Abstract

Does the rise of intangible capital affect financial stability? Intangible capital is not pledgeable, so firms hoard liquidity in the form of bank debt (e.g., deposits) for intangible investments. Firms’ liquidity demand pushes down the interest rate, giving banks a funding cost advantage. In booms, banks bid up asset prices as they grow. Higher asset prices induce firms to invest more in intangibles and hoard more liquidity. Stronger liquidity demand leads to an even lower interest rate, enabling banks to bid up asset prices even further. This paper builds a model of macroeconomy that demonstrates this feedback mechanism. It helps explain several concurrent phenomena in the run-up to the Great Recession, and reveals how endogenous risk accumulates in booms and materializes into severe and stagnant crises.
1 Introduction

Five trends in the United States have attracted enormous attention in the two decades leading up to the Great Recession:

(1) The economy was transforming to an intangible-intensive economy. Production of goods and services increasingly relies on intangible capital, such as brand name, technologies, and organizational capital. Intangible investment overtook physical investment as the largest source of growth in the period of 1995-2007 (Corrado and Hulten (2010)).

(2) An increasing share of non-financial corporations’ assets were cash holdings (Bates, Kahle, and Stulz (2009)). “Cash” is mainly financial intermediaries’ debts, such as bank deposits and repurchase agreements (held through money market funds). The rise of corporate cash holdings is largely due to a growing R&D-intensive sector (Falato and Sim (2014); Begenau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015)).

(3) Assets held by the financial sector increased dramatically (Adrian and Shin (2010a); Gorton, Lewellen, and Metrick (2012); Greenwood and Scharfstein (2013)). Schularick and Taylor (2012) find that in advanced economies, bank loan-to-GDP ratio doubled in the last two decades. The growth of financial sector was largely financed by issuing money-like debt securities (Adrian and Shin (2010b); Gorton (2010); Gorton and Metrick (2012); Pozsar (2014)).

(4) Interest rate declined steadily.

(5) The prices of risky assets increased across asset classes.

Despite extensive debates on the causes and implications of these phenomena, there are few theories that analyze them jointly. Motivated by Fact (1), I build a continuous-time model of macroeconomy that highlights the illiquidity of intangible capital and offers a coherent account of Fact (2) to (5). Moreover, the model reveals a new mechanism of endogenous risk accumulation that helps understand recent findings in a surging empirical literature: a longer period of bank expansion precedes a more severe banking and economic crisis (Jordà, Schularick, and Taylor (2013); Baron and Xiong (2016)). At the center of the mechanism are firms’ needs to hoard money for future investments and banks’ role as inside money creators.1

1The term, inside money, is borrowed from Gurley and Shaw (1960). From the private sector’s perspective, gold,
The model is built upon a simple narrative. The increasing reliance on intangible capital in the production sector implies a shrinkage of pledgeable assets (physical capital, such as properties, plants, and equipments). As a result, producers hoard more cash in anticipation of liquidity needs (e.g., R&D). Cash takes the form of intermediaries’ short-term debts, such as deposits, which are directly used as means of payment, and close substitutes (e.g., repurchase agreements held through money market funds). Corporate money demand feeds leverage to the financial sector, so intermediaries acquire more assets and push up asset prices. The rising corporate money demand also pushes down interest rates, i.e., yields or expected returns on holdings of money (e.g., deposit rate) or substitutes (e.g., asset-backed commercial paper rate).

The model has two types of agents, bankers and entrepreneurs. They consume generic goods produced by firms using tangible and intangible capital. Each entrepreneur manages one firm. Tangible capital is perfectly liquid in the sense that all of its cash flows are pledgeable and any claims on them can be traded in secondary markets. To simplify the exposition, it is assumed that a firm’s tangible capital is traded without any friction between the entrepreneur and bankers. We can think of tangible capital as inventory, equipments, and real estate. In reality, such assets may not be traded directly, but securities backed by them as collateral are traded actively. The market of tangible capital captures such liquidity. Intangible capital is illiquid, representing human capital, organizational capital, and certain proprietary technologies that are inalienable.

Entrepreneurs choose consumption and tangible capital holdings, and receive goods produced by their intangible capital. Capital depreciates stochastically, loading on a Brownian shock, which is the only source of aggregate risk in this economy. Entrepreneurs also face idiosyncratic liquidity shocks whose arrival follows a Poisson process. When hit by this shock, a firm loses all capital, but the entrepreneur is endowed with a technology that transforms goods into a fixed mixture of new tangible and intangible capital. This mixture reflects the necessity of having both types of capital for firms’ production. New tangible capital can be pledged for external funds, but intangible capital is illiquid. Therefore, intangible investment requires entrepreneur to hold a liquidity buffer. Entrepreneurs hold liquidity in the form of bank deposits (short-term safe debt) so

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fiat money, or government securities are in positive supply (“outside money”), while as bank liabilities, deposits are in zero net supply (“inside money”). See Lagos (2008) for a brief review of the related literature.
that when the liquidity shock hits, they may use deposits to buy goods as investment inputs.

Banks hold a diversified portfolio of different firms’ tangible capital, and issue deposits ("inside money") that earn a liquidity premium from entrepreneurs because they serve as a hedge against liquidity shocks. Banks add value by issuing deposits, the liquid store of value that entrepreneurs need to hold for intangible investments. This stands in contrast with the current literature that emphasize the asset side of bank balance sheet – banks are important because they extend credit. The model shares one feature with other macro-finance models, that is banks’ equity issuance constraint. Specifically, banks cannot raise equity in the model. This is a simple method to condition the equilibrium dynamics on banks’ balance-sheet capacity, so that we can discuss the implications of intangible capital, and the associated liquidity demand, on financial crises.

This setup helps explain the fact (2) to (5) listed in the beginning of this paper, and reveal a feedback mechanism that is anchored at the illiquidity of intangible capital and generates amplified boom-bust cycles. Consider a sequence of positive shocks. Bankers’ wealth (equity) increases, and they acquire more tangible capital financed by issuing deposits. Because the liquidity premium on deposits lowers bankers’ debt cost, their required rate of return is lower than that of entrepreneurs. Therefore, as bankers get richer, they bid up the price of tangible capital. Here, tangible capital is broadly interpreted as assets in the real world whose cash flow is liquid and from firms’ production.

The rising asset price induces more liquidity holdings by entrepreneurs through two channels. First, creating new capital is more profitable. Since tangible and intangible capital must be created simultaneously, to profit from a higher valuation of tangible capital, entrepreneurs must hold enough liquidity to finance intangible investments. Second, the leverage on entrepreneurs’ liquidity holdings is higher. New tangible capital can be pledged for external financing from banks at fair price, so through this leverage, entrepreneurs enjoy more surplus from investments when tangible capital price and the leverage is higher. A stronger liquidity demand of entrepreneurs leads to a higher liquidity premium, which elevates the price of tangible capital even further by lowering bankers’ cost of debt and their required return. Finally, any increase of asset price is amplified by bank leverage and causes bank equity to rise even further. As banks expand their balance sheets, more deposits are issued and held by entrepreneurs, so more intangible investments are financed.
This positive feedback loop links and works through a rising asset price, a declining interest (de-
posit) rate, the expansion of banking sector, and the run-up of corporate liquidity holdings. When
negative shocks hit, the spiral is flipped downward, causing an amplified crisis dynamics.

At the center of this mechanism is an *intermediated liquidity premium*. Entrepreneurs assign
a liquidity premium to bank deposits, because deposits transfer wealth to the contingencies where
it is needed, working as a hedge against liquidity shocks. This premium lowers the debt cost of
banks and their required rate of return (discount rate), and thereby, translates into a higher price of
tangible capital. This transmission of liquidity premium works through banks’ balance sheet, and it
works better when bankers are wealthier and able to acquire more tangible capital. In other words,
a more well-capitalized banking sector intermediates the liquidity premium more effectively.

The channel of intermediated liquidity premium can cause endogenous risk accumulation in
booms. As previously discussed, a sequence of positive shocks lead to a declining interest rate and
rising liquidity premium by making intangible investments more important. This creates a widen-
ing gap between the required rate of return of bankers and that of entrepreneurs. This creates a widening gap between the required rate of return of bankers and that of entrepreneurs. The difference
the first-best (bankers) and second-best buyers (entrepreneurs) of tangible capital increases, so as
a boom prolongs, the price of tangible capital price becomes increasingly sensitive to the variation
of bankers’ wealth (equity). On the other hand, as bankers become richer, their wealth becomes
more robust to shocks. These two forces create a hump-shaped volatility of tangible capital price.
When the economy starts from a relatively low level of bank equity, asset price volatility rises as
the banking sector grows and the interest rate declines, making the economy increasingly fragile.
The price of tangible capital affects economic growth through entrepreneurs’ incentive to hoard
liquidity, so its volatility has strong impact on the long-run welfare.

The continuous-time framework allows a convenient characterization of the long-run be-

behavior of the economy. Specifically, the stationary density is shown together with its cumulative
probability function. Crises are rare, but severe due to the feedback mechanism. Moreover, crises
are stagnant. When a sequence of negative shocks knock the economy into recessions, banks are
undercapitalized and the price of tangible capital is depressed, which discourages entrepreneurs
from investment and liquidity hoarding. As a result, the liquidity premium declines, and banks
face higher cost of debt and lower return on equity. Therefore, to climb out of crises, bankers only accumulate wealth very slowly. The expected time to recovery is shown, and under the benchmark calibration, it takes almost ten years for the economy to grow out of recessionary states.

The last exercise of this paper is to study the model performances in response to a permanent increase of the productivity of intangible capital relative to tangible capital. The rise of intangible capital strengthens the feedback mechanism. Both interest rate and asset price are more sensitive to shocks. In particular, endogenous risk in the form of asset price volatility doubles. Entrepreneurs hold more cash in their deposit account, and the economy has a larger banking sector to cater such stronger demand for liquidity. The economy does grow faster thanks to more liquidity savings of entrepreneurs, but only with more endogenous risk. Therefore, the model characterizes a risk-return trade-off in the transition towards a more intangible-intensive economy.

**Literature.** The structural change towards a new economy that is intangible-intensive has attracted enormous attention (Corrado and Hulten (2010)). Studies explore the implications of this structural change in different areas from both theoretical and empirical perspectives, such as Atkeson and Kehoe (2005) (productivity accounting), McGrattan and Prescott (2010) (current account), Eisfeldt and Papanikolaou (2014) (asset pricing), Peters and Taylor (2016) (corporate investment), and Perez-Orive and Caggese (2017) (secular stagnation). This paper looks into the financial stability implications of intangible capital. There is a large literature on how financial development affects industrial structure (e.g., Levine (1997); Rajan and Zingales (1998)). However, the reverse question, how industrial structure affects the financial system, has not yet been well explored. This paper aims to sheds light on this question.

The model fits into the literature on heterogeneous-agent models. Many frictions manifest themselves into limits on aggregate risk-sharing between sectors. In this paper, the key friction is that between bankers and entrepreneurs, the only form of financial contracting is risk-free debt. Di Tella (2014) notes that perfect risk-sharing shuts down the balance-sheet channel (e.g., Kiyotaki and Moore (1997); Bernanke and Gertler (1989)). Many study the macroeconomic and asset pricing implications of dynamic wealth distribution among heterogeneous agents (e.g., Basak and Cuoco (1998); Krusell and Smith (1998); Longstaff and Wang (2012); He and Krishnamurthy
(2013); Brunnermeier and Sannikov (2014); Moll (2014)). This paper contributes to this literature by introducing a new type of heterogeneity, that is bankers as money suppliers and entrepreneurs as money demanders. It gives rise to the channel of intermediated liquidity premium.

A recent literature revives the money view of banking that emphasizes the liabilities of banks as inside money that lubricates the economy by facilitating trades (Kiyotaki and Moore (2000); Hart and Zingales (2014); Piazzesi and Schneider (2016); Quadrini (2017)). This paper takes a step further by modeling bankers as private money suppliers in an entrepreneurial economy that suffers from capital illiquidity, and shows how this leads to financial instability.\(^2\) The theoretical framework in this paper is inspired by Kiyotaki and Moore (2012) who study the emergence of bubbly outside money (or fiat money) from capital illiquidity (see also Farhi and Tirole (2012)).

This paper is motivated by the large literature on corporate cash holdings (e.g., Opler et al. (1999); Bates, Kahle, and Stulz (2009)). The secular increase of nonfinancial firms’ cash holdings in the last few decades was driven by the R&D-intensive sectors (e.g., Falato and Sim (2014); Begenau and Palazzo (2015); Graham and Leary (2015); Pinkowitz, Stulz, and Williamson (2015)). Physical capital relaxes financial constraints by serving as collateral (Almeida and Campello (2007)). The structural transformation towards an intangible economy shrinks the collateral pool, leading to a stronger incentive to hoard liquidity. Corporate treasuries have become a prominent component of “institutional cash pools” that lend to the financial sector, particularly through the money markets (Pozsar (2011)).\(^3\)

Theoretical models of corporate cash holdings often assume a storage technology (e.g., Froot, Scharfstein, and Stein (1993); Bolton, Chen, and Wang (2011); He and Kondor (2016)). These models characterize a rich environment of corporate decision making, but severs the link between corporate money demand and the equilibrium interest rate (i.e., the yield on money-like assets) by assuming a perfectly elastic supply of storage assets. An exception is Holmström and Tirole (1998) and (2001) who take a general equilibrium approach to study whether firms are able

\(^2\)Safe debt serves as money, which echoes the literature that links the information insensitivity of assets’ payout to assets’ monetary services (Gorton and Pennacchi (1990); Holmström (2012); Dang et al. (2014)).

\(^3\)Several papers have argued that the growth of the shadow banking sector is fueled by the demand of money-like securities (e.g., Gorton (2010); Gorton and Metrick (2012); Stein (2012); Pozsar (2014)).
to supply the assets they themselves need to hold as liquidity buffer. In this paper, firms’ liquidity demand translates into the demand for bank debt, so that interest rate, asset price, bank leverage, and corporate cash holdings can be studied in a unified framework.

The model predicts that a longer period of boom predicts more severe crisis (asset price collapse). Schularick and Taylor (2012) find the expansion of bank asset (loans) precedes financial crises in advanced economies. Jordà, Schularick, and Taylor (2013) document a close relationship between bank credit growth and the severity of subsequent recessions. Related, Baron and Xiong (2016) find that bank credit expansion predicts increased bank equity crash risk. The existing theories on endogenous accumulation of risk largely focus on belief distortions (e.g., Gorton and Ordoñez (2014); Moreira and Savov (2017)). This paper emphasizes financial frictions, and the illiquidity of intangible capital in particular, relating endogenous risk to interest rate dynamics.

The rest of the paper is organized as follows. Section 2 introduces the model and discusses the mechanisms. Section 3 solves the calibrated model and shows its quantitative performances. Section 4 concludes. Proofs and solution algorithm are provided in the appendices.

2 Model

2.1 Setup

Consider a continuous-time, infinite-horizon economy with two types of agents, bankers and entrepreneurs. Each type has a unit mass of representative agents. We fix a probability space and an information filtration that satisfy the usual regularity conditions, as defined by Protter (1990).

4 Holmström and Tirole (2001) explore the impact of corporate liquidity demand on the prices of liquid assets.

5 In line with Bordo et al. (2001) and Reinhart and Rogoff (2009), Schularick and Taylor (2012) define financial crises as events during which a country’s banking sector experiences bank runs, sharp increases in default rates accompanied by large losses of capital that result in public intervention, bankruptcy, or forced merger of financial institutions. Using the information from credit spreads, Krishnamurthy and Muir (2016) are able to achieve a sharper differentiation between financial and non-financial crises. They also find that bank credit expansion precedes financial crises. Using both the information from credit spreads and the composition change of corporate bond issuance, López-Salido, Stein, and Zakrjasjek (2015) find that when the issuance of high-yield (“junk”) bond outpaced the total bond issuance, and when corporate bond credit spreads are narrow relative to their historical norms, the subsequent real GDP growth, investment, and employment tend to decline.
Agents face idiosyncratic Poisson shocks and aggregate Brownian shocks that will be elaborated later, and make decisions under rational expectation.

**Preferences.** Agents are risk-neutral and maximize expected utility with discount rate $\rho$:

$$E \left[ \int_{t=0}^{\infty} e^{-\rho t} dc_i^t \right], \quad i \in \{B, E\}.$$  

(1)

Throughout this paper, subscripts denote time, and superscripts denote types or sectors. “$B$” is for the banking sector and “$E$” for the entrepreneurial sector. For example, $c_t^E$ is the representative entrepreneur’s *cumulative* consumption up to time $t$.

**Capital and production.** Each entrepreneur manages a firm that produces non-durable generic goods using tangible and intangible capital. In aggregate, the economy has $K_t^T$ units of tangible capital and $K_t^I$ units of intangible capital at time $t$. One unit of tangible capital produces constant $\alpha$ units of goods per unit of time. The productivity of intangible capital is $\alpha + \phi$ (where $\phi > -\alpha$). So from $t$ to $t+dt$, the aggregate output is $\alpha K_t^T + (\alpha + \phi) K_t^I dt$.

The two types of capital differ in liquidity. An entrepreneur can sell her firm’s tangible capital to bankers in a competitive market at price $q_t^T$ per unit (denominated in goods). After the sale, entrepreneurs dutifully manage the capital on behalf of banks and deliver the goods produced, so tangible capital is perfectly liquid in the sense that it is free from any agency friction or asymmetric information. We may think of tangible capital as inventory, equipments, and houses. In reality, investors may not trade physical assets directly, but they invest in and trade securities backed by such assets as collateral. The secondary market captures such liquidity.

In contrast, intangible capital is illiquid. It is attached to the firm and entrepreneur, and cannot be sold or pledged to outside investors. Intangible capital represents human capital, organizational capital, and certain proprietary technologies that are inalienable.\(^6\)

**Aggregate shock.** The only source of aggregate uncertainty in this economy is the Brownian

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\(^6\)The illiquidity of intangible capital may also arise from the difficulty to measure it. Corrado, Hulten, and Sichel (2005) and McGrattan and Prescott (2010) discuss the measurement of technology capital. See Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014) for organization capital.
As in Brunnermeier and Sannikov (2014), a fraction $\delta \, dt - \sigma \, dZ_t$ of capital, both tangible and intangible, are destroyed over $dt$. We may count capital by its output. For example, certain number of machines constitute one unit of tangible capital if they are responsible for $\alpha$ units of goods per unit of time. Intangible capital is counted likewise. Therefore, capital is productive unit, and the capital destruction shock may accordingly be interpreted as a productivity shock.

**Liquidity shock and investment.** Entrepreneurs experience liquidity shocks. The arrival of liquidity shock is independent across entrepreneurs (i.e., idiosyncratic), and follows a Poisson process with intensity $\lambda$. When hit by a liquidity shock, firms lose all capital, but entrepreneurs are endowed with a technology to transform goods into new capital instantaneously, and once investments are made, old capital is restored. Tangible and intangible investments are made simultaneously and proportionally: for every $1 - \theta$ units of tangible capital, $\theta$ units of intangible capital must be created, vice versa. As a result, the intangible fraction of aggregate capital, $K_I^t / (K_I^t + K_T^t)$, is fixed at $\theta$ under the initial condition that $K_I^0 / (K_I^0 + K_T^0) = \theta$.

$\theta$ determines the pledgeability of investment project. Specifically, if an entrepreneur invests $i_t^E$ units of goods, it creates $\kappa i_t^E$ units of capital (constant $\kappa > 0$ for investment efficiency), of which $\theta$ fraction is intangible and unpledgeable. The intangible component is a private benefit for entrepreneurs, while the tangible component can be pledged to banks for financing. Since the creation of capital is immediate, the entrepreneur repays banks instantaneously with new tangible capital. The investment scale, $i_t^E$, is constrained by the liquid resources:

$$i_t^E \leq \kappa i_t^E (1 - \theta) \frac{q_t^T + m_t^E}{\text{units of new tangible capital}},$$

where $m_t^E$ is the entrepreneur’s liquidity holdings.\(^7\) The higher $\theta$ is, the more an entrepreneur relies on $m_t^E$ to finance investments. When $\theta = 0$, i.e., only tangible investment is made, the project becomes self-financed as long as $\kappa q_t^T \geq 1$.

Entrepreneurs hold liquidity in the particular form of “bank deposits” that are short-term debt

\(^7\)Intangible investments rely heavily on firms’ internal liquidity (for example, R&D investments in Hall (1992), Himmelberg and Petersen (1994), and see Hall and Lerner (2009) for a review).
of banks with interest rate \( r_t \) (issued at time \( t \) and mature at \( t + dt \)). As will be shown later, there is a unique Markovian equilibrium where banks never default, so their debt is safe. When hit by liquidity shocks, entrepreneurs use their deposits to purchase goods as investment inputs, so bank debt effectively serves as means of payment, or “inside money”, and facilitates goods reallocation towards those who need to invest.\(^8\)

Let \( q^I \) denote the value of intangible capital to the risk-neutral entrepreneur. It is simply the discounted sum of production flows:

\[
q^I = \frac{\alpha + \phi}{\rho + \delta},
\]

where the denominator contains the discount rate (\( \rho \)) and the expected destruction rate from stochastic depreciation (\( \delta \)). On the equilibrium path, entrepreneurs’ deposit holdings, \( m^E_t \), is never zero, so some investments are always made and old capital is preserved as a result. Therefore, the denominator of \( q^I \) does not count for the arrival of liquidity shock. Per unit of investment, the new capital created is worth \( q^I \theta \kappa + q^T_t (1 - \theta) \kappa \). Going forward, we shall focus on the case where \( \theta > 0 \) and \( \kappa \) is sufficiently high so that

\[
q^I \theta \kappa + q^T_t (1 - \theta) \kappa > 1,
\]

i.e., investment is a positive-NPV, scalable project. Thus, the entrepreneur wants to maximize the scale, and, (2) holds in equality. By rearranging the equation, we can relate the investment scale to deposit holdings \( m^E_t \) through the leverage obtained from pledging tangible capital:

\[
i^E_t = \left( \frac{1}{1 - \kappa (1 - \theta) q^T_t} \right) m^E_t.
\]

Project leverage increases in the market value of tangible capital, \( q^T_t \), and investment efficiency

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\(^{8}\)The term is borrowed from Gurley and Shaw (1960). From the private sector’s perspective, fiat money and government securities are in positive supply (“outside money”), while deposits, as bank liabilities, are in zero net supply (“inside money”) See Lagos (2008) for a brief review of the related literature. The moneyness of safe debt echoes the theories that links assets’ information insensitivity to their monetary services (e.g., Gorton and Pennacchi (1990); Holmström (2012); Dang et al. (2014)).
parameter $\kappa$, and decreases in the project illiquidity parameter $\theta$.

The fundamental friction in this economy is the illiquidity of intangible capital. If intangible capital is liquid (pledgeable), the investment project can be self-financed, and thus, entrepreneurs do not need to carry the liquidity buffer $m^E_t$. Equation (4) is a critical element of the model: under the illiquidity of intangible capital, entrepreneurs’ investment is tied to deposit holdings $m^E_t$ when hit by the liquidity shock. Therefore, banks add value to this economy by issuing deposits, a liquid store of value for entrepreneurs.\footnote{Holmström and Tirole (1998) consider a more general case where firms hold liquidity in the form of other firms’ liabilities. Here, the focus is on bank debt as liquidity. There are several reasons why entrepreneurs hold \textit{intermediated liquidity}. Entrepreneurs may simply lack the required expertise of asset management. There is a large literature on households’ limited participation in financial markets (Mankiw and Zeldes (1991); Basak and Cuoco (1998); He and Krishnamurthy (2013)), but firms’ portfolio choice is relatively less studied. Duchin et al. (2017) find firms hold risky securities, but what dominate are safe debts issued by intermediaries or governments. Moreover, cross holding is regulated in many countries and industries.}

\textbf{Banks.} Let $n^B_t$ denote the wealth of a representative banker who invests in firms’ tangible capital and raises leverage by issuing risk-free debt with interest rate $r_t$. A banker maximizes the risk-neutral utility in Equation (1) subject to the following budget constraint (flow of funds).

$$dn^B_t = x^B_t n^B_t dr^T_t + (1 - x^B_t) n^B_t r_t dt - dc^B_t,$$
\hspace{1cm} (5)

where $x^B_t$ is the fraction of wealth allocated to tangible capital, i.e., the asset-to-equity ratio or bank leverage, and $dr^T_t$ is the return on tangible capital. In equilibrium, because entrepreneurs hold a positive amount of bank debt as liquidity buffer, bankers issue debt, and thus, $x^B_t > 1$.

To express the return on tangible capital, $dr^T_t$, we can conjecture that in equilibrium, the
market price of tangible capital follows a diffusion process, i.e.,

\[
dq_i^T = q_i^T \mu_i^T dt + q_i^T \sigma_i^T dt.
\]  

Let \( k_{i,t}^B \) denote the units of tangible capital that banker \( i \) holds (\( k_{i,t}^B = x_i^B n_{i,t} / q_i^T \)). A fraction \( \delta dt - \sigma dZ_t \) are destroyed every instant. So, capital holdings depreciate stochastically:

\[
dk_{i,t}^B = - (\delta dt - \sigma dZ_t) k_{i,t}^B.
\]  

Using Itô’s lemma, we can solve the return on tangible capital:

\[
dr_i^T = \frac{\alpha k_{i,t}^B dt}{q_i^T k_{i,t}^B} + \frac{d (q_i^T k_{i,t}^B)}{q_i^T k_{i,t}^B} = \left( \frac{\alpha}{q_i^T} + \mu_i^T - \delta + \sigma_i^T \sigma \right) dt + (\sigma_i^T + \sigma) dZ_t.
\]  

The term, \( \sigma_i^T \sigma \), is from quadratic covariation of Itô’s calculus.

**Liquidity creation capacity.** Figure 1 summarizes the model structure. Entrepreneurs manage firms and produce goods using capital for both themselves and bankers who own tangible capital. There does not exist any agency friction. For one unit of tangible capital owned by bankers, entrepreneurs dutifully pass a cash flow of \( \alpha \) per unit of time. While the cash flow associated
tangible capital is fully liquid, the cash flow associated with intangible capital is not, and thus, entrepreneurs hold deposits issued by banks to finance intangible investments.

Let intervals $\mathbb{B} = [0, 1]$ and $\mathbb{E} = [0, 1]$ denote the sets of banks and entrepreneurs respectively. $N^B_t = \int_{i \in \mathbb{B}} n^B_{i,t} \, di$, is the aggregate net worth of the banking sector, and $M^E_t = \int_{i \in \mathbb{E}} m^E_{i,t} \, di$ is entrepreneurs’ aggregate demand for bank deposits. We have the following deposit market clearing condition:

$$M^E_t = (x^B_t - 1) N^B_t. \tag{9}$$

Here we utilize the homogeneity of bankers, that is every banker has the same $x^B_t$.

Banks add value by issuing deposits that entrepreneurs hold as liquidity buffer. Their capacity to create liquidity depends on their wealth (or equity). Following Holmström and Tirole (1997), Bolton and Freixas (2000), Van den Heuvel (2002), Brunnermeier and Sannikov (2014), and Klimenko et al. (2016), it is assumed that banks face equity issuance friction. In this model, banks do not issue equity at all, that is $dc^B_t > 0$. By inspecting banks’ budget constraint, Equation (5), we can see that negative consumption is equivalent to issuing equity.$^{11}$ This assumption makes the equilibrium dynamics conditioned upon bankers’ wealth, so the model relates capital intangibility, and the associated liquidity demand of entrepreneurs, to the likelihood and severity of balance-sheet crisis in the banking sector.

In this economy, the ultimate source of liquidity is tangible capital. While entrepreneurs cannot diversify away the Poisson shocks that destroy all of their firms’ capital, bankers can pool tangible capital and use it to back their debt, the inside money. However, to what extent bankers can provide such diversification or liquidity transformation service depends on their wealth. In this model, entrepreneurs’ liquidity demand in Holmström and Tirole (1998) meets banks’ limited balance-sheet capacity in Holmström and Tirole (1997).$^{12}

$^{11}$Negative consumption is allowed for risk-neutral entrepreneurs (except when the liquidity shock hits), which is interpreted as dis-utility from additional labor to produce extra goods as in Brunnermeier and Sannikov (2014). Allowing negative consumption fixes entrepreneurs’ required rate of return at $\rho$ and marginal value of wealth at 1, so that entrepreneurs’ wealth does not add to the dimension of aggregate state variable. Negative consumption serves the same purpose as assuming large endowments of goods. Because goods are nondurable, entrepreneurs always consume some to clear the goods market, and thus, their marginal value is pinned down at 1.

$^{12}$See also, Holmström and Tirole (2001), Eisfeldt and Rampini (2009), and Farhi and Tirole (2012) for investment-driven liquidity demand. Entrepreneurs’ liquidity demand is likely to be important from an asset pricing perspective.
Let \( K_t = K_t^I + K_t^T \) denote the total units of capital. It has the following law of motion:

\[
dK_t = - (\delta dt - \sigma dZ_t) K_t + \left[ \kappa \left( \frac{1}{1 - \kappa (1 - \theta) q_t^T} \right) M_t^E \right] \lambda dt + K_t \chi dt \tag{10}
\]

The first component is the stochastic depreciation, and the second component is from a measure \( \lambda dt \) of firms who invest an aggregate deposit holdings equal to \( M_t^E \lambda dt \) with a leverage equal to \( 1 / [1 - \kappa (1 - \theta) q_t^T] \) (as in Equation (4)). Finally, the economy is endowed with a flow of new capital \( (\theta K_t \text{ units are intangible, and } (1 - \theta) K_t \text{ units are tangible}) \), evenly distributed among \( \chi dt \) measure of newly born entrepreneurs. Such endowments capture sources of economic growth other than the liquidity-constrained investments. To fix the population size, it is assumed that entrepreneurs exit upon idiosyncratic Poisson arrival with intensity \( \chi \) and their wealth is evenly distributed among other entrepreneurs. So, their overall discount rate \( \rho \) can be decomposed into the exit probability \( \chi \) and a time-discounting rate \( \rho - \chi \).

Substituting the deposit market clearing condition (Equation (9)) into Equation (10), we can relate economic growth directly to bank equity:

\[
\frac{dK_t}{K_t} = \left\{ \kappa \left( \frac{1}{1 - \kappa (1 - \theta) q_t^T} \right) (x_t^B - 1) \left( \frac{N_t^B}{K_t} \right) \right\} \lambda - \delta + \chi \right \} dt + \sigma dZ_t \tag{11}
\]

The expected growth rate of capital, \( \mu^K_t \), increases in the ratio of bank equity, \( N_t^B \), to capital stock, \( K_t \). When banks are well-capitalized and issue abundant deposits, the economy grows fast as entrepreneurs are able to invest more out of their liquidity holdings. \( \mu^K_t \) also increases in the market price of tangible capital, \( q_t^T \), which increases the leverage on liquidity holdings and amplifies the investment scale. Given agents’ risk-neutral preferences, the mean growth rate \( \mu^K_t \) is directly linked to overall welfare. Therefore, it is clear that under the fundamental friction of capital intangibility, banks add value by creating inside money that boosts welfare by facilitating Eisfeldt (2007) show that the liquidity premium of Treasury bills cannot be explained by the liquidity demand from consumption smoothing under standard preferences.
resources reallocation towards investing agents.

**State variable.** At time \( t \), the economy has \( K^I_t \) units of intangible capital, \( K^T_t \) units of tangible capital, and aggregate bank equity \( N^B_t \). In principle, a time-homogeneous Markov equilibrium would have all three as state variables. Because production and investment technologies have constant return-to-scale, and because the mixture of intangible and tangible capital is fixed, the Markov equilibrium has only one state variable, the ratio of bank equity to capital stock:

\[
\eta_t = \frac{N^B_t}{K^I_t + K^T_t}.
\]

Because bankers are homogeneous and form a unit mass, \( N^B_t \) follows the same dynamics as \( n^B_t \), so the instantaneous expectation and standard deviation of \( dN^B_t / N^B_t \), denoted by \( \mu^B_t \) and \( \sigma^B_t \) respectively, are \( r_t + x^B_t \left( \mathbb{E}_t \left[ d r^T_t \right] - r_t \right) \) and \( x^B_t \left( \sigma^T_t + \sigma \right) \). By Itô’s lemma, \( \eta_t \) follows a regulated diffusion process

\[
\frac{d\eta_t}{\eta_t} = \mu^\eta_t dt + \sigma^\eta_t dZ_t - dy^B_t, \tag{12}
\]

where \( dy^B_t = dc^B_t / n^B_t \) is bankers’ consumption-to-wealth ratio, \( \mu^\eta_t \) is \( \mu^N_t - \mu^K_t - \sigma^N_t \sigma + \sigma^2 \) (last two terms from quadratic covariation of Itô’s calculus), and the shock elasticity \( \sigma^\eta_t \) is \( \sigma^N_t - \sigma \). Note that \( \sigma^\eta_t = x^B_t \sigma^T_t + (x^B_t - 1) \sigma > 0 \), because in equilibrium, entrepreneurs always hold some deposits and banks issue some debt, \( x^B_t > 1 \), and tangible capital price responds positively to \( dZ_t \), i.e., \( \sigma^T_t > 0 \) (more on this later). Therefore, positive shocks increase \( \eta_t \), and banks become richer, while negative shocks deplete bank equity, and decrease \( \eta_t \).

**Definition 1 (Markov Equilibrium)** For any initial endowments of entrepreneurs’ intangible capital \( \{k^I_{i,0}, i \in \mathbb{E}\} \) and tangible capital \( \{k^E_{i,0}, i \in \mathbb{E}\} \), and bankers’ tangible capital \( \{k^B_{j,0}, j \in \mathbb{B}\} \) such that

\[
\int_{i \in \mathbb{E}} k^I_{i,0} di = K^I_0, \text{ and } \int_{i \in \mathbb{E}} k^E_{i,0} di + \int_{j \in \mathbb{B}} k^B_{j,0} dj = K^T_0,
\]

and \( K^I_0 / \left( K^I_0 + K^T_0 \right) = \theta \), a Markov equilibrium is described by the stochastic processes of agents’ choices and price variables on the filtered probability space generated by the Brownian motion \( \{Z_t, t \geq 0\} \), such that:
(i) Agents know and take as given the processes of price variables, such as $q_t^T$ and $r_t$;

(ii) Entrepreneurs optimally consume, hold deposits, trade tangible capital, and invest;

(iii) Bankers optimally consume, trade tangible capital, and issue deposits;

(iv) Price variables adjust to clear all markets with goods being the numeraire;

(v) All the choice variables and price variables are functions of $\eta_t$, so Equation (12) is an autonomous law of motion that maps any path of shocks $\{Z_s, s \leq t\}$ to the current state $\eta_t$.

2.2 Markov equilibrium

We characterize the Markov equilibrium and will highlight a distinct feedback mechanism that arises from entrepreneurs’ liquidity demand and puts asset price and interest rate at the center.

Optimal investment and liquidity holdings. Under the assumption that the investment technology is highly efficient (i.e., $\kappa$ sufficiently large), entrepreneurs maximize the investment scale when hit by the liquidity shock. From Equation (4), we know that for one more dollar of liquidity holdings, entrepreneurs can invest $1/\left[1 - \kappa (1 - \theta) q_T^T\right]$ units of goods. And, because external funds are raised against tangible capital at fair price, entrepreneurs enjoy all the net present value (NPV) of investment – for one unit of goods invested, entrepreneurs’ wealth increases by $q_I^T \theta \kappa + q_T^T (1 - \theta) \kappa - 1$. Therefore, when the liquidity shock hits, the marginal benefit of liquidity holdings is precisely the NPV per unit of goods invested multiplied by the leverage on liquidity holdings: $\left[q_I^T \theta \kappa + q_T^T (1 - \theta) \kappa - 1\right] / \left[1 - \kappa \left(1 - \theta\right) q_T^T\right]$. The following proposition states that in equilibrium, risk-neutral entrepreneurs are willing to accept a deposit rate $r_t$ that is lower than their discount rate $\rho$, and the wedge $\rho - r_t$, which can be called a liquidity premium, is equal to the expect marginal benefit of deposit holdings.
Proposition 1 (Inside Money Demand) The optimality condition for entrepreneurs’ deposits is

\[ \rho - r_t = \lambda \frac{1}{(q^T \theta + q^T_t (1 - \theta)) \kappa - 1} \left( \frac{1}{1 - \kappa (1 - \theta) q^T_t} \right) \] (13)

On the left-hand side is the marginal cost of holding deposits, that is, a lower rate of return than \( \rho \). On the right-hand side is the marginal benefit, taking into account that a liquidity shock, and investment opportunities associated with it, arrives with Poisson intensity \( \lambda \).

Proposition 1 reveals an important link from asset price, \( q^T_t \), to interest rate, \( r_t \), and will serve as a key building block of a feedback mechanism that reconciles the stylized facts stated in the introduction and reveals their implications on endogenous risk accumulation. If we consider an increase in \( q^T_t \), it translates into an increase of liquidity premium, and thus, a decrease of interest rate \( r_t \), through two channels. First, the NPV per unit of goods invested becomes higher. Second, the leverage on entrepreneurs’ deposit holdings becomes larger because tangible capital can be pledged at a higher price for external funds.

**Asset price and interest rate.** Asset price affects interest rate through entrepreneurs’ liquidity demand. Standard asset pricing theory suggests that interest rate should also affect asset price through a typical discount rate channel. This intuition still holds here. To formalize it, we need to characterize the optimality conditions for entrepreneurs’ and bankers’ tangible capital holdings.

Entrepreneurs’ first-order (indifference) condition is simple and intuitive. The expected return on tangible capital should not exceed \( \rho \), and when entrepreneurs’ holdings of tangible capital is positive, it is equal to \( \rho \).

\[
\frac{\alpha}{q^T_t} + \mu^T_t - \delta + \sigma^T_t \sigma \leq \rho, \quad \mathbb{E}_t[dr^T_t] \] (14)

where equality holds when \( k^E_t > 0 \). In contrast to entrepreneurs whose marginal value of wealth is pinned down to one, the non-negative constraint on bankers’ consumption (i.e., \( dc^B_t \leq 0 \)) suggests that their marginal value of wealth, denoted by \( q^B_t \), varies over time in \([1, +\infty)\), and \( dc^B_t > 0 \) only
when \( q_t^B \) is equal to one. Given the homogeneity nature of bankers’ problem, we can conjecture that their value function is linear in wealth \( n_t^B \), and can be written as \( q_t^B n_t^B \). In equilibrium, the marginal value of wealth follows a diffusion process:

\[
\frac{dq_t^B}{q_t^B} = \mu_t^B dt + \sigma_t^B dZ_t. \tag{15}
\]

The appendix shows that the conjectures of value function and \( q_t^B \) dynamics are confirmed.

\( \sigma_t^B \) measures the shock sensitivity of bankers’ marginal value of wealth, so the instantaneous covariance between the return on tangible capital \( dr_t^T \) and the growth rate of \( q_t^B \) is \( \sigma_t^B (\sigma_t^T + \sigma) dt \). An asset with high covariance should have a low expected return, because its realized return is high precisely when bankers’ marginal value of wealth is high. Therefore, bankers’ required expected return on tangible capital is equal to \( r_t dt - \sigma_t^B (\sigma_t^T + \sigma) dt \), and we may interpret \( -\sigma_t^B \) as the price of risk charged by bankers. Because tangible capital is the only asset that bankers hold, their first-order condition always holds in equality. A formal proof based on bankers’ Hamilton-Jacobi-Bellman (HJB) equation is provided in the Appendix. The next proposition states bankers’ optimality condition.

**Proposition 2 (Asset Pricing)** The optimality condition for bankers’ tangible capital holdings is

\[
\frac{\alpha}{q_t^B} + \mu_t^T - \delta + \sigma_t^T \sigma = r_t + \left( -\sigma_t^B \right) \left( \sigma_t^T + \sigma \right) \tag{16}
\]

The price of risk, \( -\sigma_t^B \), charged by bankers is positive because \( dq_t^B (\eta_t)/d\eta_t < 0 \). Like Tobin’s Q, bankers’ marginal value of wealth \( q_t^B \) is a forward-looking measure of profitability. Bankers profit from the spread between \( \mathbb{E}_t [dr_t^T] \) and \( r_t \), and this spread tends to be larger when the whole banking sector is undercapitalized and invest less in tangible capital. We can define the elasticity of \( q_t^B \) with respect to \( \eta_t \): \( \xi_t^B = \frac{dq_t^B}{d\eta_t}/\eta_t < 0 \). By Itô’s lemma, \( -\sigma_t^B = \xi_t^B \sigma_t^\eta > 0 \) because \( \eta_t \) responds positively to \( dZ_t \), i.e., \( \sigma_t^\eta > 0 \) as previously discussed.
We can rearrange Equation (16) to express the price of tangible capital as follows

\[ q^T_t = \frac{\alpha}{r_t - \sigma^B_t \left( T^T_t + \sigma \right) - \mu^T_t + \delta - \sigma^T_t \sigma}. \] (17)

Ceteris paribus, a decrease of interest rate \( r_t \) lowers bankers’ discount rate, and thus, tends to increase asset price. From Proposition 1 and Proposition 2 arises a feedback mechanism:

- **Asset price increases** → **stronger liquidity demand of entrepreneurs** → **lower interest rate**
- → **lower discount rate of bankers** → **higher asset price** ...

At the heart of this mechanism is entrepreneurs’ liquidity demand, which is in turn from the illiquidity of intangible capital. Recall that if intangible capital is pledgeable, the investment project becomes self-financed, and thus, there is no need for entrepreneurs to carry deposits.

**Intermediated liquidity premium.** What is still missing in the feedback mechanism is the initial driver of asset price changes. In a Markov equilibrium with bank equity as the state variable, shocks affect asset price through their impact on bank equity. The model has a feature that is common among macro-finance models, that is when intermediaries’ wealth increases, asset price rises (e.g., Brunnermeier and Sannikov (2014)). However, the reason behind this feature is different.

In contrast to the existing literature, banks do not add value by having expertise on loans, such as special monitoring or restructuring abilities (Diamond (1984); Bolton and Freixas (2000)). Instead, they add value on the liability side of their balance sheets – deposits serve as a liquid store of value for entrepreneurs, and thus, earn a liquidity premium \( r_t < \rho \). This liquidity premium gives banks a funding cost advantage in comparison with entrepreneurs. However, due to the equity issuance constraint \( dc^B_t > 0 \), bankers are effectively risk-averse and charge a price of risk equal to \(-\sigma^B_t\), which puts banks in disadvantage relative to risk-neutral entrepreneurs. In a state of the world where banks’ equity is very high and their price of risk low, bankers’ funding advantage from liquidity premium dominates the disadvantage from required risk compensation, so that banks can...
have a discount rate that is lower than entrepreneurs:

\[ r_t - \sigma_B^B (\sigma_T^T + \sigma) < \rho. \]

In this case, entrepreneurs do not hold any tangible capital, while banks hold all, i.e., \( x_t^B N_t^B = q_t^T K_t^T \). Here, tangible capital should be broadly interpreted as any asset in the real world whose cash flow is from firms’ production and is liquid (pledgeable).

Inspecting Equation (17), we see that \( q_t^T \) only moves through the discount rate ("DR") channel, because the cash flow of tangible capital is fixed at \( \alpha \) per unit of time. Note that \( \mu_t^T \) and \( \sigma_t^T \), the drift and diffusion terms of \( q_t^T \), are not the fundamental drivers of asset price variation, but part of the price dynamics itself (as defined in Equation (6)). When bank equity is low and entrepreneurs hold tangible capital, the discount rate is fixed at \( \rho \), and we have \( r_t - \sigma_B^B (\sigma_T^T + \sigma) = \rho \) – bankers’ funding cost advantage from the liquidity premium is exactly offset by their disadvantage in risk-taking. When bank equity is high and entrepreneurs retreat from tangible capital market, the discount rate becomes lower than \( \rho \), which boosts \( q_t^T \). Thus, the states of the world in this economy are divided into two regions.

In the region where banks are well capitalized, the discount rate for tangible capital is lowered by an intermediate liquidity premium. Due to the illiquidity of intangible capital, entrepreneurs demand a liquid store of value and are willing to pay a liquidity premium, \( \rho - r_t \), as in Holmström and Tirole (2001). The ultimate source of liquidity is tangible capital that backs bank deposits or, in other words, is held by entrepreneurs indirectly through banks’ balance sheets. Banks’ intermediation capacity depends on their equity, so the transmission of liquidity premium happens when banks are well capitalized. Driven by the liquidity premium they earn by issuing deposits, they chase the tangible capital, bidding up its price through a low discount rate.

Intermediated liquidity premium drives \( q_t^T \) even in states of the world with low \( \eta_t \), where entrepreneurs still hold some tangible capital, and the discount rate for tangible capital is \( \rho \). Consider a positive shock, \( dZ_t > 0 \). As \( \eta_t \) increases, the economy moves closer to the region where banks hold all tangible capital and their discount rate is below \( \rho \). Agents’ rational expectation of a higher \( q_t^T \) going forward is reflected in \( \mu_t^T > 0 \), the expected price change, which boosts the
current tangible capital price as shown in Equation (17). Therefore, $q^T_t$ always moves positively with $\eta_t$, and its shock elasticity, $\sigma^T_t$, is positive.

Intermediated liquidity premium is the key to complete the feedback mechanism. Starting with an exogenous shock ($dZ_t > 0$), we trace out the responses of endogenous variables:

$$\text{Good shocks} \rightarrow \text{bank equity increases} \rightarrow \text{asset price increases through DR channel} \rightarrow \text{intermediated liquidity premium}$$

$$\text{stronger liquidity demand of entrepreneurs} \rightarrow \text{higher liquidity premium and lower } r_t \rightarrow \text{illiquidity of intangible capital} \& \text{higher investment return}$$

$$\text{lower discount rate of bankers} \rightarrow \text{higher asset price} \rightarrow \text{bank equity increases further} ...$$

Note that an increase in $q^T_t$ boosts bank equity through the typical balance-sheet channel because as the asset value increases, bank equity increases even faster through leverage.

**Credit vs. money view of banking.** There is a long-standing debate on the function of banks. Some argue that banks add value because only they are able to finance certain projects (e.g., Bernanke (1983)), while others maintain that banks are important because their liabilities, for instance deposits, serve as a liquid store of value and means of payment, what is often call the inside money (e.g., Friedman and Schwartz (1963)).$^{13}$ This paper takes the money view of banking. Banks add value because entrepreneurs hold deposits as liquidity buffer. This inside money demand is in line with Woodford (1990) and Holmström and Tirole (1998).

However, the model entertains the credit view of banking in the sense that banks finance entrepreneurs’ investment by extending intraday loans backed by new tangible capital, i.e., supplying the leverage on entrepreneurs’ liquidity holdings. From Equation (4), banks supply more credit when the price of tangible capital $q^T_t$ (and entrepreneurs’ financing capacity) is higher. And,

$^{13}$Formalized in Bernanke and Blinder (1992), the credit view on banking has gained much traction recently. In Brunnermeier and Sannikov (2014), it is embodied by the enhanced productivity when capital is held by banks (“experts”). Other types of intermediation expertise may serve the same purpose of making banks the natural buyers of capital and capture the credit view on banking, such as lower risk aversion (Longstaff and Wang (2012)), collateralization expertise (Rampini and Viswanathan (2015)), unique ability to hold risky assets (He and Krishnamurthy (2013)), unique ability to monitor projects (Diamond (1984) and Holmström and Tirole (1997)), and advantage in diversification (Brunnermeier and Sannikov (2016)). Recent works on the money view of banking includes Hart and Zingales (2014), Quadrini (2017), Piazzesi and Schneider (2016), and Moreira and Savov (2017).
\( q_t^T \) is high when banks are well capitalized as previously discussed. Therefore, the model does exhibit a relation between credit and banks’ balance-sheet capacity, but this relation derives from the channel of intermediated liquidity premium, i.e., the negative relation between bank equity and the discount rate in the tangible capital market. Therefore, the first and foremost function of banks in this economy is still to supply inside money.

**Endogenous savings glut and risk accumulation.** The model produces endogenous risk accumulation in booms. When we start with a low level of bank equity (low \( \eta_t \)), the instantaneous volatility of tangible capital price, \( \sigma_t^T \), increases as \( \eta_t \) and \( q_t^T \) increases.

The rising \( q_t^T \), through higher the investment NPV and leverage on liquidity holdings, induces an endogenous savings glut on the part of entrepreneurs, which drives down the deposit rate \( r_t \). However, before the banking sector is large enough, entrepreneurs still hold some tangible capital, so the discount rate is held at \( \rho \). Recall that \( q_t^T \) varies only with the discount rate, so when \( \eta_t \) is low, \( q_t^T \) is relatively insensitive to shocks because the discount rate is likely to stay at \( \rho \). As \( \eta_t \) increases in booms (in response to positive shocks), the economy moves closer to the region where banks hold all tangible capital and the discount rate is below \( \rho \), so the likelihood of a discount rate change rises, making \( q_t^T \) increasingly sensitive to shocks. The instantaneous volatility \( \sigma_t^T \) measures the shock sensitivity of \( q_t^T \) as shown in Equation (6).

The rising endogenous risk in booms, reflected in asset price volatility, is a distinct feature of the model. An intuitive way to understand the mechanism is two label bankers as the “first-best” buyers of tangible capital (as in Shleifer and Vishny (2010)) whose required rate of return is lower than that of the “second-best” buyers (entrepreneurs) when \( \eta_t \) is high. The discount-rate gap between these two groups is zero initially, and then widens, as \( \eta_t \) increases and banks dominate the tangible capital thanks to a rising liquidity premium. A widening difference between the first-best and second-best buyers makes asset price increasingly sensitive to the wealth of first-best buyers (bank equity). Even in the region where the discount-rate gap is zero, i.e., \( r_t - \sigma_t^B (\sigma_t^T + \sigma) = \rho \), the sensitivity of \( q_t^T \) to bank equity increases smoothly as \( \eta_t \) increases, because future discount-rate gap feeds into the current price volatility through agents’ rational expectation.

Asset price volatility is important from a welfare perspective. As shown in Equation (11),
the economic growth rate is directly tied to asset price through the scale of investment (leverage on entrepreneurs’ liquidity holdings). A high level of asset price enlarges entrepreneurs’ financing capacity, and a low level reduces it. Therefore, the volatility of asset price translates into the volatility of economic growth rate, and agents’ long-run welfare.

**Stationary distribution and recovery time.** To study the long-run behavior of the economy, it is useful to characterize the stationary probability distribution of the state variable. Another object of interest is the time to recovery, i.e., how long does it take in expectation for \( \eta_t \) to travel from a low level with low or even negative economic growth rate to a state with high economic growth rate. Proposition 3 solves the stationary probability density of \( \eta_t \) and the expected time to reach \( \eta \in (0, \bar{\eta}] \) from a low level \( \eta_t = \frac{1}{2} \) (“recovery time”). Here, endogenous variables are explicitly written as functions of state variable \( \eta_t \). Since we study a time-homogeneous Markov equilibrium, time subscripts are suppressed.

**Proposition 3**  
The stationary probability density of state variable \( \eta_t \), \( p(\eta) \) can be solved by:

\[
\mu^n(\eta) p(\eta) - \frac{1}{2} \frac{d}{d\eta} \left( \sigma^n(\eta)^2 p(\eta) \right) = 0,
\]

where \( \mu^n(\eta) \) and \( \sigma^n(\eta) \) are defined in Equation (12). The expected time to reach \( \eta \) from \( \eta_t \), \( g(\eta) \) can be solved by:

\[
1 - g'(\eta) \mu^n(\eta) - \frac{\sigma^n(\eta)^2}{2} g''(\eta) = 0,
\]

with the boundary conditions \( g(\eta) = 0 \) and \( g'(\eta) = 0 \).

**Solving the equilibrium.** In the next section, equilibrium properties of the model are presented by the calibrated solution. To solve the Markov equilibrium, we can convert agents’ optimality conditions and market clearing conditions into a system of ordinary differential equations (ODEs). In particular, bankers’ optimality condition for tangible capital holdings and their HJB equation form a pair of second-order ODEs for the two forward-looking variables, \( q^B(\eta_t) \) and \( q^T(\eta_t) \). Once these two variables are solved, other endogenous variables can be derived as functions of the level and first derivatives of \( q^B(\eta_t) \) and \( q^T(\eta_t) \). Details are provided in the Appendix. The next proposition summarizes the equilibrium solution.
Proposition 4 (Markov Equilibrium) There exists a unique Markov equilibrium with state variable $\eta_t$ that follows an autonomous law of motion in $(0, \bar{\eta}]$, where $\bar{\eta}$, the upper bound, is pinned down by bankers’ optimal consumption. Specifically, we have the boundary conditions:

At $\bar{\eta}$: (1) $\frac{dq^T(\eta)}{d\eta} = 0$; (2) $q^B(\bar{\eta}) = 1$; (3) $\frac{dq^B(\eta)}{d\eta} = 0$;

As $\eta_t$ approaches zero: (4) $\lim_{\eta_t \to 0} \frac{dq^T(\eta)}{d\eta} = 0$; (5) $\lim_{\eta_t \to 0} q^B(\eta_t) = +\infty$.

(1) guarantees that $q^T_t$ does not jump at the reflecting boundary $\eta$ (no arbitrage). (2) and (3) are the value-matching and smooth-pasting conditions for bankers’ consumption choice respectively. As $\eta_t$ approaches zero, the likelihood of a discount rate change for tangible capital shrinks to zero, so $q^T_t$ becomes insensitive to state variable variation as stated in (4). When banks are extremely undercapitalized, the their marginal value of equity, or Tobin’s $Q$, approaches infinity as stated in (5). Given these boundaries conditions, functions $q^B(\eta_t)$ and $q^K(\eta_t)$ are solved by a pair of second-order ODEs constructed from bankers’ optimality condition for tangible capital holdings and their HJB equation. Other variables can be solved as functions of the level and first derivatives of $q^B(\eta_t)$ and $q^K(\eta_t)$ based on the other optimality and market-clearing conditions.

3 Solution

We now turn to the quantitative analysis of the model, and in particular, how the model generates the stylized facts summarized in the introduction and how the feedback mechanisms lead to the accumulation of endogenous risk in booms. The model solution characterizes the full dynamics of the economy instead of perturbations around the steady state.

3.1 Calibration

One unit of time is set as one year. Entrepreneurs’ time-discounting rate is 6%, roughly in line with the expected return in the stock and corporate bond markets. Their exit/entry rate is $\chi = 4\%$, so overall, their discount rate $\rho$ is $6\% + 4\% = 10\%$. The instantaneous mean of stochastic depreciation rate $\delta$ is $= 5\%$. Together with $\chi = 4\%$, it generates a long-run mean of $\mu^K_t$, the growth rate of
the economy, equal to 1.8%. The long-run mean of a variable is calculated based on the stationary probability density. The instantaneous volatility of capital depreciation $\sigma$ is 2%, which is a pivotal parameter for banks risk-taking and generates a long-run mean of bank leverage equal to 19. The stochastic depreciation of capital can be broadly interpreted as random failure of production units. $\delta$ and $\sigma$ are comparable to the time-series mean and standard deviation of delinquency rates on commercial and industrial loans (source: FRED at Federal Reserve Bank of St. Louis).

On the investment technology, the fraction of inputs that go to intangible investment, $\theta$, is set to 70%, so the long-run mean of project leverage, defined in Equation (4), is 1.5 in line with the time-series average of US corporate leverage (source: FRED at Federal Reserve Bank of St. Louis). Liquidity shock arrives every ten years, so $\lambda = 1/10$. The economy has a constant source of growth from the entry of new entrepreneurs, governed by $\chi$, and a state-dependent source of growth from liquidity-constrained investments. Therefore, $\lambda$ is a pivotal parameter for the range of variation of $\mu_t^K$. Under the current calibration, the economic growth rate $\mu_t^K$ varies between $-1\%$ to $4\%$ under $\lambda = 1/10$. The parameter of investment efficiency, $\kappa$, is critical for marginal benefit of liquidity holdings, so it is set to 3 to generate a long-run cash-to-asset ratio of firms equal to 15%, roughly in line with average cash-to-total asset ratio in the Compustat sample.

A key parameter of the model is $\phi$. It measures the productivity gap between tangible and intangible capital. Thus, the rise of intangible capital is captured by a high value of $\phi$, meaning that more output is attributed to intangible capital as documented by Corrado and Hulten (2010). In the baseline calibration, $\phi$ is set to 0.01, which is a 20% productivity advantage of intangible capital over tangible capital ($\alpha = 0.05$). The productivity of tangible capital $\alpha$ is set to 0.05, implying that the long-run mean of $q_t^T/\alpha$ is 7.5, in line with the EV/EBITDA ratio of tangible industries such as construction, mining, and materials. In the end of this section, we will discuss how the equilibrium dynamics change when $\phi$ increases, and in particular, the decline of interest rate declines and the rise of endogenous volatility in response.
3.2 Equilibrium dynamics

Endogenous variables are plotted against the ratio of bank equity to aggregate output per year, i.e.,

\[
\frac{N^B_t}{\alpha K^T_t + (\alpha + \phi) K^T_t} = \frac{N^B_t}{[\alpha (1 - \theta) + (\alpha + \phi) \theta]} K_t = \frac{\eta_t}{\alpha (1 - \theta) + (\alpha + \phi) \theta}.
\]

Figure 2 reproduces the stylized facts summarized in the introduction. Figure 3 decomposes the feedback mechanism that puts the intermediated liquidity premium at the center. Figure 4 shows the accumulation of endogenous risk in booms, and the strength of shock amplification mechanism of the model. Stationary density and recovery time are shown in Figure 5. Figure 6 compares the model performances under different levels of $\phi$, and shows that the rise of intangible capital (higher $\phi$) generates higher economic growth rate, low interest rate, and higher endogenous risk.

**The stylized facts.** Panel A of Figure 2 plots $q^T_t/\alpha$, a price-to-cash flow ratio of tangible capital. In this economy, tangible capital represents all liquid assets that are traded and pledgeable. Therefore, the increase of $q^T_t$ should be interpreted as broad increase of asset prices when the banking sector expands following positive shocks. Recall that in the model, asset price does not move with cash-flow news because dividend is fixed at $\alpha$ per unit of time. What drives asset price is the discount rate, which is fixed at $\rho$ when banks are not large to hold all tangible capital, and below $\rho$ in states of high $\eta_t$ due to the intermediated liquidity premium. Asset price increases smoothly in $\eta_t$ even in states where $\eta_t$ is low and the discount rate is equal to $\rho$, because the future possibility of a discount rate below $\rho$ is incorporated into the current asset price through agents’ rational expectation.

Panel B of Figure 2 plots the deposit rate $r_t$. As shown in Proposition 1, the rising asset price translates into an increasing liquidity premium, $\rho - r_t$, through entrepreneurs’ investment-driven liquidity demand. This model produces an endogenous savings glut in the corporate sector that drives down the interest rate, and drives up asset price, which in turn feeds into corporate liquidity hoarding.\(^{14}\) This endogenous mechanism stands in contrast with the existing literature that emphasizes exogenous savings glut, for example savings from foreign countries, and its implications on

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\(^{14}\)Investment need is a key determinant of the cross-sectional variation in corporate cash holdings (e.g., Denis and Sibikov (2010); Duchin (2010)), especially for firms with less collateral (e.g., Almeida and Campello (2007); Li, Whited, and Wu (2016)) and more intensive R&D activities (Falato and Sim (2014)).
Figure 2: The Stylized Facts. This figure plots the price-to-dividend ratio of tangible capital (Panel A), deposit rate $r_t$ (Panel B), the ratio of firms’ liquidity holdings to capital value (Panel C), and the fraction of tangible capital held by banks (Panel D) against the ratio of aggregate bank equity to output.

Panel C of Figure 2 plots the ratio of deposit holdings (“cash”) to firms’ total asset value $q_t^{T} K_t^{T} + q_t^{I} K_t^{I}$. Following positive shocks, $\eta_t$ increases and firms’ cash-to-asset ratio rises. The pattern reminisce what we observe in the past two decades in the United States. Many have attributed the enormous corporate cash holdings to an increasing share of firms whose production heavily relies on intangible capital (Falato and Sim (2014); Begenau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015)). Note that as shown in Figure 1, intangible capital, tangible capital owned by entrepreneurs, and tangible capital owned by bankers are all
incorporated in a firm managed by the corresponding entrepreneur. Bankers’ ownership over tangible capital represents in a very general form their claims on the corporate sector. In line with the assumption of perfect liquidity of tangible capital, an entrepreneur dutifully delivers the constant stream of cash flow $\alpha dt$ to banks per unit of tangible capital they own, so there does not exist any agency friction. In the model, the run-up of corporate cash holdings stops when banks take over the whole market of liquid assets and any further growth of their equity outpaces asset price increase and crowds out their debt (deposits).

Panel D of Figure 2 plots the fraction of tangible capital owned by banks. It illustrates the expansion of banking sector, and reminisces such a trend in data that has attracted tremendous attention in the literature (Adrian and Shin (2010b); Graham and Leary (2015); Greenwood and Scharfstein (2013); Schularick and Taylor (2012)). Many drivers behind have been proposed, and among them, a very prominent one is the incentive of banking sector to earn the liquidity premium by manufacturing money-like securities (Adrian and Shin (2010a); Gorton (2010); Pozsar (2014)). This explanation echoes the theme of this paper. Banks add value by supplying liquid store of value to the entrepreneurial sector, and the endogenous savings glut of the latter feeds cheap leverage to banks. A feedback mechanism arises, putting the intermediated liquidity premium at the center and linking all together the increase of asset price, the decline of interest rate, corporate liquidity hoarding, and banking expansion in a booming economy. Next, this feedback mechanism is analyzed graphically based on the calibrated solution.

**The feedback mechanism.** Consider a state of the world where $\eta_t$ is relatively low, for example, at 5% bank equity-to-output ratio. Following positive shocks, bank equity increases, and through the channel of intermediated liquidity premium, the price of tangible capital increases. As shown in Proposition 1, the impact on entrepreneurs’ incentive to hold liquidity is two-fold. First, when the liquidity shock arrives, the profit per unit of goods invested becomes higher when $q_t^T$ is higher. Second, the leverage of liquidity holdings is higher, because the external financing capacity of the investment project increases in $q_t^T$. Panel A and B of Figure 3 exhibit these dynamics.

As the banking sector grows and the liquidity premium enlarges, banks acquire all the tangible capital when bank equity-to-output ratio reaches around 22%, because now banks’ required
rate of return becomes lower than entrepreneurs’ $\rho$. In a prolonged boom, $q_t^T$ keeps rising, driving up the liquidity premium and driving down the deposit rate $r_t$. As shown in Panel C of Figure 3, banks’ discount rate declines with $r_t$.

Throughout the process, agents’ expectation of future asset price change $\mu_t^T$ keeps rising (Panel D of Figure 3), because, for example, from a bank equity-to-output ratio of 10% to 20%, the higher $\eta_t$, the closer the economy is to the region where the discount rate for tangible capital is below $\rho$. The expectation of low discount rate translates into the expectation of high asset price, which in turn elevates the current level of asset price as shown in Equation (17).

The positive relation between bank equity and asset price arises from the intermediated liq-
uidity premium, and given that, any positive shock triggers a feedback mechanism: asset price rises, driving up the liquidity premium through both investment profit and entrepreneurs’ leverage on liquidity; the liquidity premium lowers interest rate, feeding banks with cheap financing and lowering their discount rate, so asset price increases even further.

**Endogenous risk.** The dynamics of banks’ discount rate has implications not only on the level of asset price but also on its volatility. Let us consider that the economy moves from an bank equity-to-output ratio of 10% to 20% following a sequence of positive shocks in Panel C of Figure 3. In the process, agents rationally change their belief of the distribution of future discount rate in the tangible capital market. At the beginning, it is unlikely that the discount rate will move away from $\rho$ in the near future, because it takes sufficiently large shocks to increase $\eta_t$ all the way to the cutoff point of 22% bank equity-to-output ratio. But as we move to the right, $\eta_t$ approaches the cutoff point, and thus, the likelihood of a discount rate change increases. As a consequence, asset price $q_t^T$ becomes more sensitive to shocks (higher $\sigma_T^t$). This dynamics is shown in Panel B of Figure 4 where the ratio of total volatility of tangible capital return, $\sigma_T^t + \sigma$, to exogenous volatility, $\sigma$, is plotted against bank equity-to-output ratio.

The accumulation of endogenous risk sheds light on the findings that a long period of boom and banking expansion precedes severe crises (e.g., Jordà, Schularick, and Taylor (2013); Baron and Xiong (2016)). The existing theories attribute to the accumulation of endogenous risk to learning (Moreira and Savov (2017)) or the lack of information production (Gorton and Ordoñez (2014)). This paper emphasizes financial friction instead of informational distortions, and moreover, it offers a model that accounts for several phenomena in the pre-crisis period and links them to the illiquidity of intangible capital.

The volatility amplification effect is strong. At the peak, the total volatility is almost 3.5 times of the exogenous volatility. Here, $\sigma$ is defined as the measure of exogenous risk. Without any feedback mechanism, the return on tangible capital always loads on the aggregate shock $dZ_t$ through the stochastic depreciation of capital (Equation (8)). As previously discussed, asset price moves because of the intermediated money premium. Such movement is captured by $\sigma_T^t$. This endogenous risk arises from agents’ optimal choices under constraints and frictions.
entrepreneurs’ liquidity demand, the illiquidity of intangible capital leads to the endogenous risk of tangible capital.

In Panel B of Figure 4, endogenous risk rises first and then declines when the banking sector becomes sufficiently large. The eventual decline can be understood by examining the dynamics of banks’ discount rate in Panel C of Figure 3. After the cutoff point of 22% bank equity-to-output ratio, banks’ discount rate declines but the rate of change dies out as the banking sector further expands. Therefore, asset price becomes increasingly less sensitive to shocks.

Another way to understand the hump-shaped asset price volatility is to decompose $\sigma^T_t$. By Itô’s lemma, $\sigma^T_t = \epsilon^T_t \sigma^\eta_t$, where $\epsilon^T_t$ is the elasticity of $q^T_t$ to $\eta_t$, i.e., $\epsilon^T_t = \frac{dq^T_t}{q^T_t} \frac{d\eta_t}{\eta_t}$. Another source

Figure 4: **Endogenous Risk Accumulation and Volatility Trap.** This figure plots the shock elasticity of state variable (Panel A), the ratio of total return volatility to exogenous volatility of tangible capital (Panel B), banks’ price of risk (Panel C), and the instantaneous expected growth rate of the economy (Panel D) against the ratio of aggregate bank equity to output.
of the eventual decline of $\sigma_t^T$ is the gradual stabilization of the state variable itself as it increases ($\sigma_t^\eta$ in Panel A of 4). This decomposition of endogenous risk separates the volatility from state variable and the sensitivity of asset price $q_t^T$ to state variable $\eta_t$. The latter roughly measures the dependence of tangible capital price on bank equity. As $\eta_t$ increases, the economy moves closer to a region where the discount-rate gap is positive and widening between bankers, the first-best buyers of tangible capital, and entrepreneurs, the second-best buyers, so the price of tangible capital becomes increasingly sensitive to bankers’ wealth. A caveat on this decomposition is that the dynamics of state variable itself is endogenous, as shown in Equation (12).

Another message from Figure 4 is that as the banking sector expands, banks charge an increasingly lower price of risk (Panel C). This is quite an intuitive result, because from Proposition 2, we can express $-\sigma^B$ as the Sharpe ratio earned by banks taking a long position in tangible capital and a short position in deposits, i.e., a measure of risk-adjusted profit in equilibrium. When the banking sector expands, its profit per unit of risk shrinks. The declining price of risk and deposit rate together dominates the hump-shaped quantity of risk, $\sigma_t^T + \sigma$, so overall, banks’ discount rate is a non-increasing function of $\eta_t$ as shown in Panel C of Figure 3.

Panel D of Figure 4 plots $\mu^K_t$, the expected growth rate of the economy against the ratio of aggregate bank equity to output. $\mu^K_t$ increases in tangible capital price and the amount of inside money created by the banking sector, so naturally, we expect a positive relation between $\mu^K_t$ and $\eta_t$. However, through the leverage on entrepreneurs’ liquidity holdings, the endogenous volatility of $q_t^T$ creeps into the dynamics of $\mu^K_t$, and influences its long-run distribution and agents’ welfare. Next, Figure 5 displays the stationary probability density of $\eta_t$.

**Stationary distribution and recovery time.** Panel A of Figure 5 shows the stationary probability density of $\eta_t$. It measures the relative amount of time the economy spends in different levels of $\eta_t$ over the long run. A key feature is that recessionary states (i.e., $\mu^K_t < 0$) are rare events. Reading from Panel B, the stationary C.D.F., recession happens less than 5% of the time. However, recessions in the model can be very stagnant. Panel C of Figure 5 shows the expected time to reach different levels of $\eta_t$ from $\eta = 1e-5$. Combining this graph with Panel D of Figure 4, we see that it takes almost ten years to climb out of a deep recession. Slow recovery is consistent the
Figure 5: Stationary Distribution and Recovery Time. This figure plots the stationary probability density of $\eta_t$ (Panel A), the stationary cumulative probability function (Panel B), and the expected number of years to reach different levels of $\eta_t$ from $\eta = 1e^{-5}$ against the ratio of aggregate bank equity to output.

relatively high probability density in a region of extremely low bank equity in Panel A. One reason for slow recovery is that in recessionary states, $q_t^T$ is low, so the investment profit and the leverage of entrepreneurs’ liquidity holdings is low. This reduces the liquidity premium, a key source of profits for banks, so banks recover their equity slowly with a relatively low return on equity.

In crises, the upward feedback mechanism flipped into a vicious cycle. When negative shocks destroy bank equity, asset price declines, which discourages entrepreneurs from hoarding liquidity for investments, and thus, causes an increase of $r_t$, bankers’ marginal cost of financing. Higher interest rate further depressed asset price, and the decrease of asset price is amplified by bank leverage and further erodes bank equity through the typical balance-sheet effect.

A key assumption is that bankers cannot raise equity from entrepreneurs. Otherwise, the shock impact will be dissipated across the whole economy.\textsuperscript{15} In fact, if bankers can raise equity from entrepreneurs, they always hold all the tangible capital and maximize their supply of deposits. Allowing banks to issue equity may improve the quantitative performance of the model, and may introduce new mechanisms as in He and Krishnamurthy (2013), but as long as there are

\textsuperscript{15}A limit on equity issuance is a limit on risk-sharing. Di Tella (2014) show that perfect risk-sharing shuts down the balance-sheet channel.
certain frictions on bankers’ equity issuance, the qualitative implications carry through, that is the equilibrium dynamics will still be conditioned upon the net worth of bankers.

**Fragile new economy.** $\phi$ is a key parameter in the model. It represents a productivity gap between tangible and intangible capital. A permanent increase of $\phi$ captures a transition towards a more intangible-intensive economy (more output attributed to intangible capital). Here, we consider a 10% increase of the productivity of intangible capital to 0.066, i.e., an increase of $\phi$ from 0.01 to 0.016. The first impact is that overall, the economy becomes more productive. However, as intangible becomes more valuable, entrepreneurs’ incentive to hoard liquidity becomes stronger.
Panel A of Figure 6 plots the deposit rate. The pattern still holds – as the banking sector expands, the liquidity premium increases and the interest rate declines. However, the level of interest rate becomes lower. This is because entrepreneurs assign a higher value to the more productive intangible capital, and hoard more liquidity for investments. Moreover, the banking sector expands to a higher level of bank equity-to-output ratio than the benchmark case. When more liquidity is needed, the economy will have a larger banking sector that issues deposits to meet such demand. Thus, the model predicts that a transition towards intangible-intensive economy is accompanied by the expansion of banking sector. In the past few decades, bank expansion has been documented in most advanced economies (e.g., Schularick and Taylor (2012)).

A more intangible-intensive economy has a stronger risk amplification mechanism, as shown in Panel B of Figure 6. The ratio of total return volatility to exogenous volatility of tangible capital climbed above 6 as the economy goes through a boom and bank expansion. With a stronger liquidity demand from entrepreneurs, the economy now has an even more functioning feedback mechanism that works through interest rate and asset price. In particular, Panel A shows that interest rate is more sensitive to bank equity, i.e., a steeper slope, when $\phi$ is higher.

Asset price also becomes more responsive to bank expansion, as shown by the leverage on entrepreneurs’ liquidity holdings in Panel C, which is a monotonic transformation of $q_t^T$. Not only the slope of $q_t^T$, but also its level becomes higher in a more intangible-intensive economy. This is due to a large liquidity premium and lower discount rate in the tangible capital market. When intangible capital becomes more productive, the price of tangible capital increases thanks to stronger liquidity demand of entrepreneurs intermediated by banks. A higher leverage on liquidity holdings induces entrepreneurs to hold even more liquidity.

The new economy is more fragile. Endogenous risk is generated by a stronger feedback mechanism that works around the intermediated liquidity premium. The new economy also grows faster, as shown in Panel D of Figure 6. The intermediated liquidity premium leads to a higher price of tangible capital, so entrepreneurs are able to increase the leverage on their liquidity holdings and invest more. Therefore, the model characterizes a risk-return trade-off associated with the transition to a more intangible-intensive economy – faster growth and more endogenous risk.
4 Conclusion

This paper aims to provide a coherent account of several trends in the two decades leading up to the Great Recession, such as the rising corporate cash holdings, the expansion of financial sector, the declining interest rate, and the rising prices of risky assets. At the center is the dynamic interaction between inside money suppliers (banks) and demanders (entrepreneurs).

The money demand of firms arises from investment and capital illiquidity, which are in turn motivated by the increasing reliance on intangible capital in advanced economies. The financial stability implications of this structural change has not yet been explored in the existing literature. This paper proposes a channel of intermediated liquidity premium that links rising asset price and declining interest rate with the accumulation of endogenous risk in booms. It sheds light on the recent empirical findings that a long period of banking expansion precedes severe crises. This new channel of instability and a typical balance-sheet channel reinforce each other, creating a quantitatively significant amplification mechanism for macro shocks.

The model leaves out the provision of outside money (liquid government securities or central bank liabilities) for future research. By expanding money supply in booms, the government can crowd out bank leverage (Krishnamurthy and Vissing-Jorgensen (2015)), and thereby, weaken the upward spiral in asset price that leads to endogenous risk accumulation. In contrast to the current practice of monetary expansion in recessions, the optimal supply of outside money can be procyclical in this setting. However, since entrepreneurs’ incentive to invest is tied to asset price, a government faces the trade-off between growth and stability.

The model also leaves out banks’ default. The empirical literature on financial crises commonly use banks’ default or a high possibility of default as a crisis indicator. A theoretical model of crisis should ideally accommodate default, and by doing so, it opens up the question of optimal government intervention, for instance, through equity injection into the banking sector, in order to prevent a sudden evaporation of inside money. To finance the intervention in bad times, the government may increase the issuance of outside money (e.g., short-term government securities), suggesting countercyclical supply of outside money.
Appendix I - Proofs of Propositions

Proof of Proposition 1. Entrepreneurs (“firms”) maximize life-time utility, $E \left[ \int_{t=0}^{+\infty} e^{-\rho t} dc_t^E \right]$, subject to the following wealth (equity) dynamics:

$$dw_t^E = -dc_t^E + \mu^w_t w_t^E dt + \sigma^w_t w_t^E dZ_t + \left( \bar{w}_t^E - w_t^E \right) dN_t,$$

$\mu^w_t w_t^E$ and $\sigma^w_t w_t^E$ are the drift and diffusion terms that depend on choices of tangible capital and deposit holdings and will be elaborated later. $dN_t$ is the increment of the idiosyncratic counting (Poisson) process. $dN_t = 1$ if a liquidity shock arrives. At the Poisson time, an entrepreneur’s wealth jumps to

$$\bar{w}_t^E = w_t^E + \left( \frac{q^I_1 \theta + q^T_1 (1 - \theta) \kappa - 1}{1 - \kappa (1 - \theta)} q^I_1 \right) m^E_t.$$

Note that $w_t^E$ is the *liquid* wealth of entrepreneurs. When analyzing entrepreneurs’ decisions, we can simply regard cash flows from intangible capital as streams of consumption. We conjecture that the value function is linear in equity $w_t^E$: $v_t^E = \zeta_t^E w_t^E + v^I$, where $\zeta_t^E$ is the marginal value of liquid wealth, and $v^I$ is the present value of consumption flows from intangible capital. In equilibrium, $\zeta_t^E$ follows a diffusion process:

$$d\zeta_t^E = \zeta_t^E \mu_{\zeta_t} dt + \zeta_t^E \sigma_{\zeta_t} dZ_t,$$

where $\zeta_t^E \mu_{\zeta_t}$ and $\zeta_t^E \sigma_{\zeta_t}$ are the drift and diffusion terms respectively. Entrepreneurs’ marginal value of wealth, $\zeta_t^E$, is a summary statistic of their investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual is hit by liquidity shocks.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho v_t^E dt = \max_{dc_t^E \in \mathbb{R}, k_t^E \geq 0, m_t^E \geq 0} dc_t^E - \zeta_t^E dc_t^E + \{ w_t^E \zeta_t^E \mu_t^E + w_t^E \zeta_t^E \sigma_t^w + \lambda_t^E [\bar{w}_t^E - w_t^E] \} dt.$$

Entrepreneurs can choose any $dc_t^E \in \mathbb{R}$, so $\zeta_t^E$ must be equal to one, and thus, I have also confirmed the value function conjecture.
Since $\zeta_t^E$ is a constant equal to one, $\mu_t^c$ and $\sigma_t^c$ are both zero. The HJB equation is simplified:

$$\rho v_t^E dt = \max_{k_t^E \geq 0,m_t^E \geq 0} \mu_t^w w_t^E dt + \lambda dt \left( \frac{q_t^T \theta \kappa + q_t^T (1 - \theta) \kappa - 1}{1 - \kappa (1 - \theta) q_t^T} \right) m_t^E.$$  \hspace{2cm} (18)

Wealth drift has production, value change of tangible capital holdings, and deposit return:

$$\mu_t^w w_t^E dt = \alpha k_t^E dt + \mathbb{E}_t \left( q_t^T k_{t+dt}^E k_t^E - q_t^T k_t^E \right) + r_t m_t^E dt.$$  

Let $d\psi_t^F$ denote the Lagrange multiplier of the budget constraint, $q_t^T k_t^E + m_t^E \leq w_t^E$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t^E \geq 0$, and

$$m_t^E \left\{ r_t dt + \lambda dt \left( \frac{q_t^T \theta \kappa + q_t^T (1 - \theta) \kappa - 1}{1 - \kappa (1 - \theta) q_t^T} \right) - d\psi_t^E \right\} = 0.$$  

The F.O.C. for optimal tangible capital holdings is: $k_t^E \geq 0$, and

$$-\mathbb{E}_t \left[ dr_t^T \right] + d\psi_t^E = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho v_t^E dt = w_t^E d\psi_t^E.$$  

Because $\zeta_t^E = 1$, $v_t^E = w_t^E$, and $d\psi_t^E = \rho dt$. Substituting $d\psi_t^E = \rho dt$ into the F.O.C. for $m_t^E$, we have

$$\rho - r_t = \lambda \left( \frac{q_t^T \theta \kappa + q_t^T (1 - \theta) \kappa - 1}{1 - \kappa (1 - \theta) q_t^T} \right).$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for $k_t^E$ and rearranging the equation, we have

$$\mathbb{E}_t \left[ dr_t^T \right] = \rho dt.$$
**Proof of Proposition 2.** Conjecture that the bank’s value function takes the linear form: \( v_t^B = q_t^B n_t^B \). In equilibrium, the marginal value of equity, \( q_t^B \), evolves as follows

\[
dq_t^B = q_t^B \mu_t^B dt + q_t^B \sigma_t^B dZ_t.
\]

Define \( dy_t^B = dc_t^B / n_t^B \), the consumption-to-wealth ratio of bankers. Under the conjectured functional form, the HJB equation is

\[
\rho v_t^B dt = \max_{dy_t^B \in \mathbb{R}} \left\{ (1 - q_t^B) \mathbb{I}_{\{dy_t^B > 0\}} n_t^B dy_t^B \right\} + \mu_t^B q_t^B n_t^B + \max_{x_t^B \geq 0} \left\{ r_t + x_t^B \left( \mathbb{E}_t \left[ dr_t^T \right] - r_t \right) + x_t^B \sigma_t^B \left( \sigma_t^T + \sigma \right) \right\} q_t^B n_t^B,
\]

where \( \gamma_t^B = -\sigma_t^B \). Dividing both sides by \( q_t^B n_t^B \), we eliminate \( n_t^B \) in the HJB equation,

\[
\rho = \max_{dy_t^B \in \mathbb{R}} \left\{ \frac{(1 - q_t^B)}{q_t^B} \mathbb{I}_{\{dy_t^B > 0\}} dy_t^B \right\} + \mu_t^B + \max_{x_t^B \geq 0} \left\{ r_t + x_t^B \left( \mathbb{E}_t \left[ dr_t^T \right] - r_t \right) + x_t^B \sigma_t^B \left( \sigma_t^T + \sigma \right) \right\},
\]

and thus, confirm the conjecture of linear value function. The indifference condition for \( x_t^B \) gives the equation in Proposition 2.

**Proof of Proposition 3.** Following Brunnermeier and Sannikov (2014), I derive the stationary probability density. Probability density of \( \eta_t \) at time \( t \), \( p(\eta, t) \), has Kolmogorov forward equation

\[
\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta} \left( \eta \mu^\eta(\eta) p(\eta, t) \right) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left( \eta^2 \sigma^\eta(\eta)^2 p(\eta, t) \right).
\]

Note that in a Markov equilibrium, \( \mu^\eta_t \) and \( \sigma^\eta_t \) are functions of \( \eta_t \). A stationary density is a solution to the forward equation that does not vary with time (i.e. \( \frac{\partial}{\partial \eta} p(\eta, t) = 0 \)). So I suppress the time variable, and denote stationary density as \( p(\eta) \). Integrating the forward equation over \( \eta \), \( p(\eta) \) solves the following first-order ordinary differential equation within the two reflecting boundaries:

\[
0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} \left( \eta^2 \sigma^\eta(\eta)^2 p(\eta) \right), \quad \eta \in [\underline{\eta}, \overline{\eta}].
\]

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The integration constant $C$ is zero because of the reflecting boundaries. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\eta}^{\eta_0} p(\eta) \, d\eta = 1$).

Next, I solve the expected time to reach from $\eta$. Define $f_{\eta_0}(\eta)$ the expected time it takes to reach $\eta_0$ starting from $\eta \leq \eta_0$. Define $g(\eta_0) = f_{\eta_0}(\eta)$ the expected time to reach $\eta_0$ from $\eta$. One has to reach $\eta \in (\eta, \eta_0)$ first and then reach $\eta_0$ from $\eta$. Therefore, $g(\eta) + f_{\eta_0}(\eta) = g(\eta_0)$. Since $g(\eta_0)$ is constant, we differentiate both sides to have $g'(\eta) = -f'_{\eta_0}(\eta)$ and $g''(\eta) = -f''_{\eta_0}(\eta)$.

From $\eta_t$, the expected time to reach $\eta_0$, denoted by $f_{\eta_0}(\eta_t)$, is decomposed into $s - t$, and $E_t \left[ f_{\eta_0}(\eta_s) \right]$, i.e., the expected time to reach $\eta_0$ from $\eta_s$ ($s \geq t$) after $s - t$ has passed. We have $f_{\eta_0}(\eta_t)$ equal to $E_t \left[ f_{\eta_0}(\eta_s) \right] + s - t$. Therefore, $t + f_{\eta_0}(\eta_t)$ is a martingale, so $f_{\eta_0}$ satisfies the ordinary differential equation: $1 + f'_{\eta_0}(\eta) \mu(\eta) + \frac{\sigma(\eta)^2}{2} f''_{\eta_0}(\eta) = 0$. Therefore, $g(\eta)$ must satisfy

$$1 - g'(\eta) \mu(\eta) - \frac{\sigma(\eta)^2}{2} g''(\eta) = 0.$$ 

It takes no time to reach $\eta$, so $g(\eta) = 0$. Moreover, since $\eta$ is a reflecting boundary, $g'(\eta) = 0$.

**Proof of Proposition 4.** The system of ordinary differential equations is constructed in detail in Appendix II. The boundary conditions are explained in the main text.

**Appendix II - Solving the Equilibrium**

The fully solved Markov equilibrium is a set of functions that map $\eta_t$ to the values of endogenous variables, such as tangible capital price, interest rate, bank leverage, and entrepreneurs’ deposit holdings. In the description of solution method, time subscripts are suppressed to save notations.

First, we construct a mapping from $\eta$, $q^B(\eta)$, $q^T(\eta)$, $dq^B(\eta)/d\eta$, and $dq^T(\eta)/d\eta$ to the second-order derivatives, $d^2q^B(\eta)/d\eta^2$ and $d^2q^T(\eta)/d\eta$, i.e., a system of second-order ordinary differential equations.

Given $q^T$, Proposition 1 solves the liquidity premium, $\rho - r$ and deposit rate $r$. For small values of $\eta$, consider the case where banks do not hold all tangible capital, so $\rho = r - \sigma^B(\sigma^T + \sigma)$. 

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By Itô’s lemma,
\[ \sigma^T = \epsilon^T \sigma^\eta \text{ and } \sigma^B = \epsilon^B \sigma^\eta. \]

where \( \epsilon^T = \frac{dq^T/q^T}{dq^T/d\eta} \) and \( \epsilon^B = \frac{dq^B/q^T}{dq^T/d\eta} \). Since \( \rho - r \) is a function of \( q^T \), we substitute the decomposition of \( \sigma^T \) and \( \sigma^B \) into \( \rho = r - \sigma^B (\sigma^T + \sigma) \) to obtain a quadratic equation of \( \sigma^\eta \), and the roots are
\[ \sigma^\eta = \frac{-\epsilon^B \sigma \pm \sqrt{(\epsilon^B \sigma)^2 - 4\epsilon^B \epsilon^T (\rho - r)}}{2\epsilon^B \epsilon^T}. \]

Because \( \epsilon^B < 0, \epsilon^T > 0, \) and \( \rho \geq r \), the only positive root is
\[ \sigma^\eta = \frac{-\epsilon^B \sigma - \sqrt{(\epsilon^B \sigma)^2 - 4\epsilon^B \epsilon^T (\rho - r)}}{2\epsilon^B \epsilon^T}. \]

Because \( \sigma^\eta = x^B (\sigma^T + \sigma) - \sigma \), we can solve
\[ x^B = \frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma}. \] (19)

Next, we need to check whether \( x^B N^B \leq q^T K^T \), or equivalently, \( x^B \eta \leq q^T (1 - \theta) \). If this condition does not hold, we are at point where banks are large enough to hold all tangible capital, so we set \( x^B = q^T (1 - \theta) / \eta \), and solve \( \sigma^\eta \) using Equation (19). Once \( x^B \) and \( \sigma^\eta \) are solved, \( \sigma^B \) and \( \sigma^T \) can be solved by Itô’s lemma, and bankers’ required risk compensation, \(-\sigma^B (\sigma^T + \sigma)\).

Next, we solve the drift terms of bankers’ wealth,
\[ \mu^N = x^B \mathbb{E} [d\eta^T] + (1 - x^B) r, \]

and the diffusion term
\[ \sigma^N = x^B (\sigma^T + \sigma). \]

From Equation (11), we can solve \( \mu^K \). And from Equation (12),
\[ \mu^\eta = \mu^N - \mu^K - \sigma^N \sigma + \sigma^2. \]
To solve the second-order derivatives, \( d^2 q^B (\eta) / d\eta^2 \) and \( d^2 q^T (\eta) / d\eta \), we use bankers’ HJB equation (after substituting the first-order conditions),

\[
\mu^B = \rho - r,
\]

and bankers’ optimality condition for tangible capital holdings in Proposition 2,

\[
\mu^T = r - \sigma^B (\sigma^T + \sigma) - \sigma^T \sigma + \delta - \frac{\alpha}{q^T},
\]

so from Itô’s lemma,

\[
\frac{d^2 q^T}{d\eta^2} = 2q^T \left( \frac{\mu^T - \epsilon^T \mu^\eta}{\sigma^\eta \eta} \right), \text{ and, } \frac{d^2 q^B}{d\eta^2} = 2q^B \left( \frac{\mu^B - \epsilon^B \mu^\eta}{\sigma^\eta \eta} \right).
\]

The procedure constructs a mapping from \( \eta, q^B (\eta), q^T (\eta), dq^B (\eta) / d\eta, \) and \( dq^T (\eta) / d\eta \) to the second-order derivatives, \( d^2 q^B (\eta) / d\eta^2 \) and \( d^2 q^T (\eta) / d\eta \). The five boundary conditions in Proposition 4 pin down a unique solution to the two second-order ODEs and one endogenous boundary (bankers’ consumption boundary \( \overline{\eta} \)).
References


