

# Predictability Puzzles

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## **Abstract**

Dynamic equilibrium models based on present value computation imply that returns are predictable, suggesting that time-series that predict returns could be priced risk factors. Equilibrium models however, imply that returns from risk-based state-variables are predictable in the short, but not the long-run. The variables that have been shown empirically to predict returns typically do so at medium or long horizons, and never at short horizons. This contradicts the equilibrium interpretation. I develop econometric tests aimed at testing whether predictive variables show term structures of predictability that is consistent with equilibrium. Empirically, I find that the variables in question are either not significant predictors, or the predictability fails to be consistent with equilibrium.

# 1 Introduction

Very few topics in Finance is as heavily researched and hotly contested as predictability of asset returns. Fama and French (1988) and Campbell and Shiller (1988b) were among the first to document predictability of stock returns from price-dividend ratios. Fama and French (1988a) find that the price-dividend ratio predicts an increasing term structure of regression  $R^2$ s. There is no predictability at the short horizon (month or quarter) and the  $R^2$ s increase monotonically to somewhere between 13% and 49% at the five year forecasting horizon. The long range predictability of returns from price-dividend ratios has been considered so compelling that Cochrane (1999), in his essay entitled “New Facts in Finance”, declared it a fact.

Not only does predictability of returns from price-dividend ratios seem statistically and economically significant, it is also logically consistent with equilibrium under time-varying expected returns. Indeed, Fama and French (1988) recognize this and write that “The hypothesis that D/P forecasts returns has a long tradition among practitioners and academics ... The intuition of the ‘efficient markets’ version of the hypothesis is that stock prices are low relative to dividends when discount rates and expected returns are high, and vice versa, so that D/P varies with expected returns.” This intuition is precisely the main mechanism that leads to time-variation in expected returns in modern dynamic models of asset prices. Campbell and Shiller (1988b), Campbell and Shiller (1988a) specify econometric models of dividend discounting that imply that price dividend ratios predict stock returns. This is also a fundamental property of dynamic equilibrium models such as Bansal and Yaron (2004), Campbell and Cochrane (1999), Menzly, Santos, and Veronesi (2004) and numerous others. In this literature, typically, (log) dividends and (log) prices contain unit roots but are co-integrated with co-integration vector  $[0, 1]$ . That is, the equilibrium log price-dividend ratio is a linear function of stationary state-variables  $X_t$ , as the equilibrium takes on the form

$$\ln P_t/D_t = \alpha + \beta' X_t. \tag{1}$$

The unit root behavior of (log) prices and dividends and the stationarity of  $X_t$  implies that the right hand side of (1) predicts returns. Intuitively, when  $P/D$  ratios are high, the expected rate of return going forward is low and vice versa, as suggested in the above quote to Fama and French (1998a).

In this paper I take issue with the idea that the evidence presented in favor of return predictability is consistent with equilibrium. Rather, I argue that equilibrium models

1. generate more predictability at short forecasting horizons than long, which is precisely the opposite of what we find in the data, and
2. imply that positive (negative) shocks to expected return contemporaneously correlate negatively (positively) with prices. Yet, variables that have been shown to predict returns have insignificant contemporaneous return correlation. Conversely, implied volatility has significant negative correlation with returns but does not predict returns.

The intuition regarding the first point goes as follows: Imagine an economy where a time-varying risk factor (let's say volatility) generates time-variation in expected return. If risk is high today, the equilibrium expected rate of return must be high today as well. For equilibrium to prevail in high volatility states it must be that investors are compensated for the above-average risk. Yet, we find no evidence of such short term risk premium in the data.

Moreover, long horizon predictability is not consistent with intertemporal equilibrium. To see this, consider an investor who observes high risk today but expect to be rewarded four or five months down the road. It's clearly optimal for such an investor to wait to four months to invest, thereby avoiding low reward-to-risk regime today and exploiting a high reward-to-risk regime in the future. Clearly this cannot be an equilibrium. Another way to think about the counterintuitive nature of long-but-not-short-term predictability is to consider the investor's investment rule. He or she simply has to discard recent information and look back, say, four or five months at the market conditions at that time to make current investment decisions. This essentially implies that the the relevant state-variable(s) that govern the investors' investment decisions do not follow Markov processes. This is inconsistent with standard equilibrium models and it is hard to imagine what kind of economic environment that would support this kind of delayed price response in an rational equilibrium model.

The second bullet point above is related to the fact that equilibrium prices respond negatively (positively) to positive (negative) shocks in expected returns. In the time-varying volatility literature this is referred to as a "volatility feedback effect." Many of the variables that have been shown to predict returns have economically and statistically insignificant contemporaneous correlation with returns. For example, Bollerslev, Tauchen, and Zhou (2009) show that the difference between physical and option implied volatility, known as the variance risk premium (VRP) in the literature, significantly predicts returns at horizons four-five month horizons. But the VRP does not predict returns at short forecasting horizons. It also has fairly low contemporaneous correlation with returns. This, along with the lack of short term predictability and strong medium to long term predictability each point to the VRP as showing predictability patterns that are inconsistent with equilibrium.

To make these arguments precise I quantify the relationship between the predictable variation in returns and the contemporaneous correlation between changes in the predictor and the returns. I derive simple algebraic relations that imposes the equilibrium structure onto the predictive regression coefficients. I then derive  $t$  tests for the difference between the estimated predictive coefficients and the equilibrium implied coefficients.

I apply my empirical tests to a range of time-series variables that have been shown to predict stock returns in the extant literature. Using monthly time-series going back to the 1920ties, I show that the level of interest rates (3m TBILL rate), slope of the yield curve and default spread are either insignificant predictors at all, or if they are significantly predicting stock returns, the term structure of predictability tend to be inconsistent with equilibrium. The strongest predictors are term structure slope (at the monthly frequency) and VRP at the daily frequency. Nether of these variables predict stock returns at short horizons. The lack of short term predictability is inconsistent with equilibrium where a time-varying risk premium is proportional to some mean-reverting risk variable. Rather, most of the predictable variation I find is for horizons that exceed 3-5 months. Virtually all the predictors show a monotonically increasing predictability pattern, at least for some sub-group of forecasting horizons. These patterns are consistent with predictability generated by sampling variation as suggested by Boudoukh, Richardson, and Whitelaw (2006) rather than dynamic equilibrium.

Shocks to VIX or VIX squared are very highly negatively correlated with returns. This in itself suggests VIX squared is a good candidate for a priced risk variable. Again however, the empirical tests are not kind to this interpretation: using first a test based on the idea that shocks to the state variable (squared VIX in this case) should be negatively correlated with returns *in equilibrium*, I show that the null of equilibrium consistency is sharply rejected. In short, the VIX *ought to* predict returns at short horizons, but it does not. The result is that investors are forced to hold assets in volatile times without compensation for this additional risk - an implication that cannot be reconciled with equilibrium. The same conclusion holds whether or not I use measures of physical or option implied variance. The findings echo Moreira and Muir (2017)'s finding that selling stock in volatile times and buying in low volatility times improves the Sharpe ratio of investors.

The remainder of the paper is organized as follows. In the next section I postulate a very simple equilibrium relation between dividends, prices and a state-variables that generate fluctuations in expected returns. I show that without imposing a specific equilibrium structure I can derive equilibrium-implied predictive coefficients that can be compared to estimated, reduced form predictive coefficients. I discuss empirical tests to compare the two. In section three I apply

the tests to monthly and daily sampled data on candidate predictors/ risk-variables. Section 4 discusses the findings concludes.

## 2 A test for dynamic consistency

In the following I derive tests of whether the predictable pattern from a time-series, say  $x_t$ , to  $i$  period ahead returns  $r_{t+i}$  is consistent with dynamic equilibrium. Standard dynamic equilibrium models dictate that the price obtains as a present value of future dividends. This again implies that the price-dividend ratio is a function of the state-variable  $x_t$ ,

$$\frac{P_t}{D_t} = F(x_t). \quad (2)$$

This equation just implies that the price is degree one homogenous in dividends. Consistent with Long-Run-Risk, habit formation, and other equilibrium models we assume that  $F$  is exponential affine

$$F(x) = e^{\alpha + \beta x_t} \quad (3)$$

such that the log-price dividend ratio is

$$\ln P_t = \ln D_t + \alpha + \beta x_t. \quad (4)$$

Since dividends contain a unit root, this equation implies that log prices and log dividends are cointegrated. The cointegrating relationship between dividends and prices is not important per se in our context. However, eqn. (4) suggests that prices contain a temporary component driven by the risk variable  $x_t$ . This variable predicts stock returns. It does so by temporarily moving the price away from its steady state path. To see how shocks to  $x_t$  generates time-varying expected rates of return, assume that log-dividend growth rates are given by

$$\ln D_{t+1} - \ln D_t = \mu + \omega x_t + \epsilon_{t+1} \quad (5)$$

where a non-zero  $\omega$  implies that the state-variable drives expected dividends, as in Bansal-Yaron (2004).

The dynamics of log capital gains follow

$$\ln P_{t+1} - \ln P_t = \mu + \omega x_t + \beta(x_{t+1} - x_t) + \epsilon_{t+1}. \quad (6)$$

This equation suggests that estimates of  $\omega$  and  $\beta$  can be obtained through a regression of log capital gains onto  $x_t$  and  $\Delta x_{t+1} = x_{t+1} - x_t$ . Alternatively, estimates of  $\omega$  can be obtained by regressing log-dividend growth on  $x$ .

The  $h$  period ahead capital gain is given by

$$\ln P_{t+h} - \ln P_t = \mu h + \omega \sum_{i=1}^h x_{t+i} + \beta(x_{t+h} - x_t) + \sum_{i=1}^h \epsilon_{t+i} \quad (7)$$

The expected  $h$  period capital gain can be found by taking expectations of (7). In the case that  $x$  follows an AR(1) with autocorrelation  $\rho$  it is

$$E_t [\ln P_{t+h} - \ln P_t] = \mu h + \left[ \omega \frac{\rho^{h+1} - \rho}{\rho - 1} + \beta(\rho^h - 1) \right] x_t. \quad (8)$$

Equation (8) suggests that if one were to run the regression

$$\ln P_{t+h} - \ln P_t = a_h + b_h x_t + u_t^h \quad (9)$$

the intercept and slope would have to satisfy

$$a_h = \mu h, \quad (10)$$

$$b_h = \left[ \omega \frac{\rho^{h+1} - \rho}{\rho - 1} + \beta(\rho^h - 1) \right]. \quad (11)$$

Notice that these equations impose testable restrictions. In particular, the entire term structure of predictable variation in returns is governed by the feedback coefficient  $\beta$  and the autocorrelation,  $\rho$ , of the predictor variable. I will argue below,  $\omega$  is practically very small and I ignore it in the empirical implementation of my tests. This leaves a very simple structural restriction on the predictive coefficients

$$b_h = \beta(\rho^h - 1) \quad (12)$$

This is valid under an AR(1) assumption for the state-variable. More generally, under an AR(p) assumption we have  $E_t x_{t+h} = C_h x_t$  where  $C_h = \text{Corr}(x_{t+h}, x_t)$  in which case the restriction becomes

$$b_h = \beta(C_h - 1). \quad (13)$$

To assess the impact of shocks to returns, note that the linearized log return is

$$r_{t+1} = \kappa_0 + \kappa_1(\alpha + \beta x_{t+1}) - \alpha - \beta x_t + \omega x_t + \mu + \epsilon_{t+1} \quad (14)$$

$$= \text{const.} + \kappa_1\beta(x_{t+1} - x_t) - (\beta(1 - \kappa_1) - \omega)x_t + \epsilon_{t+1} \quad (15)$$

In the case that  $\kappa_1$  is close to unity, which is the case in many applications and especially in high frequency data, the log returns are approximately

$$r_{t+1} \approx \text{const.} + \beta(x_{t+1} - x_t) - \omega x_t + \epsilon_{t+1} \quad (16)$$

We can also write

$$r_{t+1} = \ln P_{t+1} - \ln P_t + \ln \left( 1 + \frac{D_{t+1}}{P_{t+1}} \right) \quad (17)$$

Note that this expression suggests that we can decompose predictability of log returns into two components, the log-capital gain and the forward looking log dividend yield,  $y_{t+1} := \ln \left( 1 + \frac{D_{t+1}}{P_{t+1}} \right)$ .

[Figure 1 about here.]

Figure 1 shows the daily value weighted dividend paid to S&P 500 investors from 1996-2017. Note that the series look nothing like what it would have looked like if the true daily underlying dividend process was a random walk with time-varying drift, as in eqn. (5). In the continuous time limit,  $y_t$  is a diffusion with continuous sample paths. However, Figure (1) looks nothing like process with continuous sample paths. Rather, it looks like a pure white noise process. This of course reflects the fact that firms do not pay continuous or daily dividend, but prefer lump sum payments.

[Table 1 about here.]

A second noteworthy feature of dividend yield data is how little variation there is relative to the capital gains. In Table 1 I compute the ratio of the variances of the forward looking log dividend yield and variance of returns. It shows that at the daily frequency, dividend yield variation accounts for about 5/1000th of the total variation. This is an upper bound on the  $R^2$  that we could get from running a regression of total returns onto some predictor which predicts only dividend yield variation. In other words, if the forward looking log dividend yield was perfectly predictable, it could not explain more than 5/1000th of the variation in returns. The numbers

are larger for longer horizons, but nevertheless so small that it is clear that return predictability cannot come from predictability of dividends at short horizons. For this reason, I assume  $\omega = 0$  for the remainder of the paper.

[Figure 2 about here.]

To consider further the implications of (11) on predictability, Figure 2 shows the impact of a positive shock to expected return. At the time  $h = 0$  a shock to expected returns have an impact equal to  $\beta$  on the (log) price path. Figure 2 depicts this under the assumption that  $\beta < 0$ . The negative effect on the price is reversed following the shock, as, going forward, investors generate an above-average expected rate of return in the form of a steeper than average expected (log) capital gains rate. Note that absent any interaction between future expected dividend and expected return ( $\omega = 0$ ),  $b_h$  is just  $b_h = \beta(\rho^h - 1)$ . This means that the expected  $h$  holding period expected rate of return is entirely determined by the size of the initial shock and the speed of mean reversion in the predictor/ risk variable  $x_t$ . In Figure 2 the curvature in the line segment labelled by “Expected path conditional upon the shock” is determined entirely by the persistence in the state-variable,  $\rho$ . A higher value of  $\rho$  leads to a slower speed of mean-reversion in  $x_t$  which translates one-for-one into the expected price reversal.

## 2.1 When cash flow shocks are correlated with discount rate shocks

What happens if shocks to discount rates are correlated with shocks to dividends? It is possible, perhaps, to conceive of an economy where, say, a positive shock to discount rates is correlated with a positive shock to current or future cash flows such that the initial price impact is offset, while still generating higher expected rates of return going forward. To analyze whether such effects can occur, consider first some simple back of the envelope computations. Imagine that some shock to a priced risk variable takes place at time  $t$  increases the expected rate of return going forward. Specifically, let's assume that the at time  $t - 1$  the expected rate of return equals its steady state unconditional mean  $E_{t-1}(r_t) = E(r)$  such that  $E_t(r_{t+1}) - E(r)$  is the shock. Assume further that the expected rate of return is driven by a single AR(1) state-variable with autocorrelation  $\rho$ . Thus,

$$E_t(r_{t+i+1}) = E(r) + (E_t(r_{t+1}) - E(r))(\rho^i - \rho^{i-1}). \quad (18)$$

for all  $i$ . We now ask what it will take for this shock to expected returns to not impact stock prices. That is, I am asking what would have to happen to expected future dividends to offset an



increase in expected return enough to render the stock price unaffected by the shock to expected returns.

To answer this question, note first that the log-return to an asset can be decomposed into a capital gain and a dividend yield component as

$$r_{t+1} = \ln \frac{P_{t+1}}{P_t} + \ln\left(1 + \frac{D_{t+1}}{P_{t+1}}\right) \quad (19)$$

The second term  $y_{t+1} = \ln(1 + D_{t+1}/P_{t+1})$  is essentially the log-dividend yield. Taking expectations and conditioning on the zero price impact means that  $E_t(r_{t+i}) = E_t(\ln(1 + \frac{D_{t+i}}{P_{t+i}}))$ . The unconditional expected log capital appreciation is  $E(\Delta \ln D) = \mu$ . We now have

$$E_t(y_{t+i}) = \text{const.} + (E_t(r_{t+1}) - E(r))(\rho^{i-1} - \rho^{i-2}) \quad (20)$$

That is, if we condition on a shock to expected rate of return to not affect prices, the entire term structure of expected future log dividend yields will have to change one-for-one with term structure of expected returns. This is an extremely strong implication, and almost entirely implausible. For example, it would not hold for a stock or stock index that did not continuously pay a random dividend stream that would reset to every shock to expected returns.

## 2.2 Bias in OLS estimates of the factor loading

Above I argued that shocks to expected returns will show up as price shocks and lead to a subsequent price reversal generated by time-varying expected capital appreciation. This shows that if one knows the true factor loading,  $B$ , the restrictions in (12) and (13) hold.

When shocks to cash flows and discount rates are correlated a different problem relating to the empirical implementation of our test occurs, however. The problem is that the factor loading  $B$  is difficult to estimate from simply running a regression of returns or capital gains onto the first differences in the risk factor. This follows because the error terms in the regression will be correlated with the regressor.

To see how this plays out, I consider an example long-run risk model where dividend shocks are correlated with shocks to a stochastic volatility factor. Specifically, I assume

$$\ln \frac{C_{t+1}}{C_t} = \mu + \sigma_t \nu_{t+1} \quad (21)$$

$$\sigma_{t+1}^2 = \sigma + \kappa(\sigma_t^2 - \sigma^2) + \sigma_t \sigma_w w_{t+1} \quad (22)$$

where  $\text{Corr}(\nu_{t+1}, w_{t+1}) = \rho$  is the correlation between shocks to consumption growth and its conditional variance,  $\sigma_t^2$ . One can solve this model easily with the usual long-run-risk framework. The linearized solution to the equilibrium price of an asset that pays aggregate consumption as its dividend is given by

$$\ln P_t = \ln C_t + A_o + A_\sigma \sigma_{t+1}^2 \quad (23)$$

where the  $A_\sigma$  is given by

$$A_\sigma = \frac{1 - \kappa k_1 - (1 - \gamma)k_1 \sigma_w \rho - \sqrt{(\kappa k_1 - 1 + (1 - \gamma)k_1 \sigma_w \rho)^2 - 2k_1^2 \sigma_w^2 (1 - \gamma)^2 / (1 - \frac{1}{\psi})}}{2k_1^2 \sigma_w^2 \theta} \quad (24)$$

To analyze the impact of a shock to conditional variance, consider equation (23). Before I argued that we can identify the factor loading of capital gains or returns onto the risk variable through a regression of capital gains or returns onto changes in the risk variable, as in (9). The equivalent regression here would then be

$$\Delta \ln P_{t+1} = A + \beta \Delta \sigma_{t+1}^2 + \epsilon_{t+1} \quad (25)$$

where  $\epsilon_{t+1} = \sigma_t \nu_{t+1}$  are the error terms in the regression. These are interpretable as the demeaned shocks to consumption growth, and by assumption, they are correlated with shocks to the regressor. For this reason, an OLS estimate of  $\beta$  in (25) is inconsistent for  $A_\sigma$ . At the same time, it is easy to verify that expected log capital gains are given by  $E_t(\ln P_{t+i} - \ln P_{t+i-1}) = \mu + A_\sigma(\kappa^i - \kappa^{i-1})$ , analogously to the BY model example above. Thus, tests based on a simple OLS regressions, as in (12) or (13) fail.

To gauge the bias in the OLS estimated  $\beta$ , note that it is given by

$$\begin{aligned} \beta_{OLS} &= \frac{\text{Cov}(\Delta \ln P_{t+1}, \Delta \sigma_{t+1}^2)}{\text{Var}(\Delta \sigma_{t+1}^2)} = A_\sigma + \frac{\text{Cov}(\sigma_t \nu_{t+1}, \Delta \sigma_{t+1}^2)}{\text{Var}(\Delta \sigma_{t+1}^2)} \\ &= A_\sigma + \frac{\rho}{\left[ \frac{(1-\kappa)^2}{1-\kappa^2} + 1 \right] \sigma_w} \end{aligned} \quad (26)$$

where the second term is bias in the OLS estimate  $B_{OLS}$  for  $A_\sigma$ . This bias can be very substantial. Its sign depends on the correlation  $\rho$  between dividend and volatility shocks. If shocks to volatility are positively correlated with dividend news, the OLS estimator is upwardly biased for  $A_\sigma$  and vice versa.

[Figure 3 about here.]

Figure 3 shows the impact of correlation between the volatility and dividend innovations on the population value of the regression in (25). The shaded turquoise area represents the bias, which is significant. Even for small positive values of  $\rho$  it is possible that volatility  $\sigma_t$  predicts returns even if volatility shocks have zero correlation with asset prices. It is more economically plausible however that shocks to dividends are negatively correlated with volatility shocks. In this case one would observe a sharp negative correlation between volatility changes and asset returns. This is empirically relevant in the context of VIX and other measures of volatility. In particular, one routinely finds that volatility and return shocks are sharply negatively correlated while at the same time, volatility very weakly predicts returns.

## 2.3 Testing

I suggest two types of test. The first one is contingent on the assumption that there is no correlation between shocks in dividends and volatility. In this case, one can perform simple hypothesis tests of the restrictions imposed by equations (12) and (13). Let  $x_t$  denote the regressor. The strategy involves separately identifying  $\beta$  from the contemporaneous regression of returns and changes in  $x$ , and the autocorrelation function  $C_i = \text{Corr}(x_t, x_{t+i})$ . Let  $\rho$  denote the first order autocorrelation,  $\rho = C_1$ . I follow the convention on the literature and focus on cumulative returns and thus

$$r_{t:t+h} = a_h + b_h x_t + \epsilon_{t,h} \quad (27)$$

is the predictive regression.

Each test-statistic is of the form

$$t_k = \frac{b_h - b_{k,h}}{\text{se}_h} \quad (28)$$

where  $b_k$  for  $k = 1, \dots, 4$  are the *equilibrium implied forecasting slopes*. For the first test ( $k = 1$ ) it is given by

$$b_{1,h} = \hat{\beta}(\hat{\rho}^h - 1). \quad (29)$$

where  $\hat{\beta}$  and  $\hat{\rho}$  are estimates of  $\beta$  and  $\rho$ . This test is valid under the joint null hypothesis that returns are consistent with dynamic equilibrium where expected returns are driven by an AR(1) process  $x_t$  and shocks to  $x$  and cash flows are independent.

The second version is

$$b_{2,h} = \hat{\beta}(\hat{C}_h - 1). \quad (30)$$

This test is valid under the joint null hypothesis that returns are consistent with dynamic equilibrium where expected returns are  $x_t$  which follows a Markov process such that  $E_t(x_{t+h}) = C_h x_t$  and shocks to  $x$  and cash flows are independent.

The third is

$$b_{3,h} = b_1 \frac{\hat{\rho}^h - 1}{\hat{\rho} - 1} \quad \text{for } h \geq 2. \quad (31)$$

where  $b_1$  is the unrestricted OLS estimate of the slope in the one-period ahead ( $h = 1$ ) forecast regression in (27). The test is obtained by substituting out  $\hat{\beta}$  from  $b_{1,h}$  in (29), thereby avoiding a biased test due to potential bias in the estimate of  $\beta$ . It is valid under the joint null hypothesis that returns are consistent with dynamic equilibrium where expected returns are driven by an AR(1) process  $x_t$  and shocks to  $x$  and cash flows are possibly correlated.

Finally, the fourth equilibrium implied forecasting slope is

$$b_{4,h} = b_1 \frac{\hat{C}_h - 1}{\hat{C}_1 - 1}. \quad (32)$$

This test again avoids using an estimate of  $\beta$  and is thus valid under the joint null hypothesis that returns are consistent with dynamic equilibrium where expected returns are linear in  $x_t$  which follows a Markov process such that  $E_t(x_{t+h}) = C_h x_t$  and shocks to  $x$  and cash flows are possibly correlated.

### 3 Empirical Results

I apply the logic and the tests from the previous section to examine several candidate risk variables. First and foremost, I am interested in measures of risk, in particular, volatility related variables. At the daily frequency, I use measures of physical conditional variance including realized variance (RV) computed from intraday returns, option implied volatility (VIX), and the difference between squared VIX and realized variance, known in the literature as the variance risk premium (VRP).

#### 3.1 VIX

The VIX index is computed from S&P 500 cash index options by the CBOE. In theory, the square of the VIX index represents a one-month forward looking option implied estimate of the *risk-neutral* variance of the log-return for the underlying S&P 500 index. There are strong

theoretical reasons to think that the VIX index contains information about expected returns. In one-factor models based on dynamic present value computation, including Bansal and Yaron (2004), a single economy wide volatility factor drive expected excess returns. One factor models also imply that the variance-risk-premium is proportional to the volatility factor. This again means that risk-neutral and objective conditional variance are both scaled versions of the same underlying macro factor, and therefore work equally well in predicting returns. Multi-factor models of conditional variance also imply that  $Q$  expected variance predicts returns. For example, in Bollerslev, Tauchen, and Zhou (2009)'s model, objective measure conditional variance ( $P$  variance) is a strong predictor of return. In their model, the  $Q$  variance equals the  $P$  variance plus the VRP which again depends on a separate volatility-of-volatility factor. The squared VIX index is a linear combination of these two factors in their model, and thus, squared VIX predicts returns.

Beyond the model-based theoretical justification, it is also clear that option traders look forward to known future events that can cause volatility. For example, FOMC meetings are known to move prices and it is natural that options which maturity window contain an FOMC meeting are more expensive than those who do not. Empirical evidence is mixed on the extent to which implied volatility predicts future volatility and what contribution it contains relative to physical volatility. Canina and Figlewski (1993) conclude that IV has no informational content over physical volatility while Christensen and Prabhala (1998) reach the opposite conclusion.

[Figure 4 about here.]

Figure 4 presents the empirical results for the VIX index. The upper plot presents the estimated predictive slope coefficients,  $b_i$ , for  $i = 1, \dots, 252$  trading days. The red band represents 95% of the mass of the sampling distribution, and the purple line is the theoretical equilibrium predictability coefficients.

The VIX index does predict returns but only at very long horizons. The estimated  $b_i$ s cluster around zero for the first 100 days, and then turn positive. After about 120 days the VIX significantly predicts returns and the one year  $R^2$  peaks at the one-year horizon at about 4.5%.

The upwards sloping term-structure of predictive coefficients  $b_i$  is precisely the opposite of what one would expect from an equilibrium model. As seen in the upper plot in Figure 4, the baseline equilibrium model (i.e, zero correlation between VIX and cash flow shocks and an AR(1) for squared VIX) predicts that returns are significantly predictable at the short, but not the long term horizons. Tests 1 and 2 sharply reject the null at short term horizons. The long run predictability found for VIX is also inconsistent with the tests that relax the zero correlation

assumption: as seen in the middle graph, tests 3 and 4 exceed the 1.96 line at about the 150 day forecasting horizon.

[Table 2 about here.]

## 3.2 Realized Variance

[Figure 5 about here.]

[Table 3 about here.]

Figure 5 and Table 3 contain results for one-day realized variance. The estimated predictive coefficients  $b_1, \dots, b_{252}$  are uniformly statistically insignificantly different from zero. The  $R^2$ s fail to exceed 1% at any horizon. The contemporaneous correlation between changes in one-day RV and stock returns is just -0.127. This implies that RV should essentially not predict returns. Thus the theoretical model agrees with the reduced form predictive regressions. This is counterintuitive because RV ought to be highly correlated with conditional spot variance. It is quite hard to set up a model where spot variance or realized variance does not predict stock returns and simultaneously generates a large variance risk premium.

## 3.3 Variance Risk Premium

[Figure 6 about here.]

[Table 4 about here.]

Since Bollerslev, Tauchen, and Zhou (2009) a substantial literature has emerged on the Variance Risk Premium (VRP). BTZ show that the VRP predicts stock returns with 1.07% and 6.82%  $R^2$ s at one and three month forecasting horizon, and with a decreasing term structure after that. In this respect, the pattern of predictability differs from that found from, for example,  $P/D$  ratios which show monotonically increasing  $R^2$ s. Others have found even higher  $R^2$ s. For example, Bekaert and Hoerova (2014) report an  $R^2$  of 13% for the quarterly forecasting horizon.

Before proceeding, note that the operational definition of the VRP matters for the empirical results to follow. I start by replicating the results in BTZ and I therefore define the VRP as

the difference between implied variance, as measured by the squared VIX, and 30-day backward looking Realized Variance, RV.

Table 4 and Figure 6 summarize the empirical results for the VRP. First, note that I replicate the overall empirical findings in BTZ: the bottom plot in Figure 6 shows that  $R^2$ s peak at about 8% at the quarterly forecasting horizon. There is little predictability at in the short run.

As far as being consistent with equilibrium, the news is not good for the variance risk premium. All four tests rejects the null hypothesis. Tests 1 and 2, which impose the zero correlation between cash-flow and VRP shock assumption, reject the null hypothesis overwhelmingly at short horizons. The upper plot shows why: The autocorrelation in the VRP is about 85% which is to say that it mean reverts fairly quickly. Moreover, since the contemporaneous relationship between shocks to VRP and stock returns is estimated to be sharply negative (see Table 4) Panel A), tests 1 and 2 imply a that stock returns should carry a much larger premia at the short horizon, and therefore display a fair amount of predictability at the short horizon. This is illustrated by the difference between the purple line in the upper plot of Figure 6. By contrast, the predictive relationship found in the data, represented by the yellow line, shows near zero predictability at the short horizons. This leads the large  $t$ -stats in the middle plot in Figure 6 and Table 4.

By relaxing the zero correlation assumption, tests 3 and 4 fail to reject the null hypothesis at the short horizons. However, at all horizons exceeding two months tests 3 and 4 rejects the null. Tests 3 and 4 rely on the idea that equilibrium models produce a term structure of predictability that is proportional to the autocorrelation of the predictor. The VRP however, is not persistent enough to generate equilibrium predictable variation in returns three, four or twelve months ahead. Intuitively, since there is no predictability in the short run there cannot be any in the long run either.

### 3.4 Variance Risk Premium II

The analysis above uses daily return regressions in combination with backwards looking one-month Realized Variance. I followed this approach to as closely as possible conduct the analysis using the same data definitions as in BTZ. This approach however has the somewhat unfortunate consequence that when looking at the first difference in the VRP, we get

$$\Delta x_{t+1} = VRP_{t+1} - VRP_t = VIX_{t+1}^2 - VIX_t^2 - RV_{t-21:t+1} - RV_{t-22:t} \quad (33)$$

where  $RV_{t-22:t} = \sum_{j=t-22}^t RV_j$  is the cumulative, monthly backwards looking RV. The last term becomes equals the one-month change in the realized daily variance

$$RV_{t-21:t+1} - RV_{t-22:t} = RV_{t+1} - RV_{t-22}. \quad (34)$$

This is a very noisy object because each day's RV can be noisy. For this reason, the change in the VRP as measured by backward 30 day RV is dominated by the change in the squared VIX. This is why the estimated correlation between changes in VRP and the stock return equals the large -0.721 reported in Table 4.

To overcome this issue I follow Bekaert and Hoerova (2014) and other in defining the VRP as

$$VRP_t = E_t^Q(RV_{t:t+22}) - E_t(RV_{t:t+22}) \quad (35)$$

where the last term is a forecast. I estimate this forecast using a regression

$$RV_{t+1:t+23} = \alpha + \beta_1 RV_{t-22:t} + \beta_2 r_{t-22:t} + \beta_3 r_t + \beta_4 VIX_t + \epsilon_t. \quad (36)$$

All coefficients were statistically significant and the regression produced an  $R^2$  of 56%.

[Figure 7 about here.]

[Table 5 about here.]

Repeating the equilibrium analysis from the previous section, Table 5 and Figure 7 report empirical results. The evidence against the null is now weaker at the short term horizon: test 1 does indeed reject the null at short term horizons while test 2 only rejects one day ahead at the short horizon. At longer horizons, all four test statistics reject the null. Again, long term predictability is not consistent with the estimated contemporaneous relationship and autocorrelation in VRP (tests 1 and 2) and it is not consistent with the absence of predictability at the short horizon (tests 2 and 4). Again, therefore, I conclude that the long term predictability found from VRP, which here peaks at about half a year, is inconsistent with dynamic equilibrium.

### 3.5 Tests based on monthly data

In the following I study the default spread (DS), yield-curve-slope (SLOPE), and the three-month Treasury Bill Yield (TBILL). I repeat the analysis in the previous section using data collected



at the monthly frequency for these variables. This allows me to conduct the analysis using a considerably longer sample than for the daily data. For the default spread, I use monthly data from January 1933 to October of 2016 and for the other two variables the sampling period is April 1953 to January of 2017.

### 3.5.1 Default Spread

I first look at the default spread, defined as the difference between yield to maturity of BAA rated corporate bonds and Treasury Bills. Keim and Stambaugh (1986) appears to be among the first to consider DS as a predictor. Although Keim and Stambaugh report weak evidence of its usefulness as a predictor, it has since been a standard variable in predictability studies (see for example Harvey (1989), Boudoukh, Richardson, and Whitelaw (2006)). It is easy to motivate why DS ought to be a significant predictor: There are two components to the spread. First, DS measures the expected default losses of low grade corporate debt relative to high grade. This in itself ought to be counter-cyclical. Second, since DS is a market implied measure of expected loss, it is the risk-neutral expected loss also contains a risk premium component relative to the objective measure expected loss. In many ways, DS is similar to option implied volatility in that it measures aggregate corporate risk. It is also highly correlated with option implied equity volatility.

[Table 6 about here.]

[Figure 8 about here.]

Table 6 and Figure 8 report the result for default spread. There is not much predictability to be found from the default spread. In fact, the predictability coefficients are insignificantly different from zero at all horizons, as seen in the upper plot of Figure 8. The bottom plot shows that the  $R^2$ s peak at a little above 4%. None of the tests of equilibrium consistency reject the null that the predictability is consistent with equilibrium. The lack of predictability documented here is consistent with that reported in relatively recent studies including Bollerslev, Tauchen, and Zhou (2009) and Boudoukh, Richardson, and Whitelaw (2006).

### 3.5.2 Level and slope

Short term interest rates have been shown to predict returns in Fama and Schwert (1977) and Campbell (1991) while term structure slope has shown to have predictive ability by Keim and Stambaugh (1986), Campbell (1987), Fama and French (1989).

[Table 7 about here.]

[Figure 9 about here.]

Table 7 and 9 shows the results for the 3 month treasury bill. The level of the TBILL predicts returns with a negative sign: high interest rates are associated with lower future equity premiums. The predictability is marginally significant, and the  $R^2$ 's peak at about 8%.

The predictability found from TBILL rates is not consistent with equilibrium under the zero-cash flow correlation assumption. As seen in the second plot of Figure 9, tests 1 and 2 basically reject the null at every horizon. The reason why these rejections are so strong, even though the evidence for predictability is rather weak, is that the predictive coefficients have the “wrong sign.” Under the zero cash flow correlation assumption, the sign of  $B$  which is negative -0.013 should be opposite the sign of the predictive coefficients. But as seen, the predictive coefficients are negative at all horizons. This leads to the significant rejections of the null for the first two tests. Test 3, which is the most general and in principle the least powerful, also reject the null for forecasting horizons from about 20 to 50 months.

[Table 8 about here.]

[Figure 10 about here.]

The results for the slope of the term structure (SLOPE), defined as the difference between yield-to-maturity on 10 and 1 year Treasuries, are reported in Table 8 and Figure 10. SLOPE is by far the stronger predictor of monthly stock returns of the variables considered here. The  $R^2$ 's peak at 16.3% for the 51 month forecasting horizon. The upper plot in Figure 10 shows that the predictive coefficients are statistically significantly different from zero at all but the very short and very long horizons.

The predictability of SLOPE is however not consistent with equilibrium under the no cash flow correlation assumption as tests 1 and 2 reject the null at all horizons. To see where the

rejections are coming from, consider again the contemporaneous regression slope obtained by regressing returns on changes in SLOPE reported to be 0.018 in Table 8. This implies that the predictability coefficients ought to be negative. Of course they are not, and the fact that they are statistically significantly positive lead to the sharp rejections by tests 1 and 2. Test 4 also rejects, based on the estimated AR(p) and the non-zero contemporaneous cash flow assumption.

## 4 Discussion and Concluding Remarks

Predictability is widely believed to be consistent with equilibrium models. In this paper I challenge this view. In particular, I show that the predictable patterns we find in equity return data fail to be consistent with the patterns we would find if the equity return data were anything close to what the DGP is inside of standard equilibrium models.

The puzzle pieces are as follows: Equilibrium models feature persistent risk variables that generate predictability in the short, but not the long run. Intuitively, in a single factor equilibrium model, this is so because expected excess stock returns mean revert at the same rate as the priced risk variable. Thus, if the risk variable follows an AR(1) or any other process with a decaying autocorrelation pattern, while maintaining covariance stationarity, the term structure of predictive  $R^2$ s decay at the same rate as the autocorrelation of the risk variable. This is almost entirely the opposite of what we find in the predictability literature. As shown here, the two variables that have the highest predictive power in the data are the VRP and SLOPE. Both show an increasing term structure of  $R^2$ s that is essentially inconsistent with the idea that investors get a premium for holding risky assets when the risks are at their highest. Consider for example the VRP. Using either of the two definitions of physical expected realized variance I study here, the VRP predicts returns at horizons that way exceed the time for which a shock to VRP will die out. For this reason, it simply cannot be the case that VRP predicts returns at long horizons. My empirical tests show that VRP predictability is inconsistent with the basic notion of equilibrium used here at long horizons in all four tests that I employ.

This of course begs the question, where does long horizon predictability come from? Boudoukh, Richardson, and Whitelaw (2006) show that if the slope of the one-period ahead forecasting coefficient,  $b_1$ , is positive (negative), the remaining term structure of predictive coefficients are likely to show a monotonically increasing (decreasing) pattern as well. This happens simply because the coefficients themselves are very highly correlated. Their term structure slope is positively dependent on the autocorrelation of the predictor variable. Since all the predictors that are being used in stock return predictions tend to be highly autocorrelated (that is certainly true of

the ones I consider here), the term structure of regression slopes will look monotonically increasing or decreasing. This is more or less exactly what I find for all the variables I study. Thus, the predictability I examine is much more likely an outcome of spurious correlation and what Boudoukh, Richardson and Whitelaw would call “Myth of long-horizon predictability.”

The second rather depressing part of the puzzle is that the variables that ought to predict stock returns, don’t. Specifically, there are significantly compelling arguments to suggest that some macro-wide notion of uncertainty should predict returns. In Bansal and Yaron (2004) and other single factor long run risk models, both option implied variance and conditional stock market variance will be a linear function of spot consumption variance. Thus, VIX or squared VIX or measures of physical variance ought to predict returns. Eraker and Wu (2017) develop a model where VIX strongly predicts returns. The term structure of predictability, as in this paper, is monotonically decreasing at the same rate as the predictability of the squared VIX itself.

In the data, however, these relations do not hold. Squared VIX predicts returns but only at horizons of more than 100 days. By contrast, the zero cash flow correlation assumption implies that returns ought to be strongly predictable at the short term horizon only. In fact, the sharpest rejections of the null I find are for the short maturity VIX for tests 1 and 2. The term structure of predictability found for VIX, where predictability is virtually zero at all horizons shorter than 100 days, is an example of the inconsistency between long run empirical predictability and equilibrium. The empirical evidence suggests that investors will hold equity with no additional risk premium when volatility is high (as measured by VIX), and happily wait 100 days until purchasing stocks, thereby most likely experiencing lower volatility and higher returns at the same time. Clearly it is difficult to device an equilibrium model that could support these time-variations in risk and expected return.

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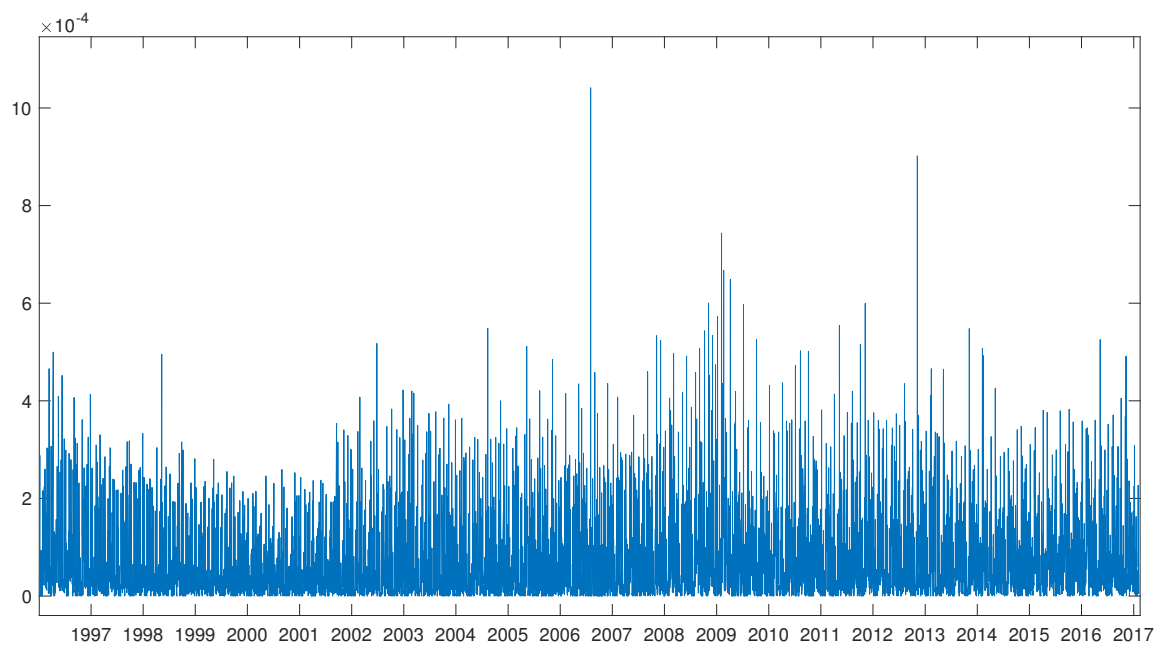


Figure 1: Plot of  $\ln\left(1 + \frac{D_{t+1}}{P_{t+1}}\right)$  from 1996-2017 using daily data.

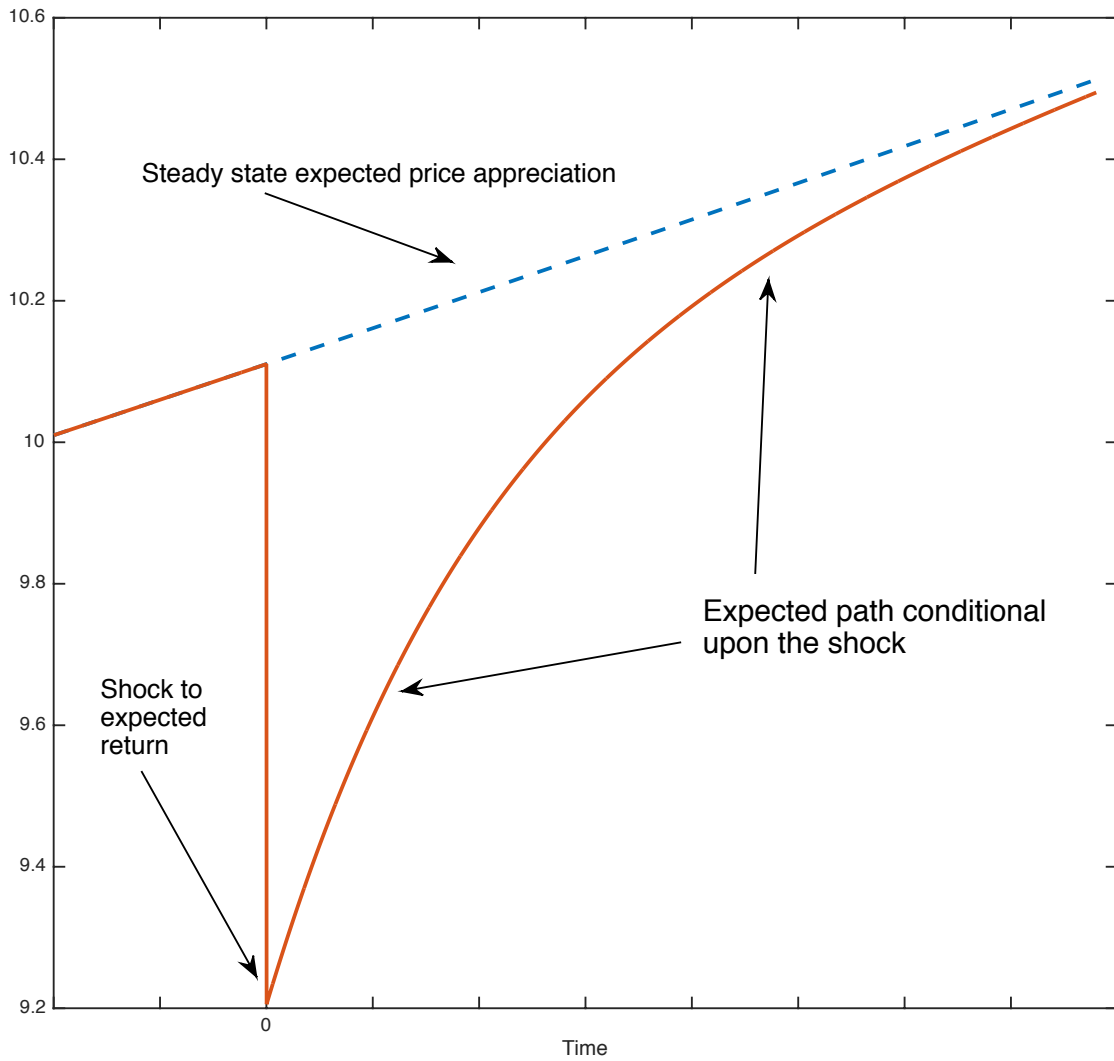


Figure 2: Price impact of a positive shock to expected return.



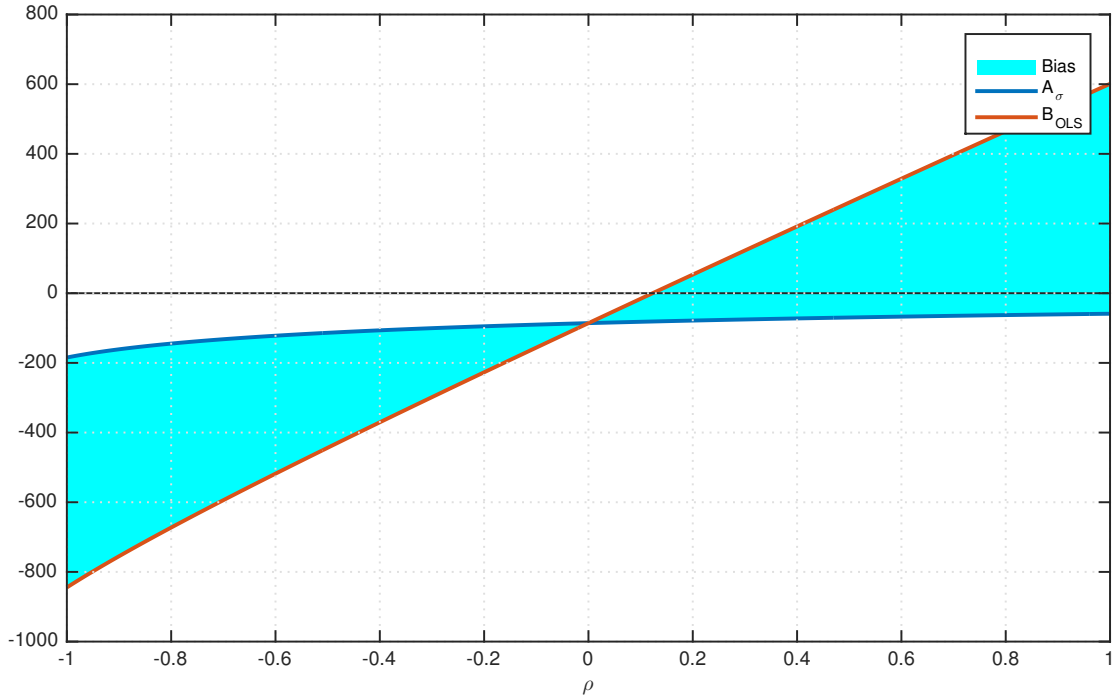


Figure 3: Bias in the OLS estimate  $B_{OLS}$  in the regression  $\Delta \ln P_{t+1} = a + B_{OLS} \Delta \sigma_{t+1}^2 + \epsilon_{t+1}$  when the error term  $\epsilon_{t+1}$  is correlated with the innovations in volatility,  $\text{Corr}(\nu_{t+1}, w_{t+1}) = \rho$ . The graph shows the theoretical factor loading,  $A_\sigma$ , and the population value of  $B_{OLS}$  as a function of  $\rho$ . The plot is generated using parameter values  $\gamma = 7.5, \psi = 2.5, \kappa = 0.97, k_1 = 0.999, \sigma_w = 0.0015, E(\sigma_t^2) = 0.0078^2$ .

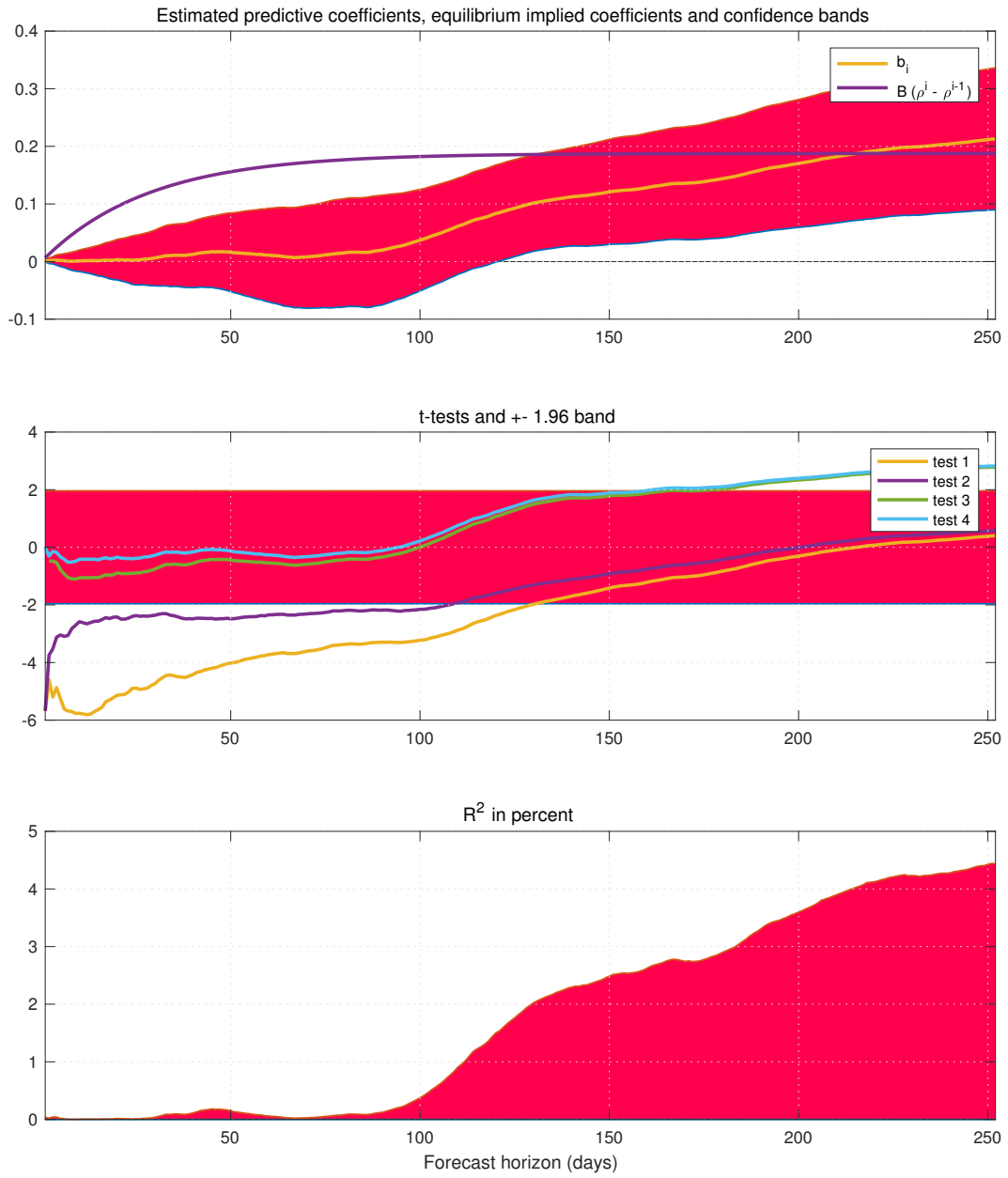


Figure 4: Empirical tests for VIX.

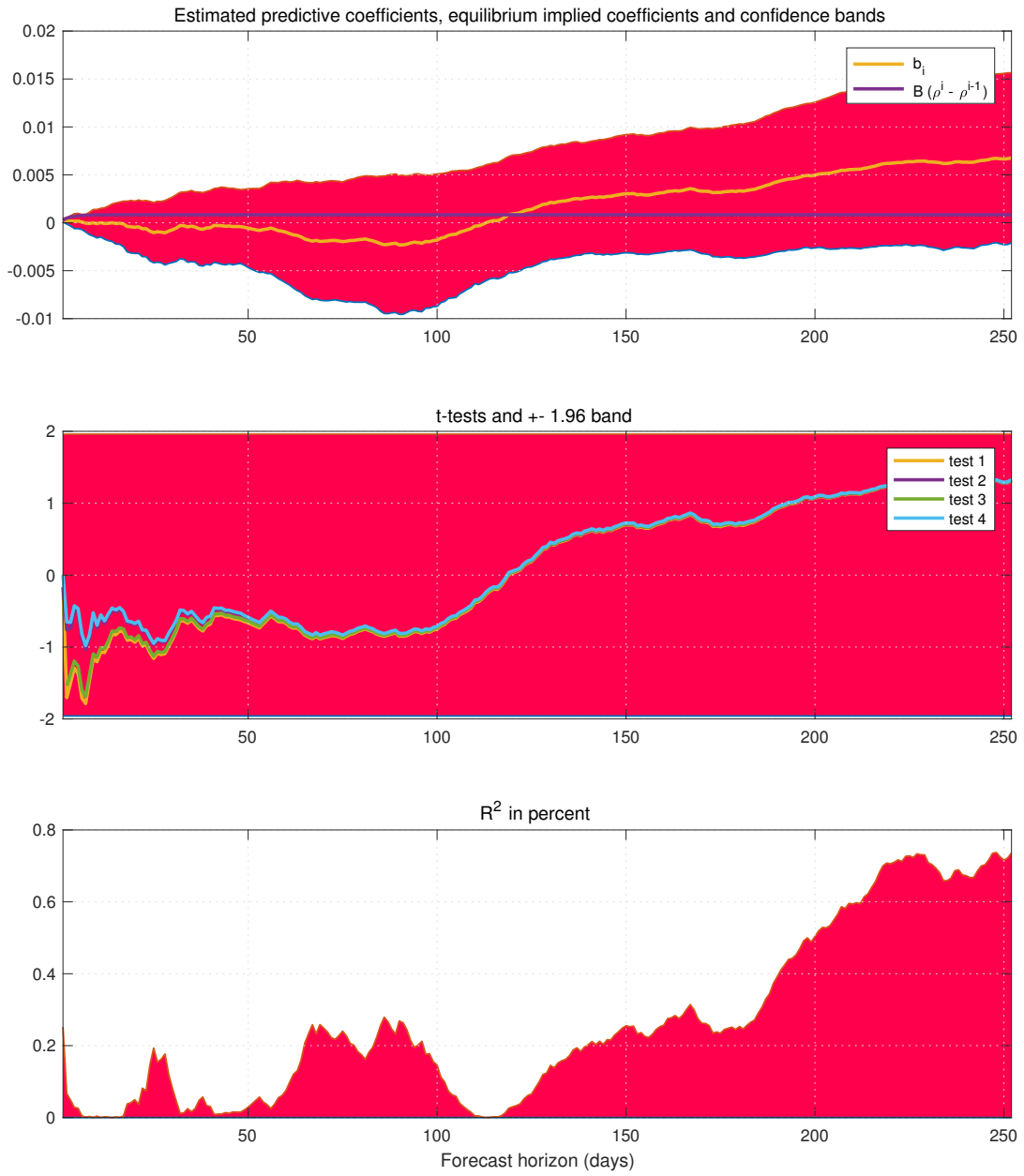


Figure 5: Empirical tests for one day Realized Variance.

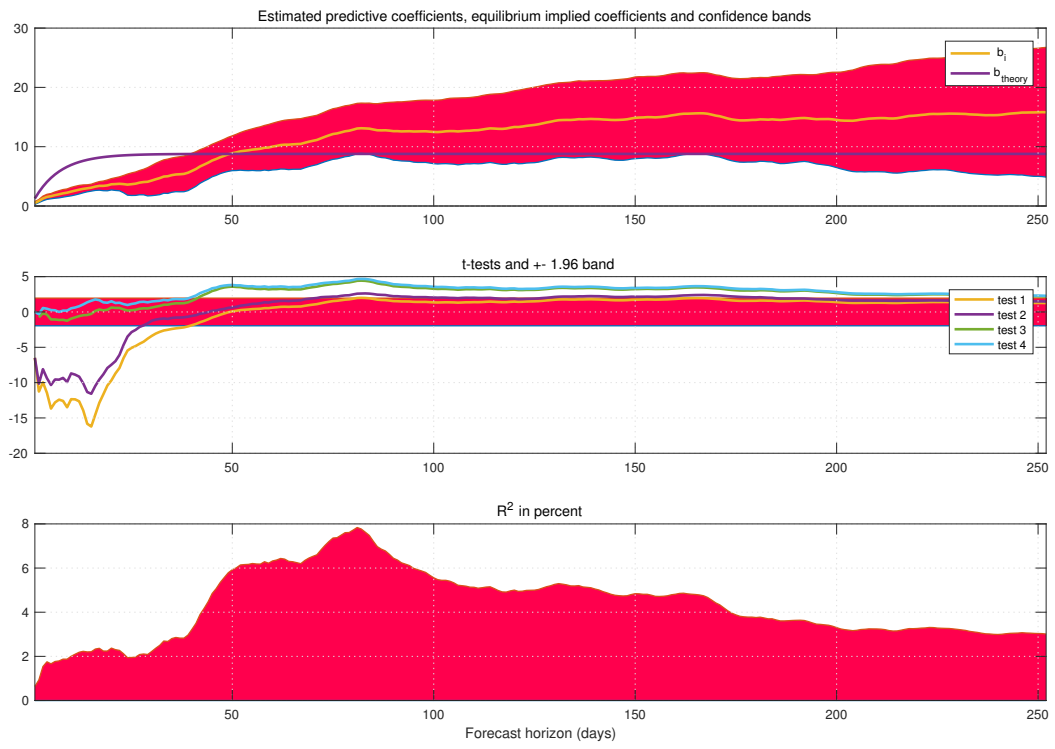


Figure 6: Empirical tests for VRP.

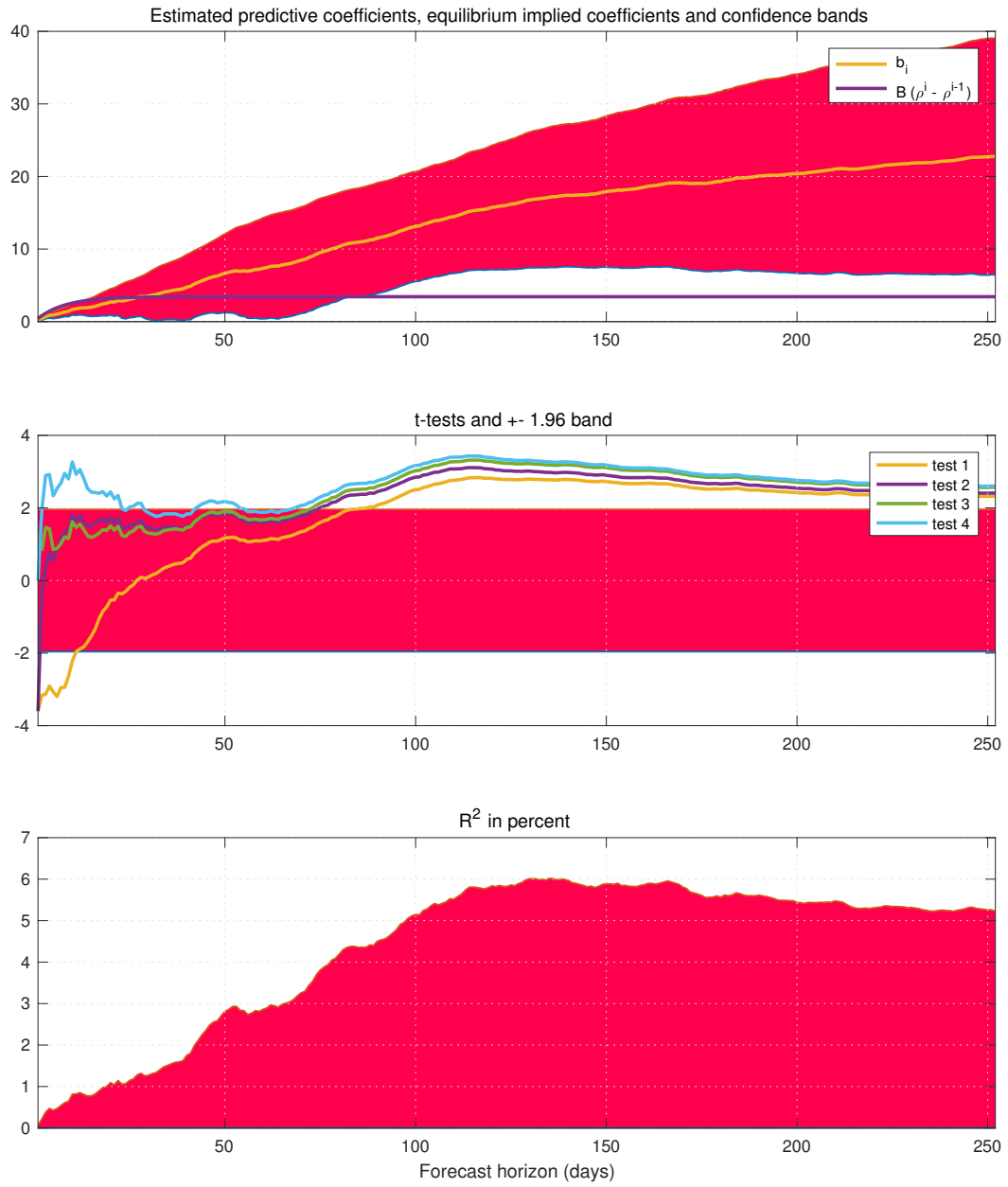


Figure 7: Empirical tests for VRP using forecasted RV in the definition of the VRP.

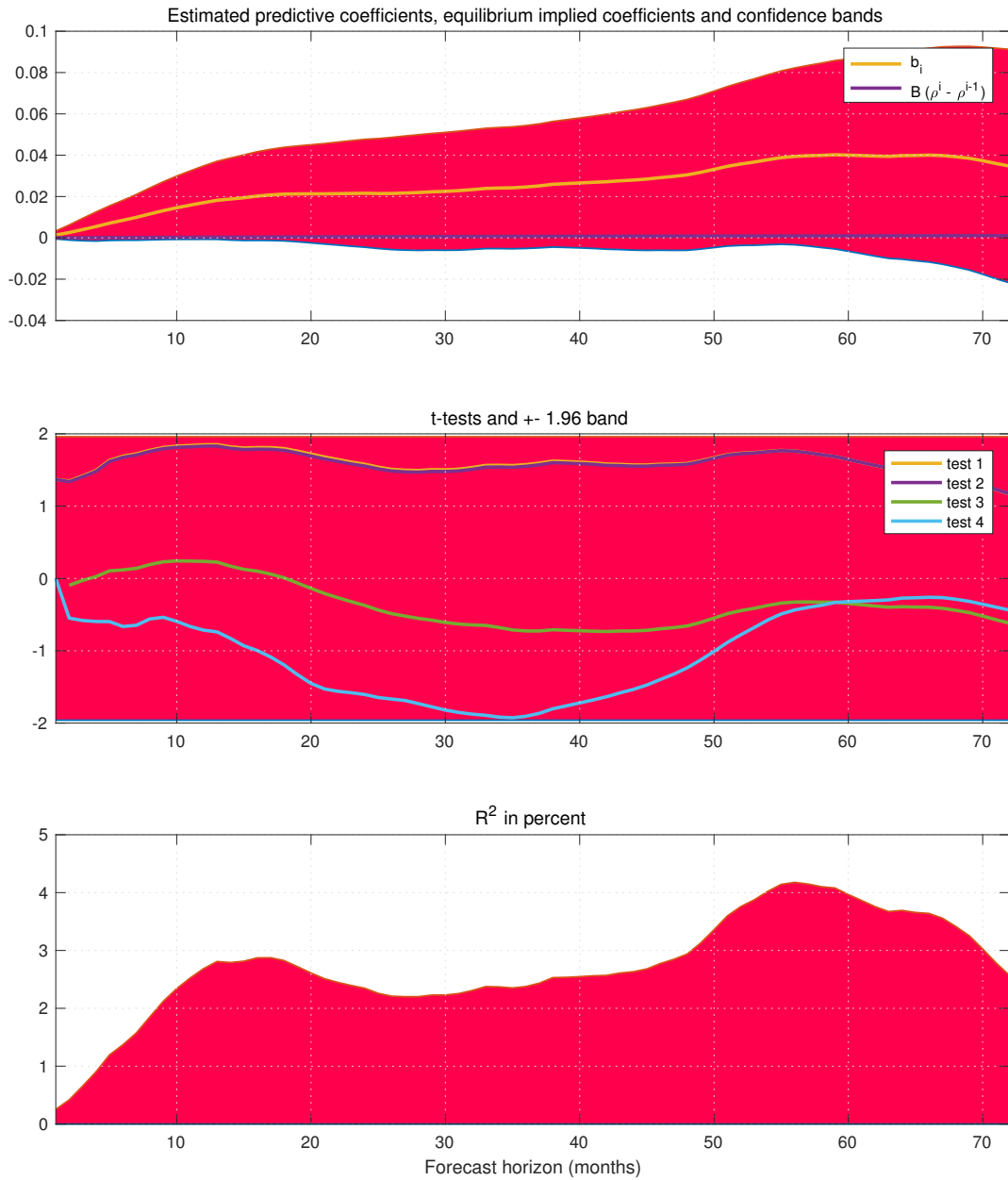


Figure 8: Empirical tests for default spread using monthly data from 1926-2016.

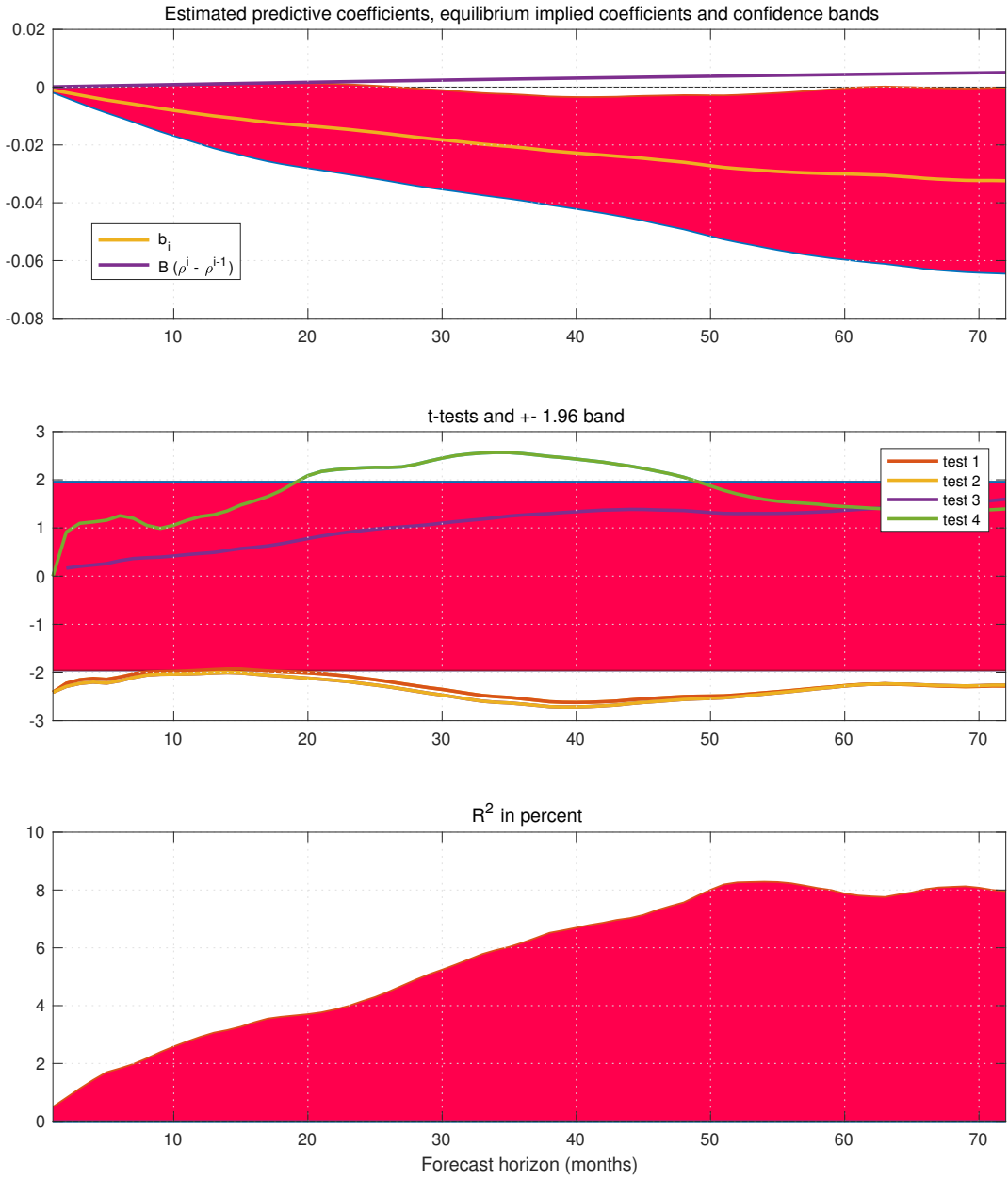


Figure 9: Empirical tests for TBILL using monthly data from 1933-2017.

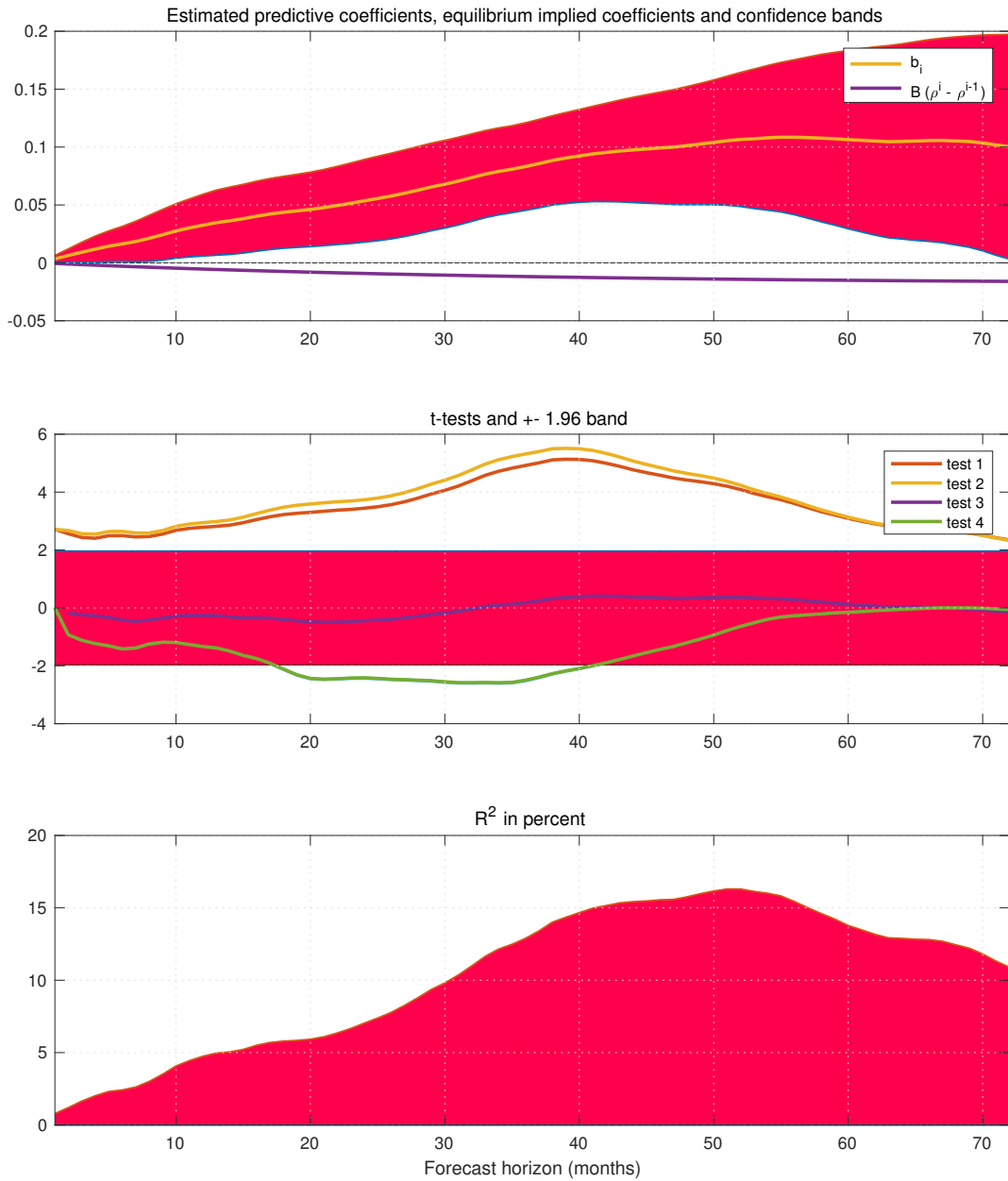


Figure 10: Empirical tests for SLOPE using monthly data from 1926-2016.



Table 1: Dividend Growth Variance Ratio

This table reports the fraction of variation in dividend growth relative to total return variation  $\text{Var}(y_t)/\text{Var}(\ln R_{t+1})$  where  $y_t = \ln\left(1 + \frac{D_{t+1}}{P_{t+1}}\right)$ . The variance ratio is shown in percent at various sampling frequencies.

Day	1M	1Q	1Y
0.049%	0.159%	0.157%	0.501%

Table 2: Predictability of VIX

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  the VIX<sup>2</sup> index in Panel A. Panel B reports the autocorrelation (AC) of VIX<sup>2</sup>, the correlation between  $h$  period ahead returns and current VIX<sup>2</sup>, and test statistics  $t_1, \dots, t_4$  as described in the test.

Panel A: Contemporaneous regression/ correlation						
	$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$				
	-0.188 (0.003)	-0.713 (0.011)				
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.965	0.019	-5.681	-5.681		0.000
3	0.928	0.020	-5.197	-3.523	-0.481	-0.142
4	0.910	0.020	-4.871	-3.119	-0.533	-0.178
5	0.901	0.014	-5.251	-3.042	-0.779	-0.332
10	0.862	0.006	-5.761	-2.583	-1.062	-0.419
22	0.727	0.010	-5.103	-2.494	-0.914	-0.386
44	0.502	0.041	-4.232	-2.474	-0.441	-0.085
66	0.409	0.016	-3.684	-2.351	-0.612	-0.342
132	0.158	0.145	-1.914	-1.261	1.552	1.684
252	0.058	0.211	0.399	0.573	2.778	2.813

Table 3: Predictability of RV

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  the Realized Variance (RV) computed from one day intraday returns in Panel A. Panel B reports the autocorrelation (AC) of RV, the correlation between  $h$  period ahead returns and current RV, and test statistics  $t_1, \dots, t_4$  as described in the test.

Panel A: Contemporaneous regression/ correlation						
	$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$				
	-0.001	-0.127				
	(0.000)	(0.016)				
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.649	0.050	-0.168	-0.168		
3	0.522	0.022	-1.501	-0.741	-1.373	-0.657
4	0.566	0.017	-1.293	-0.479	-1.194	-0.427
5	0.529	0.016	-1.370	-0.521	-1.265	-0.465
10	0.445	-0.006	-1.205	-0.731	-1.142	-0.696
22	0.363	-0.029	-0.985	-0.782	-0.953	-0.761
44	0.179	-0.012	-0.580	-0.508	-0.557	-0.489
66	0.152	-0.048	-0.864	-0.822	-0.848	-0.809
132	0.044	0.039	0.452	0.464	0.468	0.479
252	0.033	0.086	1.312	1.318	1.322	1.328

Table 4: Predictability of VRP

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  variance Risk Premium (VRP). The VRP is computed by taking the difference between squared VIX and one month lagged RV. Panel A reports the contemporaneous relationship between changes in VRP and returns. Panel B reports the autocorrelation (AC) of RV, the correlation between  $h$  period ahead returns and current RV, and test statistics  $t_1, \dots, t_4$  as described in the text.

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Panel A: Contemporaneous regression/ correlation						
		$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$			
		-8.510	-0.721			
		(0.113)	(0.010)			
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.856	0.080	-6.155	-6.155		0.000
3	0.671	0.124	-9.457	-7.647	-0.184	0.564
4	0.590	0.132	-10.879	-8.869	-0.370	0.460
5	0.546	0.129	-13.016	-9.788	-1.059	0.275
10	0.363	0.142	-11.754	-8.217	-0.943	0.519
22	0.159	0.151	-7.483	-5.697	0.572	1.310
44	0.164	0.212	-0.850	0.129	2.678	3.082
66	0.208	0.250	0.891	1.710	3.199	3.537
132	0.148	0.230	1.862	2.254	3.415	3.577
252	0.136	0.174	1.308	1.516	2.203	2.289

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Table 5: Predictability of VRP II

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  variance Risk Premium (VRP). The VRP is computed by taking the difference between squared VIX and a forecast of next month's RV. Panel A reports the contemporaneous relationship between changes in VRP and returns. Panel B reports the autocorrelation (AC) of RV, the correlation between  $h$  period ahead returns and current RV, and test statistics  $t_1, \dots, t_4$  as described in the text.

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Panel A: Contemporaneous regression/ correlation						
		$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$			
		-3.433	-0.252			
		(0.183)	(0.013)			
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.872	0.026	-3.597	-3.597		0.000
3	0.815	0.062	-3.135	0.373	1.462	2.896
4	0.790	0.069	-2.905	0.759	1.424	2.922
5	0.774	0.065	-3.072	0.541	0.859	2.336
10	0.717	0.090	-2.219	1.806	1.623	3.268
22	0.682	0.107	-0.350	1.727	1.494	2.343
44	0.568	0.150	0.928	1.742	1.777	2.110
66	0.497	0.174	1.199	1.657	1.744	1.931
132	0.300	0.245	2.785	2.998	3.205	3.292
252	0.207	0.229	2.322	2.407	2.566	2.601

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Table 6: Predictability of Default Premium

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  default premium (DP). The spread is computed by taking the difference between bonds with BAA and AAA Moodys ratings. Panel A reports the contemporaneous relationship between changes in DS and returns. Panel B reports the autocorrelation (AC) of DS, the correlation between  $h$  period ahead returns and current DS, and test statistics  $t_1, \dots, t_4$  as described in the text.

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Panel A: Contemporaneous regression/ correlation						
		$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$			
		-0.001	-0.010			
		(0.004)	(0.032)			
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.980	0.050	1.370	1.370		0.000
2	0.947	0.065	1.348	1.338	-0.094	-0.549
3	0.917	0.081	1.417	1.406	-0.027	-0.581
6	0.827	0.117	1.690	1.675	0.119	-0.664
12	0.654	0.164	1.849	1.829	0.237	-0.715
24	0.368	0.153	1.580	1.555	-0.373	-1.604
36	0.212	0.154	1.574	1.551	-0.726	-1.905
48	0.213	0.171	1.590	1.579	-0.656	-1.236
60	0.299	0.199	1.645	1.645	-0.345	-0.320
72	0.303	0.160	1.168	1.171	-0.618	-0.437

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Table 7: Predictability of TBill levels

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t = 3$  month TBILL rate. Panel A reports the contemporaneous relationship between changes in TBILL and returns. Panel B reports the autocorrelation (AC) of TBILL, the correlation between  $h$  month ahead returns and current TBILL, and test statistics  $t_1, \dots, t_4$  as described in the text.

Panel A: Contemporaneous regression/ correlation						
	$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$				
	-0.013	-0.106				
	(0.004)	(0.032)				
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.993	-0.071	-2.410	-2.410		0.000
2	0.982	-0.090	-2.225	-2.290	0.165	0.916
3	0.972	-0.106	-2.151	-2.228	0.202	1.096
6	0.944	-0.135	-2.093	-2.174	0.320	1.253
12	0.896	-0.171	-1.954	-2.020	0.468	1.238
24	0.783	-0.204	-2.116	-2.228	0.944	2.247
36	0.708	-0.249	-2.546	-2.656	1.268	2.545
48	0.672	-0.275	-2.499	-2.559	1.362	2.053
60	0.662	-0.281	-2.274	-2.281	1.366	1.444
72	0.640	-0.282	-2.282	-2.264	1.599	1.396

Table 8: Predictability of Term Structure Slope

This table reports the results of the contemporaneous regression  $r_t = A + B\Delta x_t + u_t$  for  $x_t =$  term structure slope. The slope is defined as the difference between 10 and 1 year yield on Treasuries. Panel A reports the contemporaneous relationship between changes in Slope and returns. Panel B reports the autocorrelation (AC) of Slope, the correlation between  $h$  period ahead returns and current Slope, and test statistics  $t_1, \dots, t_4$  as described in the text.

Panel A: Contemporaneous regression/ correlation						
	$B_{OLS}$	$\text{Corr}(r_t, \Delta x_t)$				
	0.018	0.110				
	(0.006)	(0.036)				
Panel B: Predictive relations						
Horizon	AC	$\text{Corr}(r_{t+i}, x_t)$	$t_1$	$t_2$	$t_3$	$t_4$
1	0.972	0.089	2.712	2.712		0.000
2	0.926	0.110	2.561	2.672	-0.182	-0.929
3	0.886	0.128	2.433	2.564	-0.233	-1.119
6	0.779	0.155	2.497	2.644	-0.424	-1.416
12	0.589	0.218	2.784	2.943	-0.263	-1.335
24	0.210	0.265	3.443	3.735	-0.444	-2.414
36	-0.064	0.359	4.921	5.317	0.179	-2.495
48	-0.074	0.397	4.418	4.646	0.335	-1.203
60	0.090	0.371	3.092	3.133	0.123	-0.152
72	0.165	0.330	2.349	2.335	-0.155	-0.056