal when it is actually good. Intuitively, with the

AoN target, rejection cascades do not occur and the type II error completely disappears if the aggregated information is precise enough, because the endogenous price and AoN target always ensure good projects are financed when N is large.

Proposition 2. *As $N \to \infty$, a good project with $V = 1$ is financed almost surely with an UP-cascade.*

Proof. See the Appendix. □

To the extent that Internet crowdfunding allows entrepreneurs to reach a large population of investors, good projects are always financed. We note that with limited number of investors such as in the case of traditional intermediaries or angel investors, good projects can fail. We thus have demonstrated one key benefit of crowdfunding.

Next, we examine the informational environment in such an up-cascaded equilibrium, and its pricing implications.

3 Pricing Implication

We start our analysis by characterizing the optimal price in the standard information cascade model (without AoN) as a benchmark (most analysis from Welch (1992) but in our framework). Pricing implications of informational cascade is important because underpricing or overpricing may affect the success or failure of the issuance, resulting in an important and direct impact on the real economy. This is especially salient in the case of IPO with limited distribution channels of investment banks (Welch (1992)).

For N large enough, the complete aggregation of investors signals gives the first-best informational environment. The main friction is that it is costly to aggregate information. The key innovation of crowdfunding is then the low-cost way (through the internet) to reach out to a greater crowd. This is also a key function performed by underwriting investment banks. Our focus is therefore on cascade with and without AoN, not on the comparison between the full information benchmark and the up-cascaded equilibrium *under the same* N.

3.1 Standard Cascades without AoN Target

If there is no AoN, then for each investor, her payoffs do not depend on what later investors do. Thus, the equilibrium is essentially the same as the one characterized in Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992). That is, each investor i chooses to support if and only if

$$E(V|X_i, H_{i-1}) \geq m. \tag{10}$$

In this equilibrium, both UP and Down cascades may occur. The aggregation of public information stops once one cascade arrives. As discussed in Bikhchandani, Hirshleifer, and Welch (1992), the impact of cascades largely depends on the private information precision. If the information is precise, then cascades would not be a big concern since a cascade only occurs when the aggregated public information is sufficiently informative to dominate one's private signal, suggesting a high probability of correct cascades. When the private signal is noisy, cascades become a serious concern since a slightly more informative public pool of knowledge is enough to cause individuals to disregard their private signals. The following proposition shows that without AoN target, the contribution is under-priced when the precision of private signals is low.

Lemma 2. *The entrepreneur always charges* $m \leq p$. *When* $p \leq \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})$, *the optimal* *contribution is* $m^* = 1 - p < \frac{1}{2} = E(V)$.

Proof. See the Appendix. □

The lemma is basically a restatement of the underpricing result in Welch (1992), especially Theorem 5. The first pricing upper bound comes from the concern for potential DOWN cascades. If entrepreneur charges $m > p$, then even with a H signal, the first investor choose rejection and so does every subsequent investor, leading to a DOWN cascade starts at the very beginning, which yields 0 benefit for sure.

The second result concerns optimal pricing when the individual signal is not very precise and cascades are a relevant concern. UP and DOWN cascades, even though they both reduce the information aggregation among investors, affect the entrepreneur's profit asymmetrically. While the entrepreneur benefited from UP cascades by attracting contributions from late investors with L signals, he is concerned with DOWN cascades since a few early rejections may doom the offering. When the private information precision is low, the concern of DOWN cascades pushes down the price to the level such that given the low price the UP cascade starts at the very beginning with probability 1. Because $m^* < \mathbb{E}[V]$, the optimal pricing entails underpricing ex ante so that the first investor finds it attractive even with a L signal. To be clear, depending on the true project quality, we still have overpricing (if $V = 0$) ex post.

3.2 Pricing with AoN Target

Now we move to the optimal pricing problem with the AoN target $T_N(m)$. This is conceptually different from the optimal pricing problem in the previous section because with AoN there would be no DOWN cascade in the equilibrium. As we shown in this section, the AoN target changes both pricing upper bound and the underpricing results.

Lemma 1 and equation (7) show that the posterior only depends on the difference between numbers of H and L signals. If the price is m_{k-1}, then an UP cascade starts once there are k more H signals. Since each investor will observe either H or L signal and in the equilibrium her decision perfectly reveals her private signal before an UP cascade starts, the arrival of an UP cascade is equivalent to the first passage time of a one-dimension biased random walk. The following lemma lays the foundation for our analysis on the distribution of UP-cascades' arrival time.

Lemma 3 (Hitting Time Theorem)**.** *For a random walk starting at $k \geq 1$ with i.i.d. steps $\{Y_i\}_{i=1}^{\infty}$ satisfying $Y_i \geq -1$ almost surely, the distribution of the stopping time $\tau_0 = \inf\{n : S_n = k + \sum_{i=1}^{n} Y_i\}$ is given by*

$$Pr(\tau_0 = n) = \frac{k}{n} Pr(S_n = 0). \tag{11}$$

Proof. See Van der Hofstad and Keane (2008). □

To characterize the distribution of UP cascades arrival time, let $\varphi_{k,i}$ be the probability that an UP cascade starts at investor i, then

Lemma 4. *If the price $m \in (m_{k-2}, m_{k-1}]$, then the probability that an UP cascade starts at investor i is*

$$\varphi_{k,i} = \frac{k}{i} \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2} \tag{12}$$

where

$$\begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} = \begin{cases} \frac{i!}{\frac{i+k}{2}!\,\frac{i-k}{2}!} & \text{if } i \geq k \text{ and } k+i \text{ even;} \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Proof. See the Appendix. □

Since for any $m \in (m_{k-1}, m_k]$, all investors make the same investment decisions, the entrepreneur can always charge $m = m_k$ and receives a higher profit. Without loss of generality, we focus our pricing analysis on $m \in \{m_{-1}, m_0, \ldots, m_N\}$. We exclude cases for $k < -1$ because $m_{-1} = 1 - p$ is low enough to induce an UP cascade from the very beginning for sure.

Now we consider the optimal pricing. An UP cascade only occurs when the posterior given another L signal is higher than m, and all subsequent investors support the project. The project is eventually implemented once an UP cascade starts. On the other hand, for any agent $i \leq N - 1$, if the UP cascade has not started yet, then there is a strictly positive possibility that the project will not be implemented. So a project is eventually funded if and only if either 1) There is an UP cascade; or 2) Investor N supports the project and the total number of supporting investors is exactly T_N. In either cases, we can compute the profit associated with m, as formalized in Proposition 3. But before going there, we illustrate the two scenarios in Figure 1, which plots the difference between supporting investors and rejecting investors when n investors have arrived. The figure also includes a sample path that leads to funding failure because AoN target is not reached.

Proposition 3. *When the price is $m = m_{-1} = 1 - p$, the entrepreneur's expected profit is $(1 - p)N$. More generally, given a price $m = m_{k-1}$, $k \in \{1, 2, \ldots, N\}$, the entrepreneur's expected profit is*

$$\pi(m_{k-1}, N) = \begin{cases} m_{k-1} \displaystyle\sum_{i=k,k+2\ldots}^{N} \varphi_{k,i}(N - \tfrac{i-k}{2}) & \text{if } k+N \text{ even;} \\ m_{k-1} \left[\displaystyle\sum_{i=k,k+2\ldots}^{N-1} \varphi_{k,i}(N - \tfrac{i-k}{2}) + \frac{1}{p}\varphi_{k,N+1}(N - \tfrac{N-k}{2}) \right] & \text{if } k+N \text{ odd.} \end{cases} \quad (14)$$

For each $k \in \{0, 1, 2, \ldots\}$, there exists a finite positive integer $\underline{N}(k)$ such that for $\forall\ N \geq$

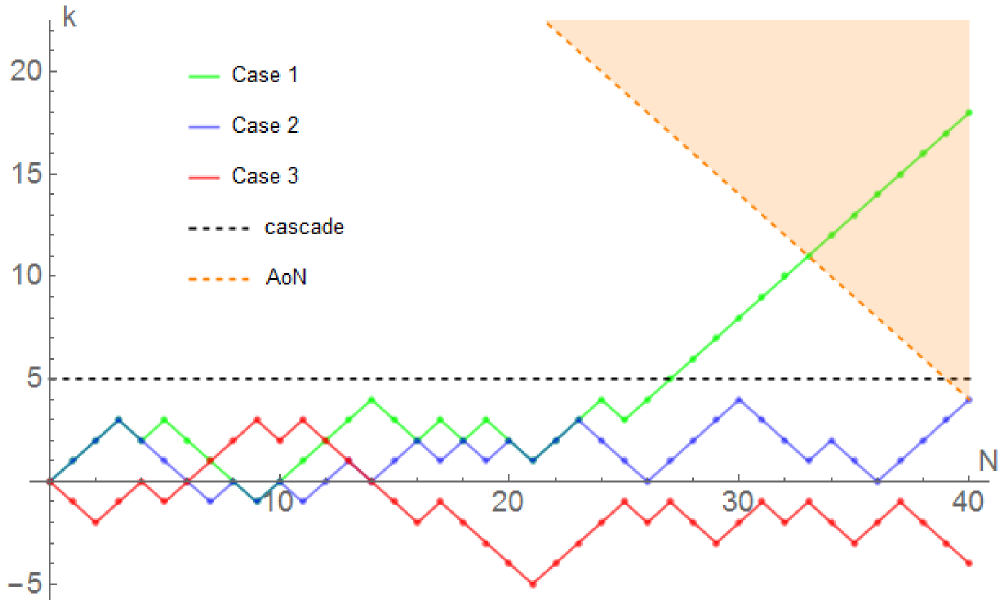

Figure 1: Evolution of Support-Reject Differential

Simulated paths for $N = 40$, $p = 0.7$, $m^* = m_5 = 0.9673$, and AoN target $T^*(N) = 22$. Case 1 indicates a path that crosses the cascade trigger $k = 5$ at the 26th investor and all subsequent investors support regardless of their private signal; case 2 indicates a path with no cascade, but the project is still funded by the end of the fundraising; case 3 indicates a path where AoN target is not reached and the project is not funded. The orange shaded region above the AoN line indicates that the project is funded.

$\underline{N}(k)$, $\pi(m_k, N) > \pi(m_{k-1}, N)$.

Proof. See the Appendix. □

Proposition 3 gives an explicit characterization of entrepreneur's expected profit as a function of price m_k and number of potential investors N. Figure 2 provides an illustration on how the profit depends on m.

More importantly, the result on $\underline{N}(k)$ suggests that, different from Lemma 2, the optimal price depends on the number of potential investors N. In the standard cascades models, a DOWN cascade hurts the entrepreneur significantly because subsequent investors all reject. The concern for DOWN cascades pushes down the optimal price, and can cause immediate start of an UP cascade, *independent of the number of investors* because the decisions of later investors have no impact on the first investor's payoffs (Welch (1992)). With the AoN target,

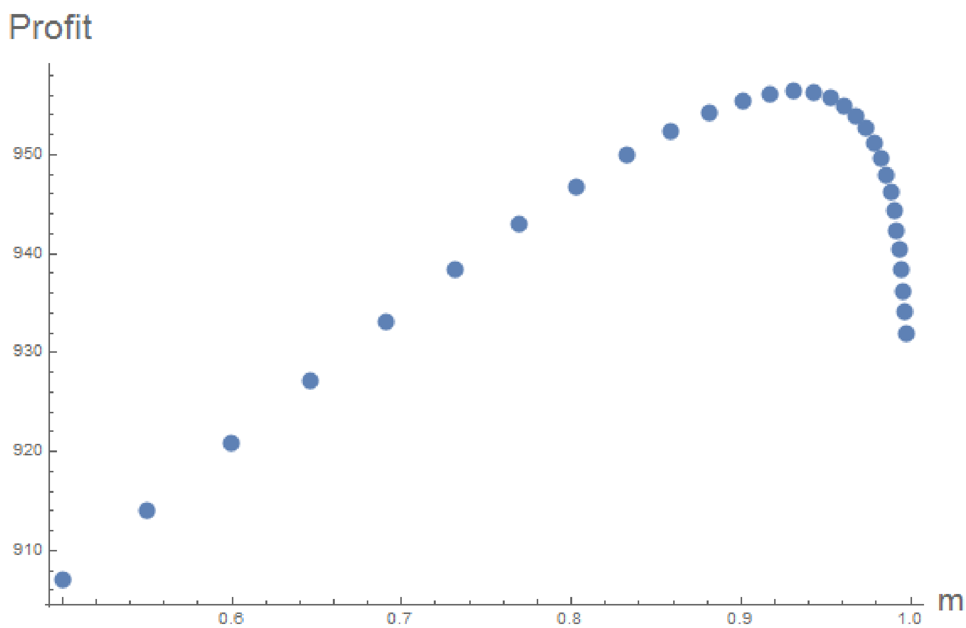

Figure 2: Optimal Pricing: An Illustration with $N = 2000$ and $p = 0.55$.

in the equilibrium there would be no DOWN cascades and one early rejection is not a big concern since all investors with H signals would still support the project. Those supporting investors may trigger an UP cascade later, especially when there are many potential investors in the market. The following corollary shows the increasing trend of optimal price m^* as the number of potential investors N grows.

Corollary 2. *For* $\forall\ m_k$*, there exists a a finite positive integer* $N_\pi(m_k)$ *such that for* $\forall\ N \geq N_\pi(m_k)$*,* $m^* > m_k$*.*

Proof. Let $N_\pi(m_k) = \max\{\underline{N}(0), \underline{N}(1), \ldots, \underline{N}(k), \underline{N}(k+1)\}$. Then for $\forall\ N \geq N_\pi(m_k)$, $\pi(m_{k+1}, N) > \pi(m_k, N) > \cdots > \pi(m_{-1}, N)$. So $m^* \geq m_{k+1} > m_k$. □

This corollary has two implications novel to the literature: first, as we reach out to more and more investors through technological innovations such as the Internet, the entrepreneur can charge a higher price; second, there would be less underpricing but more overpricing as N becomes big. The left panel in Figure 3 shows the optimal starting point of UP cascades

(kth investor) when N differs, and right panel plots the optimal pricing as a function of N. We note that $m > \mathbb{E}[V]$ in these cases.

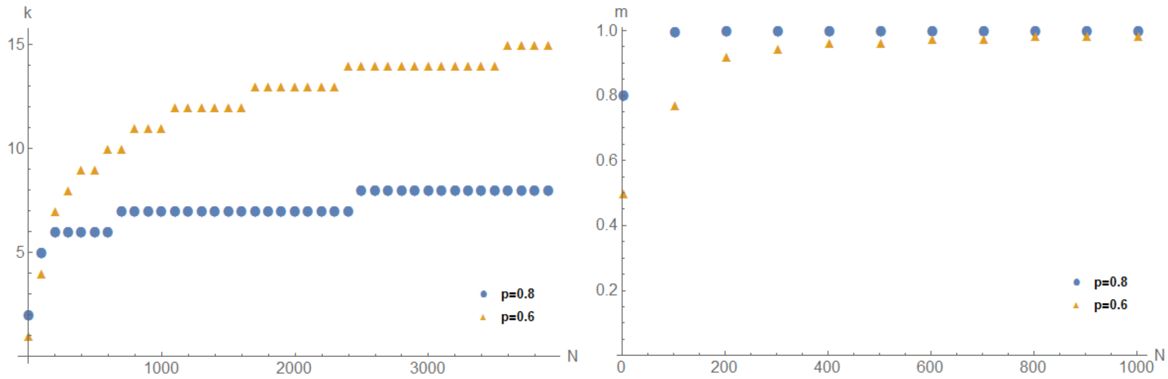

Figure 3: Cascades and optimal prices as N increases

Since for any finite integer $N \geq 2$, $m^*(N) \in \{-1, 0, 1, \ldots, N\}$. Corollary 2 implies that m^* shows an increasing trend. Since m_k is a monotonic increasing function in k and $\lim_{k\uparrow\infty} m_k = 1$, it is straightforward to see that

Corollary 3. $\lim_{N\to\infty} m^*(N) = 1$

That is to say, when there is a large base of potential investors, the optimal price approaches the highest possible value, leading to overpricing rather than the underpricing found in IPOs when N is relatively small (Welch (1992)).

4 Wisdom of the Crowd

This section discusses how AoN scheme fundamentally changes the feasibility of harnessing the wisdom of the crowd, and the resulting informational environment. We also allow the entrepreneur to carry out the project even if the target is missed, or to give up the project even if the target is met.

4.1 Feasibility of Fundraising and Information Aggregation

From Lemma 2, we see that there is a pricing upper bound in order for the fundraising or offering to be feasible. This bound becomes a serious concern when the cost ν is non-zero. In particular, when ν is too high, traditional cascade models predict a failure (rejection cascade for sure) while in our model the entrepreneur can still charge a high price and is able to implement the project when aggregated information is good.

Proposition 4. *Without AoN, no project with $\nu > p$ is financed and information aggregation is infeasible; committing to an AoN target enables fundraising and information aggregation even when $\nu > p$.*

Because of DOWN cascades, investors certainly do not finance any project with $\nu > p$. In such cases, not only do we fail to raise financing, there is also no way the entrepreneur can harness the wisdom of the crowd because no information is aggregated. This result roots from the fact that the concern for DOWN cascades imposes an upper bound on possible prices, and any project with a high cost will charge a high price and thus triggers a DOWN cascade and financing failure for sure.

The exclusion of DOWN cascades therefore has an important impact on the pricing upper bound, and hence the availability of finance. With AoN target, any price $m < 1$ is possible and there would be a strictly positive possibility that the project would be financed given there is a large enough potential investor base. Moreover, from Proposition 2 we know that the good type of project ($V = 1$) will be financed almost surely as the number of investors goes to infinity. In this sense, AoN target drives the discrete jump in financing and information aggregation feasibility.

4.2 Harnessing the Wisdom

Even when the fundraising is feasible, it serves little for information aggregation in most extant models of information cascade. For example, in Welch (1992), cascade always starts from the very beginning, and no private signals are aggregated because once a cascade starts, public information stops accumulating. Nor does the public pool of knowledge have to be very informative to cause individuals to disregard their private signals. As soon as the public pool becomes slightly more informative than the signal of a single individual, individuals defer to the actions of predecessors and a cascade begins.

With AoN target, however, the downside risk is removed, and optimal pricing does not necessarily result in information cascades from the very beginning (Lemma 4). Therefore, as long as $m^* > 1 - p$, the fundraising also aggregates some private information from the investors, allowing us to harness the wisdom of the crowd to some extent.

What is more, from Lemma 4, the probability that a cascade is correct (UP cascade when $V = 1$) is given by

$$Pr(V = 1 | cascade\ at\ i^{th}\ investor) = \frac{p^k}{p^k + q^k} \mathbb{I}_{\{i \geq k \& k+i\ is\ even\}}$$

where k satisfies $m_{k-1} < m \leq m_{k-1}$. Because k is weakly increasing in the pricing m and the optimal pricing is weakly increasing in N (Proposition 3), the following proposition ensues.

Proposition 5. *A cascade starts weakly later with higher pricing m, and thus with a larger crowd (larger N) when pricing is endogenous. The probability of a cascade being correct is increasing in p, weakly increasing in the pricing m, and weakly increasing in N when pricing is endogenous.*

AoN reduces underpricing, which in turn delays cascade and increases the probability of correct cascades. More importantly, whereas N does not matter in standard cascade models, AoN links the timing and correctness of cascades to the size of the crowd. With a large N as is the case for Internet-based crowdfunding, information cascades has a less detrimental effect, allowing better harnessing of the wisdom of the crowd.

Uni-directional cascade also means that offerings in the cascade model can fail whereas in the baseline in Welch (1992), offerings never fail. This would help us explain why some offerings fail occasionally and/or are withdrawn, without invoking insider information as Welch (1992) did in his model extension. By allowing some projects , which are mostly bad projects when N is large (Proposition 2), we put the wisdom of the crowd to use to increase social welfare.

It should be noted that our findings complement rather than contradict those in Brown and Davies (2017). In their setup, investors bid more aggressively because the project is only implemented when the total investment reaches an exogenously given AoN target, leading to "loser's blessing" and failures of aggregating information from the crowd, relative to standard auction benchmarks. We focus on sequential investments in the presence of dynamic observational learning, and the gains in informational and financing efficiency are all benchmarked to standard settings outlined in Section 3.1.

4.3 Entrepreneur's Real Option

So far in our analysis we have required the entrepreneur to implement the project according to the AoN target. In some cases in reality, especially when the entrepreneur also learns about the project's promise from crowdfunding (not knowing the true V in our model), he commits to AoN in fundraising, but still holds the real option on how to use the capital and information aggregated. For example, an entrepreneur successful on Kickstarter or Indigogo can still decide on the scale of the project and how much effort to put into developing the product. On some crowdfunding platforms, the entrepreneur can decide whether to use the capital raised explicitly or implicitly (by postponing product development indefinitely, which results in refunding the investors). Xu (2017) and Viotto da Cruz (2016) provide strong empirical evidence that the entrepreneur indeed use the information aggregated from crowdfunding platforms for real decisions.

The real option embedded in the eventual investment often comes from the fact that crowdfunding is one way to learn about aggregate demand, which is obvious in reward-based platforms. Even for equity-based crowdfunding, investors reveals information on future product demand and profit.

Similarly, in IPOs, firms successful at book-building may still occasionally withdraw and those unsuccessful may still find alternative sources of public financing. An IPO's initial pricing and trading also generates valuable information and feedback for managers. For example, van Bommel (2002) and Corwin and Schultz (2005) discuss information production at IPO through choices on pricing and underwriting syndicates.

In our baseline model, the entrepreneur's investment marginal cost ν is largely muted. One could imagine that ν is significant or there is also a fixed cost of investment for the entrepreneur. There could also be additional benefit to carrying out the project, such as the entrepreneur's private benefit of control or empire building. These forces distort the entrepreneur's ex post incentive on whether and how to implement the project. Other factors such as marketing, network effect, etc. also play a role. Given that the eventual scale of the project matters,[10] cascade can also serve as a device for coordination conditional on the project's being good.

Specifically, V can be interpreted as a transformation of the aggregate demand, which

[10] Section E in Welch (1992) considers locally increasing returns to scale.

could be high ($V = 1$) or low ($V = 0$). Suppose that after the crowdfunding, the entrepreneur considers commercialization or abandoning the project (upon crowdfunding failure), and for simplicity the commercialization or continuation decision pays V (after normalization), but incurs an effort or reputation or monetary cost represented in reduced-form by I. Then the entrepreneur's expected payoff for the real option is

$$\max \{\mathbb{E}[V - I|H_N], 0\} \tag{15}$$

recall H_N is the entire crowdfunding history, including information on the total number of supports out of N investors, and when an UP-cascade starts if there is one, etc. Even with a successful crowdfunding, the entrepreneur may still choose to forgo commercialization if his belief on V after crowdfunding is not sufficiently optimistic; likewise, despite crowdfunding failure, the entrepreneur may continue pursuing the project. In fact, Xu (2017) documents in a survey of 262 unfunded Kickstarter entrepreneurs that after failing, 33% continued as planned. He also finds that a 50% increase in pledged amount leads to a 9% increase in the probability of commercialization outside the crowdfunding platform.

Consistent with Xu (2017), one can show that the posterior of V is increasing in the equilibrium crowdfund raised. It would be interesting to understand how the entrepreneur designs AoN and pricing to not only maximize profit from the crowdfunding, but also increase the real option value, which constitutes interesting future work.

5 Conclusion

Financial processes such as crowdfunding and IPO underwriting involve aggregating information from diverse investors, sequential sales, observational learning, and most interestingly, all-or-nothing (AoN) rules that contingent the financing upon achieving certain fundraising targets. We incorporate these features into a classical model of information cascade, and find that AoN leads to uni-directional cascades in which investors rationally ignore private signals and imitate preceding investors only if the preceding investors decide to *invest*. Consequently, an entrepreneur prices issuance more aggressively, and fundraising may succeed rapidly but never fails rapidly. Information production also becomes more efficient, especially with a large crowd of investors, yielding more probable financing of good projects, and the weeding-outs of bad projects that are absent in earlier models. More generally, endogenous pricing with AoN targets leads to greater financing feasibility and better harnessing of the wisdom of the crowd under informational frictions.

References

Abrams, Eliot, 2017, Securities crowdfunding: More than family, friends, and fools?, *Working Paper*.

Agrawal, Ajay, Christian Catalini, and Avi Goldfarb, 2014, Some simple economics of crowdfunding, *Innovation Policy and the Economy* 14, 63–97.

Agrawal, Ajay K, Christian Catalini, and Avi Goldfarb, 2011, The geography of crowdfunding, Discussion paper, National bureau of economic research.

Anderson, Lisa R, and Charles A Holt, 1997, Information cascades in the laboratory, *The American Economic Review* pp. 847–862.

Bagnoli, Mark, and Barton L Lipman, 1989, Provision of public goods: Fully implementing the core through private contributions, *The Review of Economic Studies* 56, 583–601.

Belleflamme, Paul, Thomas Lambert, and Armin Schwienbacher, 2014, Crowdfunding: Tapping the right crowd, *Journal of business venturing* 29, 585–609.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch, 1992, A theory of fads, fashion, custom, and cultural change as informational cascades, *Journal of Political Economy* 100, 992–1026.

——— , 1998, Learning from the behavior of others: Conformity, fads, and informational cascades, *The Journal of Economic Perspectives* 12, 151–170.

Bond, Philip, Alex Edmans, and Itay Goldstein, 2012, The real effects of financial markets, *Annu. Rev. Financ. Econ.* 4, 339–360.

Bond, Philip, and Itay Goldstein, 2015, Government intervention and information aggregation by prices, *The Journal of Finance* 70, 2777–2812.

Brown, David C, and Shaun William Davies, 2017, Financing efficiency of securities-based crowdfunding, *Working Paper*.

Burtch, Gordon, Anindya Ghose, and Sunil Wattal, 2013, An empirical examination of the antecedents and consequences of contribution patterns in crowd-funded markets, *Information Systems Research* 24, 499–519.

Çelen, Boğaçhan, and Shachar Kariv, 2004, Distinguishing informational cascades from herd behavior in the laboratory, *The American Economic Review* 94, 484–498.

Chamley, Christophe, 2004, *Rational herds: Economic models of social learning* (Cambridge University Press).

Chang, Jen-Wen, 2016, The economics of crowdfunding, *Working Paper*.

Chemla, Gilles, and Katrin Tinn, 2016, Learning through crowdfunding, .

Corwin, Shane A, and Paul Schultz, 2005, The role of ipo underwriting syndicates: Pricing, information production, and underwriter competition, *The Journal of Finance* 60, 443–486.

Cumming, Douglas J, Gaël Leboeuf, and Armin Schwienbacher, 2014, Crowdfunding models: Keep-it-all vs. all-or-nothing, in *Paris December 2014 finance meeting EUROFIDAI-AFFI paper* vol. 10.

Ellman, Matthew, and Sjaak Hurkens, 2015, Optimal crowdfunding design, .

Feller, William, 1968, *An Introduction to Probability Theory and Its Applications* . vol. 1 (John Wiley and Sons).

Gompers, Paul, William Gornall, Steven N Kaplan, and Ilya A Strebulaev, 2016, How do venture capitalists make decisions?, Discussion paper, National Bureau of Economic Research.

Gompers, Paul Alan, and Joshua Lerner, 2004, *The venture capital cycle* (MIT press).

Guarino, Antonio, Heike Harmgart, and Steffen Huck, 2011, Aggregate information cascades, *Games and Economic Behavior* 73, 167–185.

Hakenes, Hendrik, and Friederike Schlegel, 2014, Exploiting the financial wisdom of the crowd–crowdfunding as a tool to aggregate vague information, *Working Paper*.

Hayek, Friedrich August, 1945, The use of knowledge in society, *The American Economic Review* pp. 519–530.

Herrera, Helios, and Johannes Hörner, 2013, Biased social learning, *Games and Economic Behavior* 80, 131–146.

Hung, Angela A, and Charles R Plott, 2001, Information cascades: Replication and an extension to majority rule and conformity-rewarding institutions, *The American Economic Review* 91, 1508–1520.

Kerr, William R, Ramana Nanda, and Matthew Rhodes-Kropf, 2014, Entrepreneurship as experimentation, *The Journal of Economic Perspectives* 28, 25–48.

Lau, Jonathan, 2013, Dollar for dollar raised, kickstarter dominates indiegogo six times over, *http://medium.com/p/2a48bc6ffd57* August 28.

——— , 2015, The ultimate guide to crowdfunding, *https://www.shopify.com/guides/crowdfunding/crowdfunding-infographic* September 30.

Li, Jiasun, 2017, Profit sharing: A contracting solution to harness the wisdom of the crowd, *Working Paper*.

Louis, Philippos, 2011, Standing in line: Demand for investment opportunities with exogenous priorities, Discussion paper, Mimeo.

Manso, Gustavo, 2016, Experimentation and the returns to entrepreneurship, *The Review of Financial Studies* 29, 2319–2340.

Milgrom, Paul R, and Robert J Weber, 1982, A theory of auctions and competitive bidding, *Econometrica: Journal of the Econometric Society* pp. 1089–1122.

Mollick, Ethan, and Ramana Nanda, 2015, Wisdom or madness? comparing crowds with expert evaluation in funding the arts, *Management Science* 62, 1533–1553.

Mollick, Ethan R, and Venkat Kuppuswamy, 2014, After the campaign: Outcomes of crowdfunding, .

Ritter, Jay R, and Ivo Welch, 2002, A review of ipo activity, pricing, and allocations, *The Journal of Finance* 57, 1795–1828.

Scharfstein, David S, and Jeremy C Stein, 1990, Herd behavior and investment, *The American Economic Review* pp. 465–479.

Strausz, Roland, 2017, A theory of crowdfunding: A mechanism design approach with demand uncertainty and moral hazard, *American Economic Review* 107, 1430–76.

van Bommel, Jos, 2002, Messages from market to management: The case of ipos, *Journal of Corporate Finance* 8, 123–138.

Van der Hofstad, Remco, and Michael Keane, 2008, An elementary proof of the hitting time theorem, *The American Mathematical Monthly* 115, 753–756.

Viotto da Cruz, Jordana, 2016, Beyond financing: crowdfunding as an informational mechanism, .

Welch, Ivo, 1992, Sequential sales, learning, and cascades, *Journal of Finance* 47, 695–732.

Xu, Ting, 2017, Learning from the crowd: The feedback value of crowdfunding, *Working Paper*.

Zhang, Juanjuan, and Peng Liu, 2012, Rational herding in microloan markets, *Management Science* 58, 892–912.

Appendix

A Crowdfunding Platforms

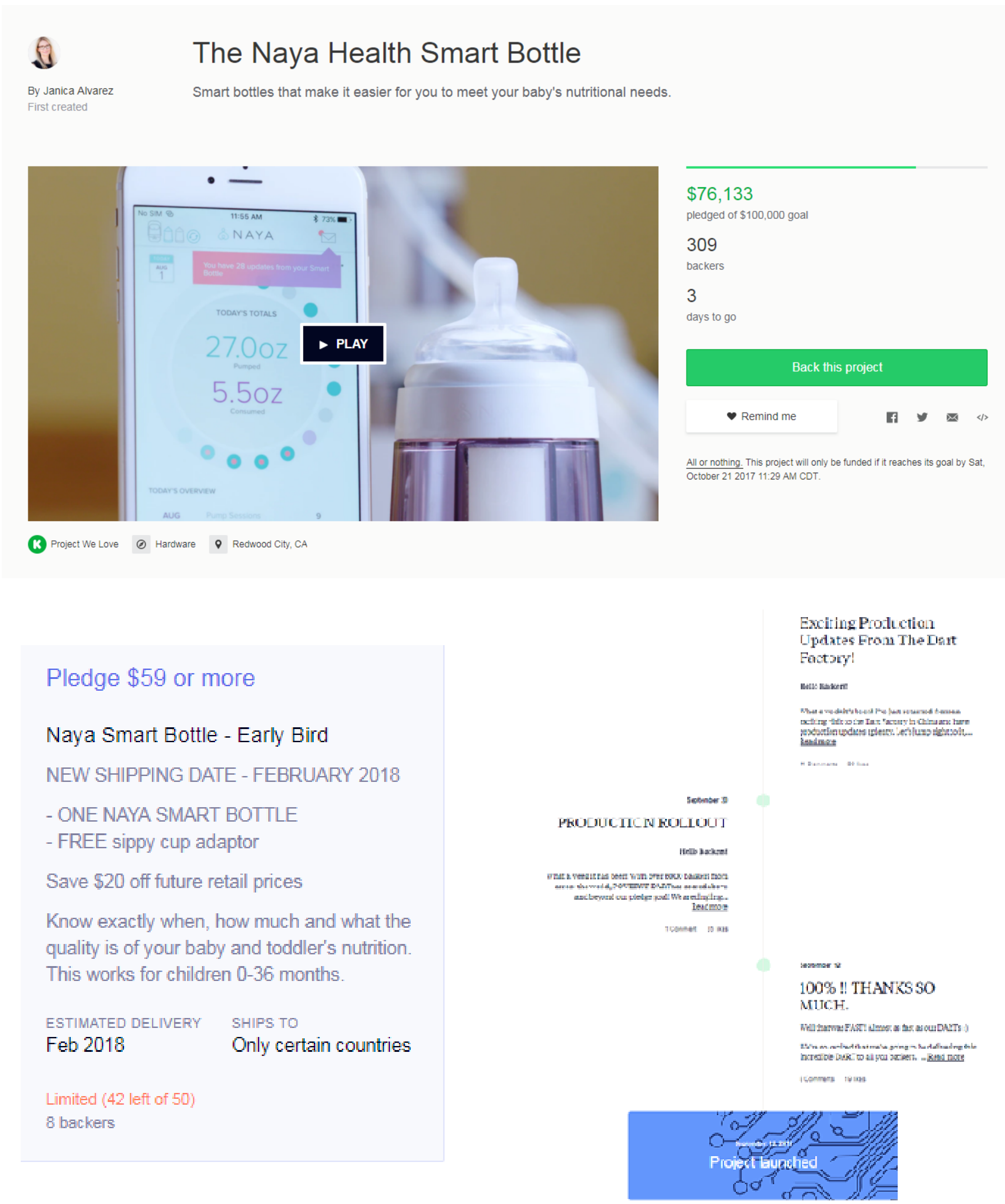

Figure 4: Example One: Kickstarter

Aside from all the details about the product, investor observes the target amount, fundraising start and end dates, pledged amount to date, and number of backers. They can also see a timeline of updates to the project (when it starts, factory production progress, etc.)

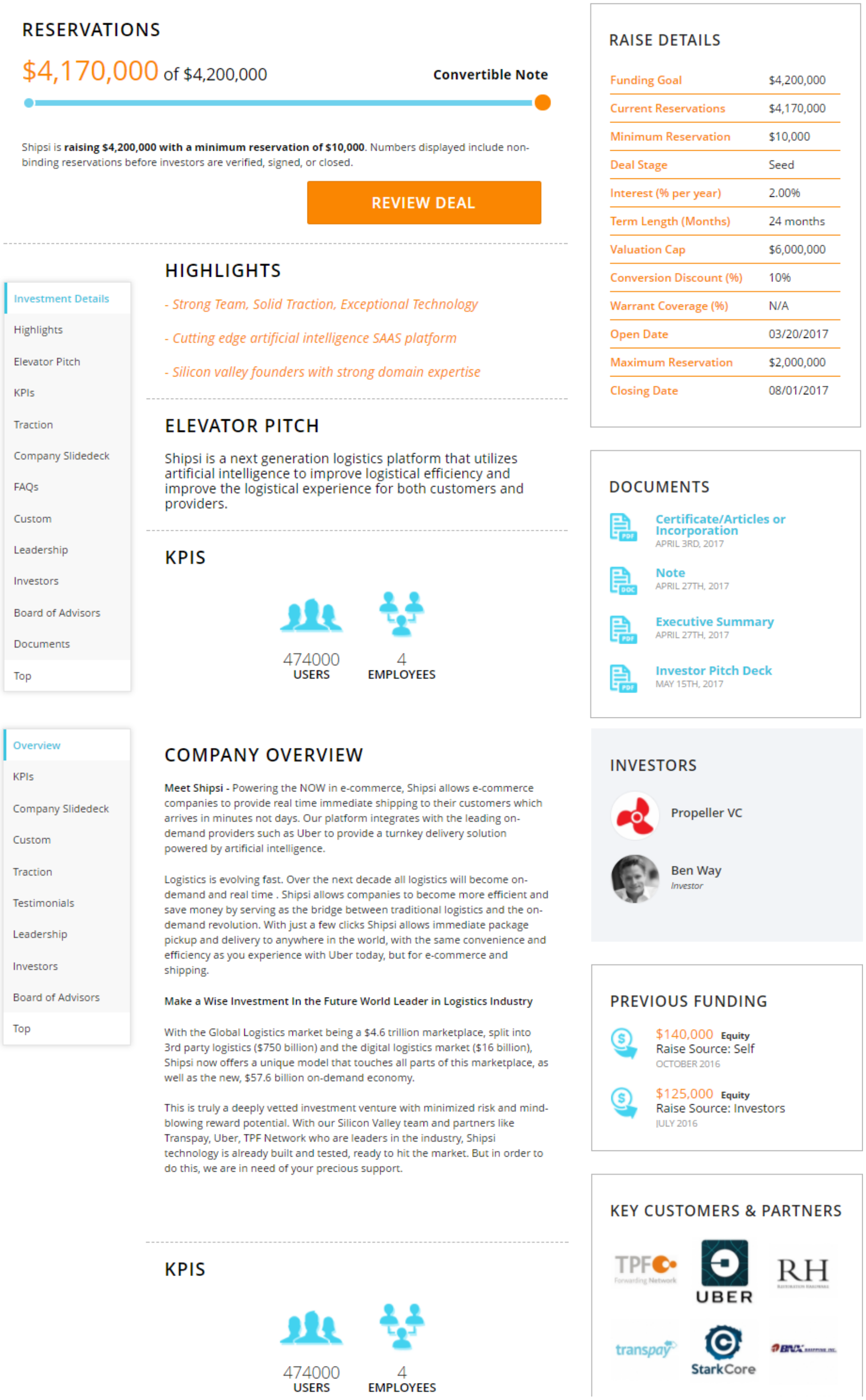

Figure 5: Example Two: Crowdfunder

Aside from all the details about the company including the equity investment contract, the companys previous funding, key customers and partners, and existing investors (only the VCs and the big players), investors also observe the target amount, fundraising start and end dates, reservation amount to date, etc.

B Derivations and Proofs

Proof of Lemma 1

Proof. Let k_n be the difference of numbers of H and L signals till the nth observation. For the prior, $k_0 = 0$, and $\frac{Pr(V=1)}{Pr(V=0)} = \frac{0.5}{0.5} = \frac{p^0}{q^0}$.

Suppose $\frac{Pr(V=1)|X}{Pr(V=0|X)} = \frac{p^{k_n}}{q^{k_n}}$ holds for $n \geq 0$, then for $n+1$:

1. If $X_{n+1} = H$, then $k_{n+1} = k_n + 1$, and

$$\begin{aligned}\frac{Pr(V=1|X)}{Pr(V=0|X)} &= \frac{\frac{Pr(X_{n+1}=H|V=1)Pr(V=1|X_1,X_2,...,X_n)}{Pr(X_{n+1}=H)}}{\frac{Pr(X_{n+1}=H|V=0)Pr(V=0|X_1,X_2,...,X_n)}{Pr(X_{n+1}=H)}}\\ &= \frac{Pr(X_{n+1}=H|V=1)p^{k_n}}{Pr(X_{n+1}=H|V=0)p^{k_n}}\\ &= \frac{p^{k_{n+1}}}{q^{k_{n+1}}};\end{aligned}$$

2. Similarly, if $X_{n+1} = L$, then $k_{n+1} = k_n - 1$, and

$$\begin{aligned}\frac{Pr(V=1|X)}{Pr(V=0|X)} &= \frac{\frac{Pr(X_{n+1}=L|V=1)Pr(V=1|X_1,X_2,...,X_n)}{Pr(X_{n+1}=L)}}{\frac{Pr(X_{n+1}=L|V=0)Pr(V=0|X_1,X_2,...,X_n)}{Pr(X_{n+1}=L)}}\\ &= \frac{Pr(X_{n+1}=L|V=1)p^{k_n}}{Pr(X_{n+1}=L|V=0)p^{k_n}}\\ &= \frac{p^{k_{n+1}}}{q^{k_{n+1}}};\end{aligned}$$

So $\frac{Pr(V=1|X)}{Pr(V=0|X)} = \frac{p^{k_{n+1}}}{q^{k_{n+1}}}$ holds for $n+1$. The lemma follows by induction. □

Proof of Proposition 1

Proof. The proof proceeds in two steps. We first show that the investment strategies in Proposition 1 is a sub-game equilibrium for a chosen price m^* and the corresponding AoN target T_N^* characterized by Equation (8). We then show that for any possible m^* in the equilibrium, the corresponding AoN target T_N^* characterized by Equation (8) is optimal.

Step 1: Investor strategy with given AoN target

Given the price m^* and the corresponding AoN target T_N^* characterized by Equation (8). Let k_m be the minimal difference of numbers of H and L signals so that $E(V|k_m \text{ more } H \text{ signals}) \geq m^*$. Without loss of generality we only consider the cases when $k_m \leq N$. It is straightforward to see that $T_N^* - (N - T_N^*) = k_m$ when $k_m + N$ is even and $T_N^* - (N - T_N^*) = k_m - 1$ when $k_m + N$ is odd. An UP cascade starts once there are $k_m + 1$ more H signals, because even if the investor currently making decision has private signal L, overall there are still k_m more H signals, and the weakly undominated strategy is to support. When there is an UP cascade, because all subsequent investors would support, there must be more than T_N^* investors supporting the project and the AoN target is reached. If there is no UP cascade, the project will also be implemented when there are exactly T_N^* more supporting investors (if there are more than T_N^* supporting investors, then an UP cascade (recall our definition) starts at the latest at investor N).

Now consider investor $i \in \{1, 2, \ldots, N\}$, given investment strategies of other investors, if there is already an UP cascade before her turn, then the project will be implemented for sure and the conditional expected payoff given her private signal is weakly higher than $E(V|k_m$ more H signals$) \geq m^*$. Her optimal decision is to support regardless of her private signal.

If there is no UP cascade yet and she chooses to support with a private H observation, then the project is implemented either when there is an UP cascade later or when there is no cascade but exactly T_N^* supporting investors.

1. When $k_m + N$ is even, then with an UP cascade, the expected payoff is $E(V|k_m + 1$ more H signals$) - m^* > 0$. If there is no cascade later but exactly T_N^* supporting investors, the conditional expected payoff given her private signal is $E(V|T_N^*, N) - m^* \geq 0$.
2. When $k_m + N$ is odd, similar to the even case, the expected payoff is $E(V|k_m + 1$ more H signals$) - m^* > 0$ if an UP cascade arrives before investor N. Now consider the scenario when no UP cascade has yet arrived at investor $N - 1$ and the project is implemented in the end. Since $T_N^* - (N - 1 - T_N^*) = k_m - 1 + 1 = k_m$, if there are less than T_N^* supporting investors until investor $N - 1$, then there are at most $k_m - 2$ more supporting investors and the project would not be implemented for sure. If there are more than T_N^* supporting investors, then there must be an UP cascade, a contradiction. Moreover, if investor $N - 1$ chooses rejection, then there must be $k_m + 1$ more supporting investors at investor $N - 2$, suggesting an UP cascade. If no UP cascade has yet arrived at investor $N - 1$ and the project is implemented in the end, then it must be the case that investor $N - 1$ supports and there are exactly T_N^* supporting decisions from the first $N - 1$ investors. Conditional expected payoff given her private signal is $E(V|k_m$ more H signals$) - m^* \geq 0$. For investor N, of course she supports if and only if her private signal is good.

Therefore it is optimal to support.

If there is no UP cascade yet and $E(V|k_m - 1$ more H signals$) < m^*$, and consider a deviation that she chooses to support with a private L observation, then the project is implemented either when 1) There is an Up cascade later or 2) There are exactly T_N^* supporting investors. In the first case, others interpret her action as having H signal on equilibrium, the conditional expected project payoff given her private signal is $E(V|k_m + 1$ more supporting investors$) = E(V|k_m - 1$ more H signals$) < m^*$, where the first expression is the point of cascade start, and $E(V|k_m - 1$ more H signals$)$ is the true value because the fact that she has L means there are two less H than what others perceive in equilibrium. This is not a profitable action. Similarly, in the second case, the conditional expected payoff given her private signal is $E(V|T_N^* - 1, N) - m^* < 0$. Therefore it is optimal to reject instead.

Step 2: Optimal AoN target

Notice that $m^* \geq E(V|1$ less H signal$)$, since $m^* = E(V|1$ less H signal$)$ guarantees an UP cascade from the very beginning and thus strictly dominates any $m < E(V|1$ less H signal$)$. Let $T_N(m^*)$ be the target that satisfies $E(V|T_N(m^*), N) \leq m^* < E(V|T_N(m^*) + 1, N)$.

When $m^* = E(V|1$ less H signal$)$, the UP cascade starts from the first investor for sure, so any AoN target $T_N^* \leq N$ is optimal.

When $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* = T_N(m^*)$: Following the proof in step 1, the project will be implemented whenever there is an UP cascade (that is to say, at some investors there are $k_m + 1$ more H signals), or when no UP cascades occur and there are exactly T_N^* supporting investors in total.

When $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* > T_N(m^*)$: Since $T_N^* \geq T_N(m^*) + 1$, $T_N^* - (N - T_N^*) \geq T_N(m^*) + 1 - (N - T_N(m^*) - 1) \geq k_m + 1$. Suppose all investors choose the same investment strategies discussed in step 1. Because there would be an UP cascade once there are $k_m + 1$ more supporting investors, if there are no less than T_N^* supporting investors in total, then there would always be an UP cascade. That is to say, the project will be financed only when there is an UP cascade and the total number of supporting investors is at least T_N^*. Similar to the discussion in step 1, it is optimal for investors to support once an UP cascade starts. If there is no cascade yet and investor i chooses support, then the conditional expected project payoff given her private H signal is at least $E(V|k_m + 1 \text{ more supporting investors}) = E(V|k_m + 1 \text{ more } H \text{ signals}) > m^*$, and the conditional expected project payoff given her private L signal is $E(V|k_m + 1 \text{ more supporting investors}) = E(V|k_m - 1 \text{ more } H \text{ signals}) < m^*$. So investor i finds it optimal to choose support with a H observation and rejection with a L signal. Now consider the deviation to the same m^* but $T_N^* = T_N(m^*)$, each of the financing success scenario with UP cascades is still there, but the entrepreneur's strategy now strictly dominates $T_N^* > T_N(m^*)$ because it yields positive profit absent UP cascade but with exactly $T_N(m^*)$ supporting investors.

Finally, we have the case of $m^* > E(V|1 \text{ less } H \text{ signal})$, and $T_N^* < T_N(m^*)$, which is further divided into two sub-cases:

1. $m^* > E(V|0 \text{ more } H \text{ signals})$, and $T_N^* < T_N(m^*)$:

   Similar to previous discussions, the project is implemented whenever there is an UP cascade before the T_N^*th supporting decision. And investors choose the same investment decisions before the total number of supporting investors reaches T_N^*. For the pivotal investor who makes the T_N^*th support decision and all subsequent investors, because the project is implemented for sure if they invest, we are back to the standard cascade models without AoN. Now we show that any $T_N^* < T_N(m^*)$ is dominated by $T_N^* + 1$, using the following arguments.

   (a) For any $T_N^* < T_N(m^*)$, let investor i be the T_N^*th investor observing a good signal. Then she invests only if there are at least $k_m - 1$ more supporting actions than rejection actions before her. If there are at least k_m more supporting actions than rejection actions before investor i, then there must already be an UP cascade before the next investor, and thus the project can also be implemented with $T_N^* + 1$ as the AoN target (note $T_N^* < N$) because the entrepreneur profit is the same. Without loss of generality, we focus on the paths when there are exactly k_m more H signals at investor i with an UP cascade.

   (b) Given equation (8) and $T_N^* < T_N(m^*)$, it must be $i \leq N - 2$ by the definition of $T_N(m^*)$. Consider investor $i + 1$, if she observes a good signal, then in both T_N^* and $T_N^* + 1$ cases she chooses support and the project would be implemented, and entrepreneur receives the same profit. If investor $i + 1$ observes signal L, she rejects in both cases because including her own signal, there are only $k_m - 1$ more H signals. Conditional on the rejection at investor $i + 1$, investor $i + 2$ chooses support if and only if her signal is good in both cases. For the support case the project would also be implemented with $T_N^* + 1$ target and entrepreneur receives the same profit. However, when investor $i + 2$ chooses rejection, it becomes a DOWN cascade and the project would be abandoned for sure. When there are $k_m - 1$ more good signals before investor i, and investors i, $i+1$ and $i+2$ observe H, L and L, respectively, we call this path HLL, and it is the only path along which T_N^* target dominates $T_N^* + 1$ target.

(c) To show $T_N^* + 1$ target dominates T_N^* in expectation, it suffices to consider the path LHH, which refers to the scenario that there are $k_m - 1$ more good signals before investor i, investors i, $i+1$ and $i+2$ observe L, H and H, respectively. With $T_N^* + 1$, this path meets AoN target, because investor $i+1$ with H signal invests, knowing that she only needs to pay if investor $i+2$, or a subsequent investor also has H signal and supports; with T_N^*, investor $i+1$ rejects even with H because she is the T_N^*th investor and she has to pay if she supports, yet there are only $k_m - 1$ more H signals (including hers). Conditional on there are $k_m - 1$ more good signals before investor i, when $k_m \geq 1$, the probability of LHH case is larger than the probability of HLL. Also notice that HLL suggests a DOWN cascade at investor $i+2$, so the expected profit of LHH is higher than the expected profit of HLL.

2. $m^* = E(V|0 \text{ more } H \text{ signals})$ (which equals $\frac{1}{2}$ in our setup) and $T_N^* < T_N(m^*)$:

Similar to the $m^* > E(V|0 \text{ more } H \text{ signals})$ case, T_N^* target only dominates $T_N^* + 1$ target along the HLL path. Let $Q_{T_N^*}$ be the event that there is no UP cascade yet and at the T_N^*th supporting investor there are exactly equal numbers of supporting and rejecting investors (that is to say, the T_N^*th supporting investor is the $2T_N^*$th investor). Let $U_{2T_N^*+1}$ be the event that the UP cascade arrives at the $2T_N^*+1$th investor. Event $U_{2T_N^*+1}$ happens if and only if $Q_{T_N^*}$ happens and the $2T_N^*+1$th investor observes a good signal, because by this point AoN target is already met and we are back to the standard cascade setting. Based on Lemma 4 for the case of $k = 1$, we have:

$$
\begin{aligned}
P(Q_{T_N^*}) =& \frac{1}{2p} P(U_{2T_N^*+1}|V=1) + \frac{1}{2q} P(U_{2T_N^*+1}|V=0) \\
=& \frac{1}{2T_N^*+1} \begin{pmatrix} 2T_N^*+1 \\ T_N^*+1 \end{pmatrix} (pq)^{T_N^*},
\end{aligned}
$$

and the expected profit from HLL path (implementable with T_N^* but not with $T_N^* + 1$) is:

$$
E_{HLL} \equiv m^* T_N^* P(Q_{T_N^*}) P(LL \textit{ for } 2T_N^*+1 \textit{ and } 2T_N^*+2) = \frac{1}{2} T_N^* P(Q_{T_N^*}) \frac{p^2+q^2}{2}.
$$

Similarly, let $Q_{T_N^*+1}$ be the event that there is no UP cascade yet and at the $T_N^* + 1$th supporting investor there are exactly equal numbers of supporting and rejecting investors (that is to say, the (T_N^*+1)th supporting investor is the $(2T_N^*+2)$th investor). When the target is T_N^*+1, the probability that the project would be implemented with the $T_N^* + 1$ target but fails in the T_N^* target (since the T_N^*th H signal investor behave differently given different AoN target) is:

$$
P_1 \equiv P(Q_{T_N^*+1}) - P(Q_{T_N^*}) pq,
$$

where the second term is the case that the event $Q_{T_N^*}$ realizes and investor $i+1$ and $i+2$ observe L and H, respectively. Note that $Q_{T_N^*+1}$ indicates the $T_N^* + 1$th supporter sees equal number of supporting and rejection actions (including her own), thus HLH meetings both funding target $T_N^* + 1$ and T_N^* with the same payoff to the entrepreneur.

The ratio of the expected profit from HLL path that meets T_N^* but not T_N^*+1, to that from paths implemented with T_N^*+1 target but not T_N^* is:

$$\frac{E_{HLL}}{m^*(T_N^*+1)P_1} = \frac{(p^2+q^2)(T_N^*+2)}{6pq(T_N^*+1)} \leq \frac{p^2+q^2}{4pq},$$

where the last inequality comes from the fact that $T_N^* \geq 1$. Since $p^2+q^2+2pq = (p+q)^2 = 1$, $\frac{p^2+q^2}{4pq} < 1$ is equivalent to $pq > \frac{1}{6}$. So when $pq > \frac{1}{6}$, any $T_N^* < T_N(m^* = \frac{1}{2})$ is strictly dominated by T_N^*+1.

When $pq \leq \frac{1}{6}$, we have $p \geq \frac{1}{2} + \frac{\sqrt{3}}{6} > \frac{3}{4}$. We now show that any target $T_N^* < T_N(\frac{1}{2})$ is strictly dominated by alternative strategy $m^* = p$ (so $k_m = 1$) and AoN target T_N^*+1. For $m^* = \frac{1}{2}$ and AoN target $T_N^* < T_N(\frac{1}{2})$, we have shown earlier that the project would be implemented either there is an UP cascade before/at investor $2T_N^*-1$ or there is no UP cascade before $2T_N^*$ but the $2T_N^*$th investor is the T_N^*th supporting investor. It suffices to show that in either scenario, the alternative strategy fares better for the entrepreneur.

(a) When there is an UP cascade before $2T_N^*$, consider the case that right after the cascade the next investor observes H signal and support. This would also result in an UP cascade for $(m^* = p, T_N^*+1)$ and the same number of supporting investors. The conditional probability that the next investor observes H is $E(V=1|1 \text{ more } H \text{ signals})p + E(V=0|1 \text{ more } H \text{ signals})q = p^2+q^2 = 1-2pq \geq \frac{2}{3}$. For the case $(m^* = p, T_N^*+1)$, for each contribution the entrepreneur charges p instead of $\frac{1}{2}$. The entrepreneur receives higher expected payoffs from UP cascades because $p(p^2+q^2) > \frac{1}{4}(p^2+q^2) \geq \frac{1}{2}$.

(b) When there is no UP cascade before $2T_N^*$ but the $2T_N^*$th investor is the T_N^*th supporting investor (event $Q_{T_N^*}$), consider two corresponding scenarios in $(m^* = p, T_N^*+1)$: (i) Event $Q_{T_N^*}$ happens and the next investor observes H and support; (ii) There is no UP cascade (corresponding to $m^* = p$, that is to say, $k_m+1 = 2$) yet, but there is one more supporting investor by (and including) the $2T_N^*-1$th investor, and the $2T_N^*$th and $2T_N^*+1$th investors observe L and H, respectively.

In both cases, funding target T_N^*+1 is met and there are at least the same number of supporting investors as in $Q_{T_N^*}$. For (i), conditional on there are equal number of supporting and rejecting investors at $2T_N^*$, the conditional probability that the next investor observes H is $E(V = 1|0 \text{ more } H \text{ signals})p + E(V=0|0 \text{ more } H \text{ signals})q = \frac{1}{2}$. For (ii), similar to the discussion on $P(Q_{T_N^*})$, the probability of scenario (ii) is:

$$\frac{1}{2T_N^*}\begin{pmatrix} 2T_N^* \\ \frac{2T_N^*+2}{2} \end{pmatrix}(pq)^{T_N^*} = \frac{1}{2}P(Q_{T_N^*}).$$

The probability that either (i) or (ii) happens equals $P(Q_{T_N^*})$, and in either case there are at least the same number of supporting investors paying $p > \frac{1}{2}$. So for $(m^* = p, T_N^*+1)$ the entrepreneur receives more payoffs when there is no UP cascade before $2T_N^*$. Thus the entrepreneur is strictly better off with strategy $(m^* = p, T_N^*+1)$.

In conclusion, $T_N^* = T_N(m^*)$ is the entrepreneur's weakly dominant strategy. □

Proof of Proposition 2

Proof. When $m^* = E(V|1 \text{ less } H \text{ signal})$, an UP cascade starts from the beginning and the project will be implemented for sure. When $m^* > E(V|1 \text{ less } H \text{ signal})$, an UP cascade starts once there are $k_m + 1 \geq 1$ more H signals. Then for arbitrary positive integer $k_m + 1$, the starting time of an UP cascade is equivalent to the standard gambler's ruin problem with asymmetric simple random walk. Because when $V = 1$, $Pr(X = H|V = 1) = p > q$, it is known that (Feller (1968), page 348 equation 2.8):

$$Pr(\text{an UP cascade starts at some finite time}) = 1.$$

□

Proof of Lemma 2

Proof. For investor 1, her posterior after observing H is $E(V|X_1 = H) = p$. If $m > p$, then investor 1 chooses rejection regardless of her private signal and a DOWN cascade starts from the beginning for sure.

Since $m = 1 - p = E(V|1 \text{ less } H \text{ signal})$ will induce an UP cascade starting form the beginning for sure, the entrepreneur has no incentive to charge $m < 1 - p$. Combine with $m \leq p$ we have $m \in [1 - p, p]$. Also, for each $m \in (m_{k-1}, m_k]$, $m = m_k$ induces exactly the same number of supporting investors, so in the equilibrium entrepreneur always charges $m^* = m_k$ for some $k \in \{-1, 0, 1, \ldots, N\}$. Without loss of generality, only three prices are possible: $m_{-1} = 1 - p$, $m_0 = \frac{1}{2}$ and $m_1 = p$. Let $S(m, N)$ be the expected profit when the price is m and there are $N \geq 2$ potential investors. It is clear that $S(m, N)$ is increasing in N.

$m = 1 - p$: The total profit is $(1 - p)N$;

$m = \frac{1}{2}$: After the first two observations, LL induces a DOWN cascade, HL and HH both induce an UP cascade at investor 1, and LH does not change the belief. The expected profit is $S(m, N) = \frac{p+q}{2}\frac{1}{2}N + \frac{qp+pq}{2}(\frac{1}{2} + S(m, N - 2)) \leq \frac{1}{4}N + pq(\frac{1}{2} + S(m, N))$. Thus $m = \frac{1}{2}$ is dominated by $m = 1 - p$ if:

$$S(m, N) \leq \frac{\frac{N}{4} + \frac{pq}{2}}{1 - pq} \leq (1 - p)N \ for \ N = 2, 3, \ldots \tag{16}$$

One can verify that the inequality holds for $p \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$;

$m = p$: After the first two observations, HH induces an UP cascade, LL and LH both induce a DOWN cascade at investor 1, and LH does not change the belief. The expected profit is $S(m, N) = \frac{p^2+q^2}{2}pN + \frac{qp+pq}{2}(p + S(m, N - 2)) \leq \frac{p^2+q^2}{2}pN + pq(p + S(m, N))$. Thus $m = p$ is dominated by $m = 1 - p$ if:

$$S(m, N) \leq \frac{\frac{p^2+q^2}{2}pN + p^2q}{1 - pq} \leq (1 - p)N \ for \ N = 2, 3, \ldots \tag{17}$$

One can verify that the inequality holds for $p \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$. □

Proof of Lemma 4

Proof. Since an UP cascade starts once there are k more H signals. Exactly k more H signals at investor i implies $\frac{i-k}{2}$ L signals and $\frac{i+k}{2}$ H signals. The number of L signals suggests that $i \geq K$, and the number of H signals implies that $i + k$ must be even. There are $C_i^{\frac{i+k}{2}}$ different potential paths to reach exactly k more H signals, and for each path, the possibility is $p^{\frac{i+k}{2}} q^{\frac{i-k}{2}}$ conditional on $V = 1$ and $q^{\frac{i+k}{2}} p^{\frac{i-k}{2}}$ conditional on

$V = 0$. Then:

$$Pr(\text{exactly } k \text{ more } H \text{ signals at investor } i) = \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2}$$

By the reflection principle and Lemma 3 one can infer that $\varphi_{k,i} = \frac{k}{i} Pr(\text{exactly } k \text{ more } H \text{ signals at investor } i)$. That is:

$$\varphi_{k,i} = \frac{k}{i} \begin{pmatrix} i \\ \frac{i+k}{2} \end{pmatrix} (pq)^{\frac{i-k}{2}} \frac{p^k + q^k}{2}.$$

□

Proof of Proposition 3

Proof. For $m = m_{-1} = 1 - p$, the project will be financed for sure. For $m = m_{k-1}$ $k \in \{1, 2, \dots, N\}$, an UP cascade starts once there are k more supporting investors. When an UP cascade occurs at investor i, all subsequent investors support the project and the financing is successful, there would be in total $N - \frac{i-k}{2}$ supporting investors, and each contributes $m = m_{k-1}$. An UP cascade occurs only when $i + k$ is even. If $N + k$ is odd and there is no UP cascade yet, then the project may still reach the AoN target if there are exactly $k - 1$ more supporting investors at investor N. Suppose there is one more round $N + 1$, then an UP cascade starts at investor $N + 1$ if and only if there are exactly $k - 1$ more supporting investors at investor N and investor $N + 1$ observes H. That is to say, when $k + N$ is odd, the probability that there is no UP cascade and the project reaches the AoN target is $\frac{1}{p}\varphi_{k,N+1}$, and there would be $N - \frac{N-k}{2}$ supporting investors in total.

To show the existence of $\underline{N}(k)$, we first prove the existence of $\underline{N}(0)$, then proceed to the $k \geq 1$ case. $\pi(m_{-1}, N) = (1 - p)N$. When $m = m_0 = \frac{1}{2}$, an UP cascade starts once there are more than 1 H signals. From standard Gambler's ruin problem we know that the conditional probability that an UP cascade occurs at sometime is 1 if $V = 1$, and $\frac{q}{p}$ if $V = 0$ (Feller (1968), page 348 equation 2.8). Because $pq = p(1 - p) < \frac{1}{4}$, we have:

$$\begin{aligned} m_0(Pr(V=1) + Pr(V=0)\frac{q}{p}) &= \frac{1}{2}(\frac{1}{2} + \frac{1 + \frac{1-p}{p}}{2}) \\ &= \frac{1}{4p} \\ &> 1 - p \\ &= m_{-1}. \end{aligned}$$

Since $\varphi_{0,i}$ is strictly positive, there exists a strictly positive integer $N_1(0)$ such that:

$$m_0 \sum_{i=1}^{N_1(0)} \varphi_{0,i} > 1 - p.$$

Let $D = m_0 \sum_{i=1}^{N_1(0)} \varphi_{0,i} - (1 - p) > 0$, $Q = m_0 \sum_{i=1}^{N_1(0)} \varphi_{0,i} \frac{i}{2}$, and $\underline{N}(0)$ be the smallest integer that is larger than

$\max\{N_1(0), \frac{Q}{D}\}$. Then for any $N \geq \underline{N}(0)$:

$$\begin{aligned}
\pi(m_0, N) &\geq m_0 \sum_{i=1}^{\underline{N}(0)} \varphi_{0,i}(N - \frac{i}{2}) \\
&= N m_0 \sum_{i=1}^{\underline{N}(0)} \varphi_{0,i} - Q \\
&\geq \frac{Q}{D} D + (1-p)N - Q \\
&= (1-p)N.
\end{aligned}$$

Now consider the case $k \geq 1$. When the price is m_{k-1}, an UP cascade starts once there are more than k H signals. It occurs once there are more than $k+1$ H signals when the price is m_k. For both cases, the conditional probability that an UP cascade occurs at sometime is 1 if $V = 1$. When $V = 0$, the conditional probability that an UP cascade occurs at sometime is $\frac{q^k}{p^k}$ for m_{k-1} and $\frac{q^{k+1}}{p^{k+1}}$ for m_k, respectively.

For each $k \geq 1$, and the time i arrival rate $\varphi_{k,i}$, there exists a corresponding $\varphi_{k+1,i+1}$ for price m_k. For each i, we have:

$$\begin{aligned}
\frac{m_k \varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}} &= \frac{m_k \frac{k+1}{i+1} \frac{(i+1)!}{\frac{i+k+2}{2}!\frac{i-k}{2}!} (pq)^{\frac{i-k}{2}} \frac{p^{k+1}+q^{k+1}}{2}}{m_{k-1} \frac{k}{i} \frac{i!}{\frac{i+k}{2}!\frac{i-k}{2}!} (pq)^{\frac{i-k}{2}} \frac{p^k+q^k}{2}} \\
&= p\frac{k+1}{k} \frac{i}{\frac{i+k}{2}+1} (1 + \frac{(pq)^{k-1}(p-q)^2}{(p^k+q^k)^2}).
\end{aligned}$$

Since $\lim_{i \uparrow \infty} p\frac{i}{\frac{i+k}{2}+1} = 2p > 1$, for each k, the ratio $\frac{m_k \varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}}$ is monotonically increasing in i and there exists an integer N_1 that $\frac{m_k \varphi_{k+1,i+1}}{m_{k-1}\varphi_{k,i}} \geq 1$ whenever $i \geq N_1$.

Because

$$\begin{aligned}
(p^{k+1} + q^{k+1})(p^{k-1} + q^{k-1}) &= p^{2k} + q^{2k} + p^{k+1}q^{k-1} + p^{k-1}q^{k+1} \\
&= p^{2k} + q^{2k} + p^{k-1}q^{k-1}(p^2 + q^2) \\
&> p^{2k} + q^{2k} + p^{k-1}q^{k-1}(2pq) \\
&= (p^k + q^k)^2.
\end{aligned}$$

We have

$$
\begin{aligned}
\lim_{N\uparrow\infty} m_k \sum_{i=1}^{N-1} \varphi_{k+1,i+1} &= m_k\left(\frac{1}{2} + \frac{\frac{q^{k+1}}{p^{k+1}}}{2}\right) \\
&= \frac{1}{2}\frac{p^k}{p^k+q^k}\frac{p^{k+1}+q^{k+1}}{p^{k+1}} \\
&= \frac{1}{2p}\frac{p^{k+1}+q^{k+1}}{p^k+q^k} \\
&> \frac{1}{2p}\frac{p^k+q^k}{p^{k-1}+q^{k-1}} \\
&= m_{k-1}\left(\frac{1}{2} + \frac{\frac{q^k}{p^k}}{2}\right) \\
&= \lim_{N\uparrow\infty} m_{k-1} \sum_{i=1}^{N} \varphi_{k,i}.
\end{aligned}
$$

Given $\lim_{N\uparrow\infty} m_k\varphi_{k+1,i+1} \downarrow 0$, there exists an integer $N_2 \geq N_1$ such that:

$$
D \equiv m_k \sum_{i=1}^{N_2-1} \varphi_{k+1,i+1} - m_{k-1}\sum_{i=1}^{N_2}\varphi_{k,i} - m_{k-1}\frac{1}{2p}\varphi_{k,N_2+1} > 0
$$

Let $Q \equiv m_{k-1}\sum_{i=1}^{N_2}\varphi_{k,i}\frac{i-k}{2} - m_k\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1}\frac{i-k}{2}$. Then for each k, let $\underline{N}(k)$ be the smallest integer that is larger than $\max\{N_2, \frac{Q}{D}\}$. Then for any $N \geq \underline{N}(0)$:

$$
\begin{aligned}
\pi(m_k, N) - \pi(m_{k-1}, N) &> \pi(m_k, \underline{N}(k)) - \pi(m_{k-1}, \underline{N}(k)) \\
&> \underline{N}(k) m_k \sum_{i=1}^{N_2-1}\varphi_{k+1,i+1} - m_{k-1}\sum_{i=1}^{N_2}\varphi_{k,i} - Q - \frac{1}{p}m_{k-1}\varphi_{k,\underline{N}(k)}\left(\underline{N}(k) - \frac{\underline{N}(k)-k}{2}\right) \\
&> \underline{N}(k)D - Q \\
&\geq \frac{Q}{D}D - Q \\
&= 0.
\end{aligned}
$$

□