Dealer Funding and Market Liquidity*

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Abstract

We consider a model in which dealers need to raise external financing to provide immediacy to their clients, and also exert unobservable effort to improve the chance of closing their positions at a profit. This moral hazard problem reduces the amount of external finance dealers can raise, and therefore reduces intermediation volume, softens competition between dealers, and widens bid-ask spreads, and in this sense has a negative effect on the market liquidity of intermediated assets. When dealers suffer losses, the problem becomes worse. Effects are stronger for riskier assets. Endogenous correlations and liquidity spillovers arise between otherwise unrelated assets. As the optimal financing arrangement involves debt, regulations that limits the leverage of bank-affiliated dealers can have adverse effects on market liquidity.

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1 Introduction

In many asset markets, it is the intermediation by dealers that determines how easily or cheaply an asset can be traded. A dealer will allow a market participant to offload a position onto his balance sheet immediately, and will then take over the chore of finding another market participant to close the position later. In this sense, dealers provide immediacy. The terms on which dealers provide this immediacy determine the cost of immediacy for clients, or, the market liquidity of the asset.

To intermediate, dealers typically need external funding. To what extent can frictions in the ability to raise such funding limit dealers' ability to provide immediacy, and what does this imply for market liquidity? Do the frictions suggest an optimal way to raise financing? Does this optimal way to raise financing explain why liquidity might co-move across markets, and does it explain why regulation (especially of bank-affiliated dealers) may have had adverse effects on market liquidity?

To address these questions, we set up a model in which dealers need some external funds in addition to their internal funds to intermediate, but in which they are also subject to an agency problem: When raising external funds, dealers need to preserve their incentives to exert effort. This has two important consequences.

First, dealers cannot raise too much external financing. The limit on the total amount of funds available for intermediation softens softens the competition between dealers, reduces intermediation volume, and widens bid-ask spreads. In that sense, market liquidity becomes worse. The problem is more severe for riskier assets, and gets worse when dealers lose money.

Second, dealers must raise financing in a way that maximizes their incentives per dollar raised. This implies that dealers optimally choose to intermediate across many markets simultaneously and to raise external finance in the form of debt. The former leads to co-movement of liquidity across markets, and the latter suggests that recent regulations such as limits on the leverage of dealers could harm market liquidity.

In the model, dealers acquire positions through the provision of immediacy, and can then exert effort in searching for a good counterparty, to increase the chance of closing the position at a profit.[1] This effort is unobservable by outsiders who provide finance to the dealers. To preserve the skin in the game of dealers, so that they exert effort, the promises to financiers cannot become too large. The agency problem therefore determines a maximum amount that dealers can raise from financiers, that is, a maximum *pledgeable income*. As a consequence, they may not be able to intermediate some large trades at all and hence overall intermediation volume is lower. In other words, due to the agency problem, dealers have limited balance-sheet capacity.

A limit on funding also affects bid-asks spreads and the degree of competition. Financially unconstrained dealers have plenty of funds to compete aggressively and drive down bid-ask spreads. However, financially constrained dealers lack the funds to compete aggressively, resulting in wider bid-ask spreads. Furthermore, these effects are more pronounced for i) larger trades, for which more funding is required; and ii) assets with higher fundamental risks, for which the associated agency problem is more severe.

When dealers can potentially intermediate in several separate markets simultaneously, we show that doing so increases the amount of funds they can raise.[2] This implies that in equilibrium, a dealer attempting to intermediate several markets will out-compete those who do not, and will be able to offer better terms. But this also means that if dealers' financial constraints become tighter, liquidity in all markets will be affected simultaneously. Also, the liquidity demanded in one market can have non-monotonic effects on liquidity supplied in another market. Our agency-based financial constraints provides new micro-foundation for why liquidity should co-move across markets and why liquidity may exhibit non-monotonic spillovers.

[1]In the model, dealers use funds either to buy a security from a client, or as cash collateral to borrow the security and then sell it to clients.

[2]The underlying optimal-contracting effect is described e.g. by Cerasi and Daltung (2000), or Laux (2001). It is sometimes referred to as a "cross-pledging" effect (see Tirole (2006), Section 4.2).

Our model also indicates how regulation of dealer financing can affect market liquidity. As in Innes (1990), external debt is typically part of the optimal financing arrangement because it leaves maximum upside to the dealer, who then has a strong incentive to expend effort so as to close positions at a profit. This motive to use debt financing is in conflict with regulators' motives to limit the leverage of dealers, especially bank-affiliated dealers. A regulator who imposes a maximum leverage ratio (or minimum equity capital requirement) limits the ability of a dealer to preserve incentives, therefore limits the amount of financing that can be raised, and therefore limits the ability of dealers to provide immediacy.

This argument provides an interpretation for recent empirical findings regarding the impact of bank regulations on the liquidity of the U.S. bond market. First, in terms of the amount of liquidity provision, regulation would reduce bank-affiliated dealers' balance-sheet capacity, resulting in smaller trade size, lower trading volume, and a decline in capital committed by bank-affiliated dealers to intermediation. These predictions are borne out in Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018); Bao, O'Hara, and Zhou (2018); Schultz (2017).[3] In addition, as dealers become more reluctant to use their balance sheet to provide immediacy, they broker (or "pre-arrange") more trades between clients (Choi and Huh (2017)) and charge higher prices of immediacy (Dick-Nielsen and Rossi (2018)). Finally, in line with the findings of most of the above studies, our model predicts that these effects are stronger for riskier bond and larger trades.

Related Literature Our paper is most closely related to a set of theoretical papers that examines the role of financially-constrained dealers or arbitrageurs in asset markets. Within this literature, it is the papers that look at intermediaries who provide immediacy that are most closely related to ours. Gromb and Vayanos (2002) consider specialists who can arbitrage the prices of the

[3]See Haselmann, Kick, Singla, and Vig (2019) for similar negative effects of capital regulations on German banks' ability to provide liquidity for equities and bond.

same asset when it trades in two segmented markets, but can only raise funds with fully collateralized debt contracts. Brunnermeier and Pedersen (2009), building on the work of Grossman and Miller (1988), consider dealers who are subject to a form of exogenous, Value-at-Risk based margin constraint, and then go on to consider the feedback loop between losses of dealers and changes in asset prices and volatility. Andersen, Duffie, and Song (forthcoming) study how outstanding debt of a dealer discourages its shareholders to provide liquidity, a form of debt-overhang problem. All these papers also emphasize the role of dealers' internal funds in determining market liquidity. However, they assume exogenous financing constraints. We contribute to this literature by considering the shape of optimal contracts that dealers use to finance themselves in the presence of a moral hazard problem. This approach provides an explanation of why dealers may find it optimal to intermediate across several markets simultaneously, and why, therefore, liquidity cross-moves across assets. It provides novel empirical predictions regarding non-monotonic spillovers between markets. Finally, it also provides a potential explanation of why regulating what securities dealers can use to finance themselves can have an effect on market liquidity.

Other papers consider financially-constrained fund managers. In He and Krishnamurthy (2012) and Brunnermeier and Sannikov (2014), households can only get exposure to a risky asset via investing in funds run by fund managers. Since households want exposure to risk, they will prefer to buy equity-like claims (rather than the debt-like claims which are optimal in our case). There is a tension, because if funds issue to much equity, this reduces the skin in the game of fund managers, and hence their incentive to exert effort. As funds the marginal investors, shocks to intermediary capital affect the dynamic of risk premium and volatility of the risky asset.

Our model contributes to the market microstructure literature on the determinants of bid-ask spreads or more broadly, market illiquidity. Stoll (1978) argues that bid-ask spreads can result from cost that a risk-averse dealer attaches to holding inventory. Kyle (1985) argues that adverse selection can produce a bid-ask spread. Duffie, Garleanu, and Pedersen (2005) suggest that search frictions in OTC markets could result in a bid-ask spread.[4] Our paper proposes another important source of illiquidity in the OTC markets, namely the agency or financing frictions of dealers.

2 The model

There are two dates $t = \{1, 2\}$, one good ("cash"), no time-discounting, and four types of risk-neutral agents: Dealers, clients, financiers, and security lenders. Our focus is on dealers, so that clients, financiers and security lenders will mostly play a passive role. We first consider the case of a single market, in which one type of asset is traded. For simplicity, in this case, we refer to this traded asset as "the asset." There is also always a risk-free asset with a zero net return, which we will call "cash." In Section 4, we consider the case of multiple markets.

There are two types of clients, who differ in their valuation of the asset, and in the date at which they are in the market: The client "Earl" arrives early at $t = 1$ but is not present at $t = 2$, and the client "Lacey" arrives late at $t = 2$, and is not present at $t = 1$. Earl can have either a high or low valuation of the asset. With probability $\frac{1}{2}$, Earl's valuation is high and equal to $V + \ell$, and he wants to buy a random quantity $\tilde{q}$ of the asset. With probability $\frac{1}{2}$, Earl's valuation is low and equal to $V - \ell$, and he wants to sell a random quantity $\tilde{q}$ of the asset. We assume that $\tilde{q}$ is uniformly distributed on $[0, 1]$. Lacey always values the asset at V, and is rich in both cash and assets.

The differential valuations between Earl and Lacey generate potential gains from trade. However, since Earl and Lacey are not present in the market at the same time, they cannot trade directly with each other. $N \geq 2$ competing dealers are present at both $t = 1$ and $t = 2$, and can intermediate, even though they attach no value to the asset per se. Dealers are initially all

[4] See Foucault, Pagano, and Röell (2013) for a comprehensive survey on market liquidity.

identical: They start with no position in the asset, and all have the same, limited amount of cash w as internal capital. Dealers may have to obtain additional cash funding and/or borrow assets in order to intermediate. Financiers are rich in cash, and do not own any units of the asset. They value the asset at $V - k$. Security lenders are rich in assets, but have no cash. They value the asset at $V + k$, and require per-unit cash collateral of $V + k$ for lending the asset.[5]

Importantly, we assume $V > k > \ell > 0$ to ensure that the only available gains from trade are between the clients Earl and Lacey. The different valuations of the asset by various agents are summarized in Figure 1. The premia ($+k$ and $+\ell$) in valuations could be motivated by agents' portfolio or hedging needs for the asset, whereas the discounts ($-k$ and $-\ell$) represent liquidity needs or the opportunity costs of cash.

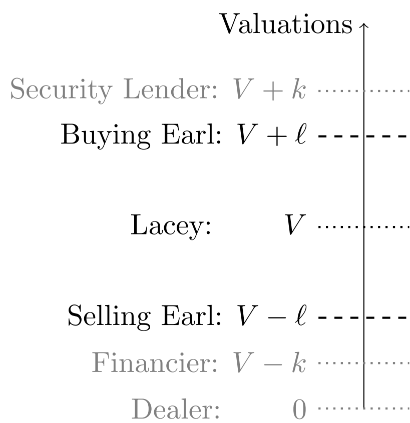

Figure 1. Valuations of the asset by different agents

The timing is as follows. At $t = 1$, the dealers post bid price(s) $\{b(q)\}$ and ask price(s) $\{a(q)\}$ that are function of trade size q. Earl arrives, and will either have a valuation of $V - \ell$ per unit

[5] There are some alternative interpretations to dealers providing cash collateral. For instance, the security lender could be thought of as selling the asset to the dealer for $(V + k)$ and promising to buy it back at the same price at $t = 2$. Or, dealers could be thought of as issuing financial claims which they then use as collateral.

of the asset and want to sell q units, or have a valuation of $V + \ell$ and want to buy q units. For simplicity, we assume that Earl cannot split up his order. Earl sells to the best bid or buys from the best ask.[6]

If Earl sells q units, then the chosen dealer has a financing need of $[qb(q) - w]^+$. If Earl buys q units, then the chosen dealer must borrow q units from the security lenders, and provide them with cash collateral $q(V + k)$. Since Earl pays $qa(q)$ to the chosen dealer, the net financing need for the dealer in this case is $[q(V + k) - qa(q) - w]^+$. The dealer raises any cash that he needs by offering a contract with contingent repayments to financiers, as described in more detail below.[7]

At $t = 2$, a dealer who has opened a position chooses (unobservable) effort $e \in \{0, 1\}$, which affects the probability of finding Lacey. Finding Lacey is useful, as it allows closing the position at a more advantageous price – not finding Lacey means that the dealer needs to close the position either by buying assets from security lenders at a high price, or selling to financiers at a low price. Once the position has been closed, repayments on the contract with financiers are made.

We assume that if the dealer exerts effort ($e = 1$), he finds Lacey with probability 1. If the dealer shirks ($e = 0$), he finds Lacey with probability $1 - \delta$ (where $1 > 1 - \delta > 0$). In terms of e, we can therefore write the probability of finding Lacey as $1 - (1 - e)\delta$. We also assume that effort incurs a non-pecuniary cost proportional to the number of units of the asset, cq.

Suppose Earl sells to a dealer's bid. If the dealer finds Lacey and sells on to her, then the dealer

[6] To break ties, we index dealers $\{1, 2, 3, \dots\}$, and assume that if there are multiple dealers with the same best bid or ask, Earl sells to or buys from the dealer with the lowest index.

[7] Here, we have described the timing as one in which dealers post bid and asks, are then potentially chosen by clients, and only then raise financing. This description may seem somewhat unrealistic. However, other, more realistic interpretations of the timing exist. For instance, dealers may first obtain a funding commitment from financiers, e.g. a credit line, at some date $t = 0$. Then, if a dealer is chosen at $t = 1$, the dealer requests the fund under the funding commitment (e.g. draws down on the credit line) to finance the intermediation. Or, dealers may use the transaction in the asset itself to obtain funds. For instance, when buying the asset, the dealer could repo it out to finance the transaction. Macchiavelli and Zhou (2019) provide evidence to support this view: the ability of dealers to finance their inventories through repos affects their bid-ask spreads in the U.S. corporate bond market.

will obtain a high gross cash flow of $x_b^H = qV$. If he does not find Lacey and has to sell on to financiers, he would obtain a low gross cash flow of $x_b^L = q(V-k)$.

Similarly, suppose Earl buys from a dealer's ask, after the dealer has borrowed the assets from some security lenders. If the dealer finds Lacey and buys back the assets from her for at price qV, and then returns the assets to his security lenders and gets back the cash collateral $q(V+k)$, he will obtain a high gross cash flow of $x_a^H = q(V+k) - qV = qk$. If he does not find Lacey, then buys back the assets from some other security lenders at a higher price $q(V+k)$, and then returns the asset to his security lenders and gets back the cash collateral $q(V+k)$, he will obtain a low gross cash flow of $x_a^L = q(V+k) - q(V+k) = 0$.

The repayments to financiers can be contingent on the gross cash flow, and on whether the dealer buys or sells from Earl. We denote these repayments as R_b^H, R_b^L, R_a^H, R_a^L for the case of high (superscript H) and low (superscript L) cash flows, in the case when the dealer buys (subscript b) and sells (subscript a), respectively.

We make the following assumptions on parameters. First, we assume that the cost of exerting effort is lower than both the gains from trade (ℓ) and the expected loss in value from shirking (δk). Hence it is efficient to exert effort.

Assumption 1. $c < \min\{\ell, \delta k\}$

We then also assume that the expected loss in value from shirking is larger than the gains from trade, implying that intermediation while shirking destroys value.

Assumption 2. $\delta k > \ell$.

We solve the single-market version of the model as outlined here in Section 3. In Section 4, we extend the model to multiple markets. In this version of the model, in each market, exactly one type of asset is traded. Each market will have their separate, unrelated clients (Earls and Laceys), so that search effort needs to be exerted independently in each market.

3 Intermediation in a single market

As a benchmark, we first discuss the case in which dealers have large cash capital w, so that no external financing is required, and hence no agency problems arise. We then turn to the more interesting case in which w is low enough to produce agency frictions.

First-best benchmark Consider $N \geq 2$ dealers with large w. These dealers do not need external financing to intermediate, and Assumption 1 guarantees that they will always exert effort once they have opened a position. We consider the case in which the dealers compete à la Bertrand, in posting functions $b_c(q), a_c(q)$ that specify how much they would bid or ask for a given quantity q sold to them or bought from them.

Suppose that in equilibrium, the two bids are different, and suppose that the dealer with the higher bid obtains positive net utility from intermediating sales from Earl. The dealer with the lower will does not intermediate, obtains zero net utility, and can deviate by out-bidding the other dealer and obtain positive net utility. So this cannot happen in equilibrium. In equilibrium, the dealer with the highest bid must obtain zero net utility from intermediating sales from Earl. Similarly, the dealer with the lowest ask must obtain zero net utility from intermediating purchases from Earl. This argument implies that in equilibrium, the highest bid and lowest ask must be $b_c = V - c$ and $a_c = V + c$, respectively.

The average bid-ask spread for an order with size q is:

$$a_c - b_c = V + c - (V - c) = 2c. \tag{1}$$

Since all trades are intermediated, the average intermediated volume is equal to:

$$E[q] = \frac{1}{2}. \tag{2}$$

In equilibrium, both dealers' net *expected* utility is 0. Earl either sells a quantity q at price $V - c$ but has valuation $V - \ell$, and hence obtains utility $q(V - c - (V - \ell)) = q(\ell - c)$, or buys a quantity q at price $V + c$ but has valuation $V + \ell$, and hence obtains utility $q(V + \ell - (V + c)) = q(\ell - c)$. Earl's expected utility is therefore $E[q](\ell - c) = \frac{1}{2}(\ell - c)$. Lacey's expected utility is zero. The social surplus, defined as the unweighted sum of utilities of all agents, is therefore

$$\mathcal{S} = \frac{1}{2}(\ell - c). \tag{3}$$

Pledgeable income/ Funding liquidity Consider now dealers who need to raise external finance in order to be able to intermediate. The repayments to financiers under the contract are contingent on the cash flows to the dealer, and hence, contingent on whether the dealer succeeds in finding Lacey. For any given bid $b(q)$ or ask $a(q)$, the dealer can consider the repayments R_b^H, R_b^L, R_a^H, R_a^L that maximize his utility, subject to the relevant constraints.

Formally, we can write the utility of the buying and selling dealer, respectively, as follows:[8]

$$U_j(e, \{R_j\}) = (1 - (1 - e)\delta)\left[x_j^H - R_j^H\right] + (1 - e)\delta\left[x_j^L - R_j^L\right] - cqe, \text{ for } j \in \{b, a\}, \tag{4}$$

where $\{R_j\}$ denotes the state-contingent payoffs to financiers, R_j^H and R_j^L, and the gross cash flows are $x_b^H = qV$, $x_b^L = q(V - k)$, $x_a^H = qk$, $x_a^L = 0$, respectively, as explained in Section 2.

Under Assumption 2, intermediation will destroy value unless effort is exerted. Contracts under which effort is not exerted can therefore never be optimal, so that we can state the dealer's problem,

[8]The utility of the buying dealer is actually $U_b - qb(q) + [qb(q) - w]^+ + w$, but this is equal to U_b, since here, $[qb(q) - w]^+ = qb(q) - w$. Similarly, the utility of the selling dealer is $U_a + qa(q) - q(V + k) + [q(V + k) - qa(q) - w]^+ + w = U_a$.

for $j \in \{b, a\}$ as follows:

$$\max_{\{R_j\}} U_j(1, \{R_j\}), \text{ for } j \in \{b, a\} \tag{P}$$

subject to the incentive compatibility constraint

$$U_j(e = 1, \{R_j\}) \geq U_j(e = 0, \{R_j\}), \text{ for } j \in \{b, a\}. \tag{5}$$

After some manipulation, this can be written as

$$R_j^H - R_j^L \leq q\left(k - \frac{c}{\delta}\right), \text{ for } j \in \{b, a\}. \tag{IC}$$

In addition, the contract must satisfy a limited liability constraint,

$$R_j^H \leq x_j^i, \text{ for } j \in \{b, a\} \text{ and } i \in \{H, L\}, \tag{LL}$$

as well as the financiers' break-even constraint:

$$R_j^H = \begin{cases} qb(q) - w & \text{if } j = b, \\ q(V + k) - qa(q) - w & \text{if } j = a. \end{cases} \tag{BE}$$

We note that the break-even constraint takes a particularly simple form because the exertion of effort implies that the dealer will find Lacey with probability 1.

The optimal contract that satisfies these constraints and solves the dealer's problem (P) is potentially not unique. In much of our analysis, however, we will not focus on the contract per se, but on the *pledgeable income* $\mathcal{P}_j(q)$, defined as the maximum amount of cash that can be raised from financiers:

$$\mathcal{P}_j(q) = \max_{\{R_j\}} R_j^H \tag{6}$$

subject to the constraints (IC), (LL), (BE).

The amount raised from financiers is equal to R_j^H, as can be seen from the break-even constraint (BE). Maximizing this amount requires that the constraint (IC) bind with equality, and that R_j^L is set as high as possible given the limited liability constraints (LL).

When the dealer buys, we therefore obtain

$$\mathcal{P}_b(q) = \max_{R_b^H} R_b^H = q\left(V - \frac{c}{\delta}\right). \tag{7}$$

Similarly, when the dealer sells, we have

$$\mathcal{P}_a(q) = \max_{R_a^H} R_a^H = q\left(k - \frac{c}{\delta}\right). \tag{8}$$

The pledgeable income per unit of asset $\mathcal{P}_j(q)/q$ corresponds to the *funding liquidity* in Brunnermeier and Pedersen (2009).

Agency frictions matter if and only if dealers cannot simply raise all the cash that they need to intermediate from financiers. We therefore make the following assumption:

Assumption 3. *Dealers must use some of their own cash for intermediation,* $\frac{c}{\delta} > \ell$.

To see why this assumption implies that dealers must use some of their own cash, consider first a dealer who buys. Under Assumption 3, the per unit pledgeable income $\mathcal{P}(q)/q = V - \frac{c}{\delta}$ is smaller than the lowest valuation at which Earl is willing to sell $(V - \ell)$, implying that the dealer cannot raise all the required cash from financiers and must contribute at least $(\frac{c}{\delta} - \ell)$ of his own cash per intermediated unit. A similar argument applies for a selling dealer.

Balance-sheet capacity and market liquidity Assumption 3 implies that dealers may not be able to set high bids or low asks, because they may not be able to raise the cash required to finance these high bids or low asks. The maximum total amount that a dealer can bid is his own cash capital w, plus the cash that can be raised from financiers $\mathcal{P}(q)$. In other words, $w + \mathcal{P}(q)$ is the dealer's *balance-sheet capacity.* This implies that the maximum bid that a dealer can finance (with an incentive compatible contract) is

$$b_{IC}(q, w) = \frac{w + \mathcal{P}(q)}{q} = \frac{w}{q} + \left(V - \frac{c}{\delta}\right). \tag{9}$$

Similarly, the maximum total amount that a dealer can raise to finance the cash collateral required for a sale (net of the ask price) is equal to the dealer's own cash, plus the cash that can be raised from financiers, or $q(V+k) - qa(q) = w + \mathcal{P}_a(q) = w + q\left(k - \frac{c}{\delta}\right)$. Hence the minimum ask price that a dealer can offer is

$$a_{IC}(q,w) = -\frac{w}{q} + \left(V + \frac{c}{\delta}\right). \tag{10}$$

The bid and ask measure the *market liquidity* of the asset. As shown above, agency frictions affect funding liquidity $\mathcal{P}_j(q)/q$, which in turns determine market liquidity.

Competitive equilibrium with financial constraints In the absence of financial constraints, competitive dealers can raise bids and lower asks until they obtain zero utility, and bids and asks are equal to $b_c = V - c$ and $a_c = V + c$, respectively. Now, however, the maximum bid and minimum ask is constrained by pledgeable income, as described in equations (9) and (10). Since more cash is required for intermediating trades of larger size, there is a level of q above which these constraints bind. We denote this level as $\bar{q}(w)$. It is implicitly defined by $b_c \equiv b_{IC}(\bar{q}(w), w)$ and $a_c \equiv a_{IC}(\bar{q}(w), w)$ which implies that

$$\bar{q}(w) = \frac{w}{\frac{c}{\delta} - c}. \tag{11}$$

For any $q > \bar{q}(w)$, even dealers who are competing with each other cannot set a bid above $b_{IC}(q,w)$ or an ask below $a_{IC}(q,w)$. In that sense, financial constraints limit competition.

In addition, since a dealer has to use $\left(\frac{c}{\delta} - 1\right)$ units of his own cash per intermediated unit of the asset, the maximum number of units which a dealer can intermediate is now potentially limited, and given by

$$q_{\max}(w) = \begin{cases} 1 & \text{if } w \geq \frac{c}{\delta} - \ell \\ \frac{w}{\frac{c}{\delta} - \ell} & \text{otherwise.} \end{cases} \tag{12}$$

An alternative way to interpret $q_{\max}(w)$ is to note that for trades with $q > q_{\max}(w)$, a dealer would only be able to pay a bid below Earl's reservation value $V - \ell$ (would only be able to sell at an ask above Earl's reservation value $V + \ell$), so intermediation cannot occur. Below, we will refer to the maximum size $q_{\max}(w)$ for which intermediation occurs as the *depth* of the market. Financial constraints reduce market depth.

Noting that $\bar{q}(w) < q_{\max}(w) = \frac{w}{\frac{c}{\delta}-\ell}$, we can summarize the discussion in the following proposition:

Proposition 1 (Equilibrium with single market)**.** *In a competitive equilibrium, the bids and asks are as follows:*

$$b_c(q,w) = \min\{b_{IC}(q,w), b_c\} = \min\left\{\frac{w}{q} + \left(V - \frac{c}{\delta}\right), V - c\right\} \tag{13}$$

$$= \begin{cases} b_c = V - c & \textit{for} \quad q < \bar{q}(w) \\ b_{IC}(q,w) = \frac{w}{q} + \left(V - \frac{c}{\delta}\right) & \textit{for} \quad q \in [\bar{q}(w), q_{\max}(w)] \\ < V - \ell \textit{ and thus no trade} & \textit{for} \quad q > q_{\max}(w) \end{cases} \tag{14}$$

and

$$a_c(q,w) = \max\{a_{IC}(q,w), a_c\} = \max\left\{\left(V + \frac{c}{\delta}\right) - \frac{w}{q}, V + c\right\} \tag{15}$$

$$= \begin{cases} a_c = V + c & \textit{for} \quad q < \bar{q}(w) \\ a_{IC}(q,w) = \left(V + \frac{c}{\delta}\right) - \frac{w}{q} & \textit{for} \quad q \in [\bar{q}(w), q_{\max}(w)] \\ > V + \ell \textit{ and thus no trade} & \textit{for} \quad q > q_{\max}(w) \end{cases} \tag{16}$$

where

$$\bar{q}(w) = \frac{w}{\frac{c}{\delta} - c} \qquad \textit{and} \qquad q_{\max}(w) \textit{ is defined in (12)} \tag{17}$$

In equilibrium, the bid-ask spread is

$$a_c(q,w) - b_c(q,w) = \begin{cases} 2c & \textit{for} \quad q < \bar{q}(w) \\ 2(\frac{c}{\delta} - \frac{w}{q}) & \textit{for} \quad q \in [\bar{q}(w), q_{\max}(w)] \\ \textit{no trade} & \textit{for} \quad q > q_{\max}(w) \end{cases} \tag{18}$$

Proof. See the preceding discussion. □

We can see that for small trades, the bid-ask spread is as in the first-best benchmark. For medium trades, the spread is wider than in the first-best benchmark. This is because agency frictions restrict the amount of cash that dealers can raise from financiers, restricting their ability to aggressively compete to tighten bid-ask spreads. It may not be possible to intermediate large trades at all, as these require too much financing.

If $q_{\max}(w) < 1$, large trades with $q \in [q_{\max}(w), 1]$ cannot be intermediated. This has an effect on the average intermediated volume, which is now

$$E[q] = \int_0^{q_{\max}(w)} q dq = \begin{cases} \frac{1}{2} & \text{if } w \geq \frac{c}{\delta} - \ell \\ \frac{q_{\max}(w)^2}{2} & \text{otherwise} . \end{cases} \tag{19}$$

Lacey's utility is zero, and Earl's utility is

$$\begin{cases} q(V-c) - q(V-\ell) = q(\ell - c) & \text{for} \quad q \leq \bar{q}(w) \\ [w + q(V - \frac{c}{\delta})] - q(V-\ell) = w - q(\frac{c}{\delta} - \ell) & \text{for} \quad q \in [\bar{q}(w), q_{\max}(w)] \\ 0 \text{ due to no trade} & \text{for} \quad q > q_{\max}(w). \end{cases} \tag{20}$$

We can see that Earl's utility is highest when bid-ask spreads are tightest, and that it decreases as bid-ask spreads widen. It is zero when there is no trade.

Dealers who are not chosen have a net utility of zero. A dealer who is chosen has net utility

$$U^c(q,w) = \begin{cases} q(V-c) - q(V-c) = 0 & \text{for} \quad q \leq \bar{q}(w) \\ q(V-c) - [w + q(V - \frac{c}{\delta})] = q(\frac{c}{\delta} - c) - w & \text{for} \quad q \in [\bar{q}(w), q_{\max}(w)] \\ 0 \text{ due to no trade} & \text{for} \quad q > q_{\max}(w). \end{cases} \tag{21}$$

This is zero for very tight bid-ask spreads with unconstrained Bertrand competition, becomes positive as agency frictions restrict pledgeable income and hence lead to wider bid-ask spreads and a positive agency rent for a chosen dealer.

Hence the social surplus generated by any trade of size $q \leq q_{\max}(w)$, defined as the sum of Earl's utility, Lacey's utility, and the chosen dealer's utility, is $\ell - c$. Social surplus is therefore given by the following expression:

$$\mathcal{S} = \begin{cases} \frac{1}{2}(\ell - c) & \text{if } w \geq \frac{c}{\delta} - \ell \\ \frac{(q_{\max}(w))^2}{2}(\ell - c) & \text{otherwise.} \end{cases} \tag{22}$$

We illustrate the bid-ask spreads in Figure 2. Competition leads to tight bid-ask spreads for smaller orders, with size up to $q \leq \bar{q}(w)$. For medium sized orders, the amounts of external finance that dealers can raise limits the extent to which they can compete on bid-ask spreads, meaning that spreads are wider. Large order with $q > q_{\max}(w)$ are not intermediated because the spread becomes so wide that clients find it unprofitable to trade.

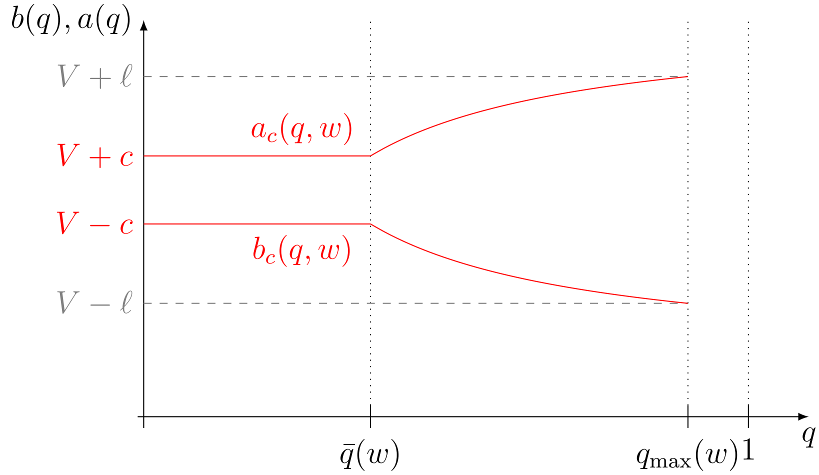

Figure 2. Bid-ask spreads with financial constraints
Competitive bid and asks $(b_c(q, w), a_c(q, w))$ as functions of size of trades q. Competition drives down bid-ask spreads to zero-profit level for smaller orders, with size up to $q \le \bar{q}(w)$. For medium sized orders, the limits to the ability to raise external finance, caused by agency problems, mean that dealers cannot compete aggressively, so that competitive bid-ask spreads become wider. Large order with $q > q_{\max}(w)$ are not intermediated because the spread becomes so wide that clients find it unprofitable to trade.

The comparative statics of the bid-ask spreads and market depth with respect to w are described in the following corollary to Proposition 1:

Corollary 1 (Dealers' capital increases market liquidity)**.** *Consider a situation in which financial constraints have an effect on market liquidity, in the sense that $\bar{q}(w) < 1$, and $q_{\max}(w) < 1$. Then an unexpected increase in dealers' internal capital w produces an increase in $\bar{q}(w)$, an increase in market depth $q_{\max}(w)$, and an increase in expected volume. The bid-ask spread for small trade sizes $(q < \bar{q}(w))$ is unaffected and remains at $2c$. The bid-ask spreads for larger trade sizes narrow.*

Corollary 1 illustrates how in the model, dealers' internal capital are positively related to measures of market liquidity such as market depth, expected volume, and bid-ask spreads. In this sense, the model suggests that when dealers lose money and therefore have less internal capital, market liquidity suffers: bid-ask spreads for larger trades widen, market depth decreases, and volume decreases. This is consistent e.g. with the empirical finding of Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010) on NYSE specialists' ability to provide liquidity.

4 Intermediation in multiple markets

In this section we consider an extension of the model in which dealers can intermediate in two markets simultaneously. We assume that dealers need to exert independent search effort to find a good counterparty in each market. There are three key results: 1. Intermediating across markets relaxes incentive constraints, and allows dealers to raise more financing per market; 2. As a result, in equilibrium, a dealer who intermediates across markets outbid two dealers who do not; 3. When dealers are financially constrained, there are endogenous correlation and non-monotonic spillovers in liquidity between different assets. These results arise entirely for reasons related to agency frictions and properties of the optimal contract, and in spite of the fact that all agents are risk-neutral and that assets payoffs are uncorrelated (they are riskless).

4.1 Setup

We consider a setup with with two markets. We assume that each market has its separate, unrelated set of clients (Earls and Laceys), so that search effort needs to be exerted independently in each market. In market A, an "Earl$_A$" arrives at $t = 1$ to trade a quantity $q^A \in [0, 1]$ of asset A. In market B, an "Earl$_B$" arrives at $t = 1$ to trade a quantity $q^B \in [0, 1]$ of asset B. A dealer may choose to post bid-ask spreads only in one market, or in both markets. A dealer posting bid-ask spreads in market A may be chosen by the Earl$_A$ in market A, and will then have to search for

Lacey$_A$ in market A. A dealer posting bid-ask spreads in market B may be chosen by the Earl$_B$ in market B, and will then have to search for the Lacey$_B$ in market B.

Fundamental values (Laceys' valuation) of the assets are fixed constants V_A, V_B, and so are uncorrelated. To reduce notational clutter, we set $V_A = V_B = V$. Furthermore, we assume that a dealer intermediating in both markets has to make two separate search effort decisions to find the counterparties in two different markets. Conditional on the effort choices, the probabilities of finding a counterparty in either market are independent. The setting is intentionally stark to ensure that the only commonality between the two markets is that they are potentially both intermediated by the same dealer.

We consider $N \geq 3$ identical dealers who compete à la Bertrand to intermediate the two markets.[9] As a convention to break ties, we implicitly assume that Dealer 1 has an infinitesimal capital advantage over the other two dealers.

Finally, as we explain below, dealers who intermediate in both markets have higher pledgeable income per market. To ensure that such dealers still need to use some of their internal capital w for intermediation, we need to replace Assumption 3 with the following, tighter assumption:

Assumption 4. *(Replaces Assumption 3.)*
$\frac{c}{\delta(2-\delta)} > \ell$.

4.2 Analysis

Below, we will refer to a dealer who intermediates in both markets simultaneously as a *cross-market dealer*, and a dealer who only intermediates in one market as a *specialized dealer*. In this subsection, we first show that a cross-market dealer has higher pledgeable income than the combined pledgeable income of two dealers who specialize in the two markets. This result has important implications.

[9]With only two dealers, an equilibrium in which each dealer monopolizes intermediation in one asset is possible. Assuming at least three dealers rules out this equilibrium.

First, we show that it implies that a cross-market dealer can sometimes intermediate trades with sizes that cannot be intermediated by two specializing dealers. We then show that it also has an effect on the terms trade, i.e., liquidity in the two markets.

The following proposition characterizes the pledgeable income of a cross-market dealer:

Proposition 2 (Cross pledging)**.** *The total pledgeable income of a dealer intermediating two markets in the case of bid and, respectively ask, are*

$$\mathcal{P}_b(q^A, q^B) = \begin{cases} (q^A + q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) & \textit{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A V + q^B\left(V - \frac{c}{\delta}\right) & \textit{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B V + q^A\left(V - \frac{c}{\delta}\right) & \textit{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta} \end{cases} \tag{23}$$

and

$$\mathcal{P}_a(q^A, q^B) = \begin{cases} (q^A + q^B)\left(k - \frac{c}{\delta(2-\delta)}\right) & \textit{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A k + q^B\left(k - \frac{c}{\delta}\right) & \textit{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B k + q^A\left(k - \frac{c}{\delta}\right) & \textit{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta}. \end{cases} \tag{24}$$

This is strictly higher than the sum of the pledgeable incomes of two specialized dealers, i.e., $\mathcal{P}_j(q^A, q^B) > \mathcal{P}_j(q^A) + \mathcal{P}_j(q^B)$ *for* $j \in \{b, a\}$*.*

Proof. See Appendix C. □

Proposition 2 shows that there are "economies of scope" when a dealer intermediates in multiple markets, in the sense that such a dealer can obtain more funding per market than dealers intermediating each of the markets individually. This type of result is well-known in the contracting literature (Cerasi and Daltung, 2000; Laux, 2001), and is sometimes described as "cross-pledging" (Tirole, 2006, Sec. 4.2). The intuition is as follows. When intermediating in multiple markets, the dealer can design a financing contract under which the dealer only gets positive payoffs when counterparties are found in all markets. That is, the dealer can effectively pledge the profit of intermediating in each individual market as collateral. As a result, the dealer's incentives to search are strengthened and the pledgeable income per market is increased.

We now turn to the question as to what combination of trade sizes (q^A, q^B) can be intermediated by a cross-market dealer. Trade can only take place if the dealer sets a bid and ask that are at least as high and as least as low as Earl's reservation values $V - \ell$ (when Earl sells) and $V + \ell$ (when Earl buys), respectively. The dealer must be able to finance such trades, requiring

$$w + \mathcal{P}_b(q^A, q^B) \geq (q^A + q^B)(V - \ell). \tag{25}$$

and (after some algebra)

$$w + \mathcal{P}_a(q^A, q^B) \geq (q^A + q^B)(k - \ell). \tag{26}$$

for the case of bids and asks respectively. Using the expression for pledgeable income in (23) and (24), we have the following results.

Proposition 3 (Combination of trade sizes that can be intermediated by a cross-market dealer)**.** *Consider a pair of orders (q^A, q^B), for which $q^A \leq q^B$. A cross-market dealer can intermediate both assets when $q^B \leq q^B_{max}(q^A, w)$, where*

$$q^B_{\max}(q^A, w) = \begin{cases} \dfrac{w + q^A \ell}{\frac{c}{\delta} - \ell} & \text{for} \quad q^A \in \left[0, \dfrac{(1-\delta)w}{\frac{c}{\delta} - \ell(2-\delta)}\right] \\ \dfrac{w}{\frac{c}{\delta(2-\delta)} - \ell} - q^A & \text{for} \quad q^A \in \left(\dfrac{(1-\delta)w}{\frac{c}{\delta} - \ell(2-\delta)}, \dfrac{\frac{1}{2}w}{\frac{c}{\delta(2-\delta)} - \ell}\right] \end{cases} \tag{27}$$

The expression for $q^A_{\max}(q^B, w)$, for trades with sizes $q^A \geq q^B$ is symmetric.

Any pair of orders (q^A, q^B) that can be intermediated by a pair of specialized dealers can also always be intermediated by a cross-market dealer. The converse is not true.

Proof. See Appendix C. □

The result in Proposition 3 implies that there are combinations of trade sizes (q^A, q^B) that can be intermediated by a cross-market dealer, but which cannot both be intermediated by two specialized dealers. We illustrate this in Figure 3.

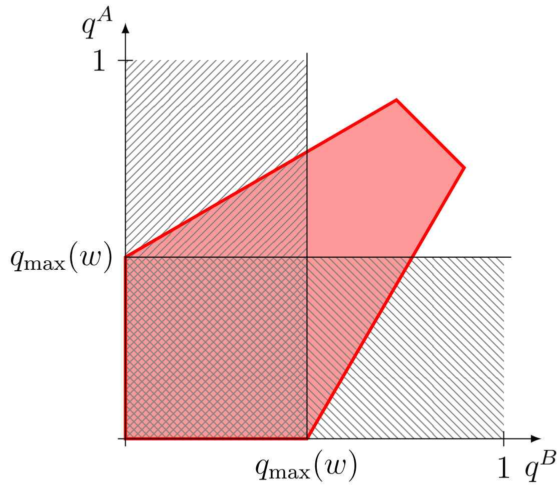

Figure 3. Trades sizes, cross-market dealers, and specialized dealers
Specialized dealers who intermediate only in market A or market B, respectively, can intermediate trades of at most size $q_{\max}(w)$, as indicated by the hatched regions. A cross-market dealer who intermediates in both markets can intermediate the trades indicated in the dark shaded region.

Proposition 3 highlights the novel effects of cross pledging on market depth of two unrelated assets, when they are intermediated by financially-constrained dealers. First, a relaxation of financial constraint (higher w or lower $\frac{c}{\delta}$) increases the (cross-market) depths of the two assets *simultaneously.* In addition, the depth of an asset is non-monotonic in the demand for immediacy in the other asset, due to the opposite forces of expanding total pledgeable income via cross pledging, and of exhausting available pledgeable income. When the demand for immediacy for asset A is small $\left(q^A \leq \frac{(1-\delta)w}{\frac{c}{\delta}-\ell(2-\delta)}\right)$, the cross-pledging benefit dominates and the depth of asset B is *increasing* in q^A. In contrast, when q^A is large, further increases in demand for immediacy in asset A deplete pledgeable income, and hence reduce the depth of asset B. Finally, these phenomena of correlated market depth and spillovers across asset would not emerge if dealers were unconstrained (high enough w and low enough $\frac{c}{\delta}$, such that $w > \frac{c}{\delta} - \ell$ or $q^B_{\max}(q^A, w) > 1$).

We now turn to how cross-market dealers may affect the terms of trade, i.e., market liquidity for the two assets. For a cross-market dealer to out-compete specializing dealers, such a dealer must have sufficient funds to outbid the specializing dealers. For bids, this requires

$$w + \mathcal{P}_b(q^A, q^B) \geq q^A b_c^A(q^A, w) + q^B b_c^B(q^B, w), \tag{28}$$

where $b_c^i(q^i, w) = \min\{V - c, \frac{w}{q^i} + V - \frac{c}{\delta}\}$. Similarly, for asks, this requires

$$w + \mathcal{P}_a(q^A, q^B) \geq q(V + k) - q^A a_c^A(q^A, w) - q^B a_c^B(q^B, w), \tag{29}$$

$a_c^i(q^i, w) = \max\left\{V + c, V + \frac{c}{\delta} - \frac{w}{q_i}\right\}$.

If these conditions are satisfied, all dealers will attempt to become cross-market dealers, to intermediate in both markets simultaneously. Our tie-breaking assumption means that Dealer 1 wins this competition. The following proposition characterizes equilibrium:

Proposition 4 (Equilibrium with multiple markets)**.** *Consider a pair of orders (q^A, q^B) that can be intermediated by a cross-market dealer, as described in Proposition 3.*

There always exists an equilibrium in which all such pairs of orders will be intermediated by a cross-market dealer. This equilibrium has the following properties:

- *For any w, bid-ask spreads posted by the cross-market dealer are tighter than those that could be posted by dealers specializing in each market.*
- *The bid (the ask) is weakly increasing (weakly decreasing) in w. For sufficiently high w, the bid is equal to $V - c$ (the ask is equal to $V + c$).*

For sufficiently high w, there also exists an equilibrium in which specialized dealers post bids and ask equal to the first-best levels of $V - c$ and $V + c$, respectively.

Proof. See Appendix C. □

Proposition 4 highlights the importance of financial constraints (low w) on market liquidity and structure in a competitive equilibrium. In equilibrium, a single dealer will emerge to dominate both asset markets, thanks to the enhanced pledgeable income from cross pledging. On the one hand, this improves liquidity provision for both assets, as shown in the tighter bid-ask spreads posted by a cross-market dealer than two specialized dealers. On the other hand, however, the presence of a cross-market dealer also has other implications: First, when internal capital w is low enough, an unexpected shock to w would cause bid-ask spreads (or, liquidity) to co-move in both markets. In other words, the presence of cross-market dealers in equilibrium produces positive correlation in liquidity between the two assets. Second, a high demand for immediacy in one asset can lead to both a higher and a lower bid (respectively, ask) in the other asset. In that sense, shocks to the demand for immediacy in one asset can have positive and negative spillovers on the liquidity in the other asset. We summarize these observations in the following corollary.

Corollary 2 (Correlation and spillovers in liquidity, as well as negative price impact when dealers are constrained)**.** *Consider an equilibrium in which a cross-market dealer intermediates trades* (q^A, q^B)*, where* $q^A \leq q^B$*. The equilibrium has the following properties regarding the liquidity of the two assets: (for brevity, only bids are presented below.)*

1. *For* $q^A < (1-\delta)q^B$,

outcomes $\setminus$ w	$w \in [0, w_1)$	$w \in [w_1, w_2)$	$w \geq w_2$
Correlation $\left(\frac{\partial b^A}{\partial w} \times \frac{\partial b^B}{\partial w}\right)$	+	0	0
Spillovers from B to A $\left(\frac{\partial b^A}{\partial q^B}\right)$	−	0	0
Spillovers from A to B $\left(\frac{\partial b^B}{\partial q^A}\right)$	+	+	0

2. *For* $q^A \geq (1-\delta)q^B$,

outcomes $\setminus$ w	$w \in [0, w_1')$	$w \in [w_1', w_2')$	$w \geq w_2'$
Correlation $\left(\frac{\partial b^A}{\partial w} \times \frac{\partial b^B}{\partial w}\right)$	+	0	0
Spillovers from B to A $\left(\frac{\partial b^A}{\partial q^B}\right)$	+	0	0
Spillovers from A to B $\left(\frac{\partial b^B}{\partial q^A}\right)$	+	−	0

The equilibrium also features negative price impact, $\frac{\partial b^A}{\partial q^A} > 0$ *and* $\frac{\partial a^A}{\partial q^A} < 0$ *for* $q^A < (1-\delta)q^B$ *and* $w < \min\left\{\frac{c(q^A)^2}{\delta(2q^A+q^B)}, w_1\right\}$.

Proof. See Appendix C □

5 Extensions and empirical predictions

In this section, we discuss how the predictions of our model with a single market relate to the debate in the empirical literature about the link between dealer/market maker funding and market liquidity.

We first re-iterate the interpretation that dealer losses imply lower market liquidity. We then argue that this relationship between agency frictions and market liquidity should be stronger for riskier assets, on the basis of an extended version of our model. We then turn to the effect of post-crisis regulation on market liquidity. We show that in the context of our model, tightened leverage ratio and capital requirements should reduce the ability of bank-affiliated dealers to provide immediacy, and hence should have negative impact on market liquidity. Finally, we sketch a version of the model in which such dealers may post tight bid-ask spreads while at the same time providing less immediacy due to post-crisis regulations.

Dealers losses and liquidity The analysis at the end of Section 3 indicated that dealers' internal funds are positively related to measures of market liquidity such as market depth, expected volume, and bid-ask spreads. In this sense, the model suggests that when dealers lose money and therefore have less internal funds, market liquidity suffers: bid-ask spreads for larger trades widen, market depth decreases, and volume decreases (although bid-ask spreads for smaller trades might be unaffected). This prediction is consistent with e.g. the empirical results of Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010), who find that when NYSE specialists have incurred loss in a previous day, aggregate market-level and specialist firm-level bid-ask spreads widen. The effects of losses on bid-ask spreads is smaller when specialist firms merge, consistent with larger internal capital easing financing constraints.[10]

There are also many papers that show that during the global financial crisis, there was a sharp drop in U.S. corporate bond market liquidity (See e.g. Trebbi and Xiao, forthcoming; Anderson and Stulz, 2017; Bessembinder, Jacobsen, Maxwell, and Venkataraman, 2018). A consistent explanation from our model is that dealer losses during this time reduced their ability to intermediate, which had a negative effect on market liquidity, as in our model.

Fundamental risk and liquidity Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010) also show that the effect of specialist losses on liquidity is more pronounced for high volatility stocks. In Appendix A, we consider an extension of the model in Section 3, in which the fundamental value V can change randomly from $t = 1$ until $t = 2$, while the dealer has an open position. With this risk of a change in the fundamental value, a dealer could lose out e.g. if he buys the asset from Earl, and the fundamental value of the drops from $t = 1$ until $t = 2$ while the dealer is looking for Lacey. We show that for riskier assets, agency frictions are more severe, resulting in lower bids, higher asks, less market depth and lower trading volume.

This result can also be seen as a micro-foundation of risk-based margins in a collateralized debt contract, which is exogenously assumed in Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). To see this, consider the case when the dealer buys from Earl. As we show next, the optimal contract can be implemented by a debt contract with the purchased asset as collateral. Then the margin is simply

$$\frac{\text{Market value of the asset}}{\text{Loan amount}} - 1 = \frac{q b_c(q, w)}{\mathcal{P}(q)} - 1 = \frac{w}{\mathcal{P}(q)}$$

Note that the market value of the asset is the equilibrium bid price. As pledgeable income for riskier assets is lower, margins are higher.

[10]See Macchiavelli and Zhou (2019) for evidence that dealer funding liquidity affects the market liquidity of corporate bonds.

The effect of tightened regulation There is a debate around whether the tightened regulation in the aftermath of the global financial crisis has worsened bond market liquidity relative to pre-crisis levels. Trebbi and Xiao (forthcoming) argue that there is no deterioration in a variety of liquidity measures. Anderson and Stulz (2017) present a mixed view on price-based liquidity metrics, but show that there has been a drop in post-crisis turnover. Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) point out that turnover, block trade frequency, and average trade size decreased post-crisis. They also point out that the decline in liquidity provision in the U.S. bond market is coming from a reduction in capital committed to intermediation by bank-affiliated dealers, whereas non-bank affiliated dealers have increased their capital commitment.

There are many regulatory changes that could have had an effect on the ability of bank-affiliated dealers in particular to intermediate. Potential candidates include the Volcker rule, net stable funding ratios, liquidity coverage ratios, supplementary leverage ratios, and tightened capital requirements.

Our model highlights one problem in particular with post-crisis regulation. In the model, dealers chose debt as a form of external finance because this form of financing leaves more of an upside to dealers, and hence is good for incentives to exert search effort. When regulators force (bank-affiliated) dealers to use less external debt, e.g. via a supplemental leverage ratio or tightened capital requirements, this is bad for incentives and the ability of the dealers to raise money. (In the context of our model, these two types of regulation are equivalent.) Hence market liquidity decreases. There is some evidence that e.g. tightened capital requirements have had this effect (Haselmann, Kick, Singla, and Vig, 2019).

In Appendix B, we sketch what would happen in our model if a regulator where to put a limit on the amount of debt a dealer could use. Suppose, for instance, that a bank-affiliated dealer is subject to maximum leverage requirement, that says that the ratio of debt to total assets Λ cannot exceed some maximum $\Lambda_{\max}$. (For simplicity, we only consider only the case in which Earl sells to the dealer.)

To pin down the leverage ratio, we need to introduce debt and equity into the model. First note that in our model, any repayment to financiers $\{R_H, R_L\}$ can be implemented by a safe, standard debt contract with promised repayment D in both states $\{H, L\}$, and a standard equity contract that pays a fraction α of the remaining cash flows in the two states, $\{\alpha(x^H - D), \alpha(x^L - D)\}$. That is, $R_H = D + \alpha(x^H - D)$, $R_L = D + \alpha(x^L - D)$. We illustrate this in Figure 4.

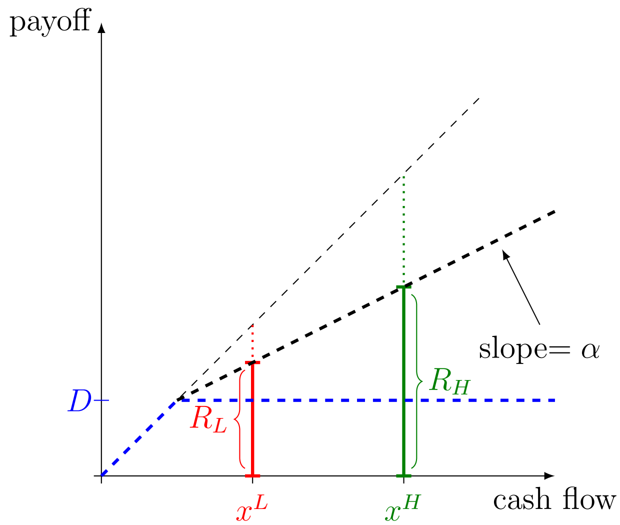

Figure 4. Repayments to financiers in terms of debt and equity
Any repayment to financiers $\{R_H, R_L\}$ can be implemented by a safe, standard debt contract that promises a payment D in both states $\{H, L\}$, and a standard equity contract that pays a fraction α of the remaining cash flows in the two states, $\{\alpha(x^H - D), \alpha(x^L - D)\}$. Therefore, $R_H = D + \alpha(x^H - D)$, $R_L = D + \alpha(x^L - D)$.

To raise as much funding as possible while satisfying the leverage constraint, the dealer would choose to implement the optimal contract with as much equity as possible, and as little debt as possible. We describe the resulting leverage ratio in Lemma B.2. When intermediating trades of small size q, a dealer can rely entirely on external equity, without causing incentive problems for the dealer, so that the leverage ratio can be zero. When the dealer wants to intermediate trades with larger and larger size q, he will have to use increasing amounts of debt (and therefore, leverage) so as to preserve incentives (see also Innes, 1990).

The leverage ratio constraint will therefore start to bind for trades beyond some critical size $q_\Lambda(w)$. Once the constraint binds for such large trades, the dealer will have to use more external equity than under the optimal contract, which reduces the pledgeable income (Proposition B.6). Essentially, the constraint hinders the ability of the dealer to finance large trades.

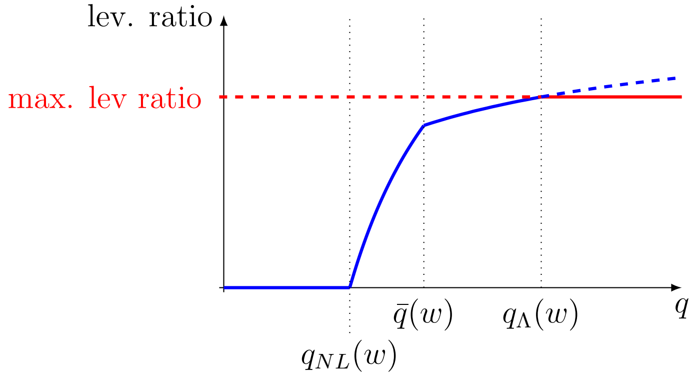

Figure 5. Leverage ratio constraint and trade size q
This figure describes Lemma B.2 in Appendix B. A dealer can finance the very smallest trades ($q \leq q_{NL}(w)$) with equity only, so that the resulting leverage ratio is zero. For slightly larger trades ($q_{NL}(w) < q \leq \bar{q}(w)$), the dealer will have to use some debt in the external financing mix, to improve incentives, so the leverage ratio becomes positive. For even larger trades $q \leq \bar{q}_\Lambda(w)$, the bids and asks, and hence the overall financing need, is affected by the incentive problem. For trades with a size that exceeds a critical level $q_\Lambda(w)$, the leverage ratio constraint will bind, which will increase bid-ask spreads. Market depth is reduced.

Given the results in Section 3, it is relatively straightforward to appreciate that such a reduced ability to finance large trades will lead to lower market liquidity for such trades, in the form of wider bid-ask spreads, and, potentially, lower market depth and expected volume (see Corollary 3).

Bank v.s. non-bank dealers We return to the empirical evidence of Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018). They find that the decline in liquidity provision in the U.S.

bond market is coming from reduction in capital committed to intermediation by bank-affiliated dealers, whereas non-bank-affiliated dealers have increased their capital commitment. Such an evolution of bond market liquidity provision can be rationalized in the context of our model, if bank-affiliated dealers are now subject to e.g. the maximum leverage ratio, but non-bank-affiliated dealers are not.

Suppose Dealer 1 is a bank-affiliated dealer and Dealer 2 is a non-bank dealer. Further assume that Dealer 1 is more efficient than Dealer 2 in liquidity provision, e.g., $c_1 < c_2$. Without any additional regulatory constraint, Dealer 1 intermediates the asset in equilibrium. Suppose the equilibrium changes, as Dealer 1 is now subject to a maximum leverage ratio requirement. This can reduce the pledgeable income of Dealer 1 to the extent that Dealer 2, the non-bank dealer can now out-compete Dealer 1, the bank-affiliated dealer.

Customer liquidity provision Choi and Huh (2017) document a substantial rise in "customer liquidity provision" in corporate bond markets post-crisis. By customer liquidity provision, they mean that dealers become more unwilling to use their own balance sheet to provide immediacy, and instead broker more trades between clients. In that sense, it is clients who provide liquidity to each other.[11] They also find the bias for brokered trades stronger for larger orders and riskier bonds. They conclude that customers demanding liquidity from dealers rather than other clients need to pay 35 to 50 percent higher spreads post crisis, suggesting that dealers' ability to provide liquidity is significantly reduced. Dick-Nielsen and Rossi (2018) also find evidence that the cost of immediacy in corporate bond market has doubled after the 2008 financial crisis.

Our model can be easily modified to incorporate customer liquidity provision and rationalize these empirical findings. Modify the model by assuming that with some probability $\pi \in (0, 1)$, Lacey arrives at the market early, and hence the dealer can intermediate between Earl and Lacey, but

[11]See also the papers of Goldstein and Hotchkiss (2018) and Schultz (2017).

without having to hold the asset over time. This would be "customer liquidity provision," or "pre-arranged trades," or brokerage. If a dealer is unconstrained, the ratio of customer liquidity provision to dealer liquidity provision is $\frac{\pi}{1-\pi}$. We can interpret the post-crisis regulations as tightening financing constraints of the dealers, so that some dealer-intermediated trades do not take place anymore. Thus the ratio will increase, and more so for larger trades and riskier bonds because they require more dealers' internal capital.

6 Concluding remarks

We present a model in which dealers need external finance to conduct their market-making activities, specifically to provide immediacy for their clients. Once they take over a position from a client, dealers need to exert unobservable effort in searching for a counterparty, to increase the chance of closing the position at a good intermediation profit. This moral hazard problem affects how and how much external finance dealers can raise. It limits intermediation volume, softens competition between dealers, and widens bid-ask spreads. When dealers suffer losses, the problem becomes worse, especially for riskier assets and larger orders.

We show that dealers can alleviate the agency frictions by intermediating across several markets. This provides a new micro-foundation for why liquidity should co-move across markets, with potentially non-monotonic spillovers, when dealers are financially constrained.

The model can also provide a rationale for many of the observed changes in market liquidity during and after the financial crisis. External financing for dealers optimally takes the form of debt, to leave sufficient upside so that they have incentives to exert effort. Regulations that limit the use of debt, such as maximum leverage ratios or minimum capital requirements for bank affiliated dealers, will affect the ability of dealers to intermediate larger, riskier trades, and may prompt them to switch from providing immediacy with their balance sheet to brokering trades between clients.

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A Risky assets

In this appendix, we extend the model in Section 3 to allow for random changes in fundamental value V during intermediation. Our main result is that the agency friction makes riskier assets less liquid.

We assume that after the effort decision at $t = 1$, the fundamental asset value will change. The fundamental value can either go up to $V + z$ or down to $V - z$, with equal probability. Correspondingly, Lacey will now have a valuation of either $V + z$ or $V - z$, financiers will have a valuation of either $V + z - k$ or $V - z - k$, and security lenders will have a valuation of either $V + z + k$ or $V - z + k$.[12] We refer to z as the *risk* of an asset.

There are now four possible net cash flows to the dealer from intermediation, which depend on whether the fundamental value went up or down, and whether the dealer found Lacey or did not find Lacey. We assume that contracts can be contingent on these four states. We use the subscripts U and D to refer to whether the fundamental value went up or down, and the subscripts H and L to refer to whether Lacey was found, or not found, respectively, as before.

If the dealer buys from Earl, he can subsequently sell either at price $V + z$ or $V - z$ if he finds Lacey, and will have to make repayments R_b^{HU} or R_b^{HD}, or will be forced to sell at price $V + z - k$ or $V - z - k$ to financiers and make repayments R_b^{LU} or R_b^{LD} if he does not.

If the dealer sells to Earl, he provides cash collateral $V + z + k$ to borrow the asset first, then sells, and can buy back later either at price $V + z$ or $V - z$ and make repayments R_a^{HU} or R_a^{HD} if he finds Lacey, or will be forced to buy back at price $V + z + k$ or $V - z + k$ from security lenders and make repayments R_a^{LU} or R_a^{LD} if he does not.

The possible cash flow orderings are illustrated in Figure 6.

For brevity, we again let $\{R_j\}$ denote the set of repayments R_j^{HU}, R_j^{HD}, R_j^{LU}, R_j^{LD} for $j \in \{b, a\}$. We can then write the utility of the buying and selling dealer, respectively, as follows:

$$U_j(e, \{R_j\}) = (1 - (1 - e)\delta) \left[\frac{1}{2} \left(x_j^{HU} - R_j^{HU} \right) + \frac{1}{2} \left(x_j^{HD} - R_j^{HD} \right) \right] + \delta(1 - e) \left[\frac{1}{2} \left(x_j^{LU} - R_j^{LU} \right) + \frac{1}{2} \left(x_j^{LD} - R_j^{LD} \right) \right] - cqe \text{ for } j \in \{b, a\}. \quad (30)$$

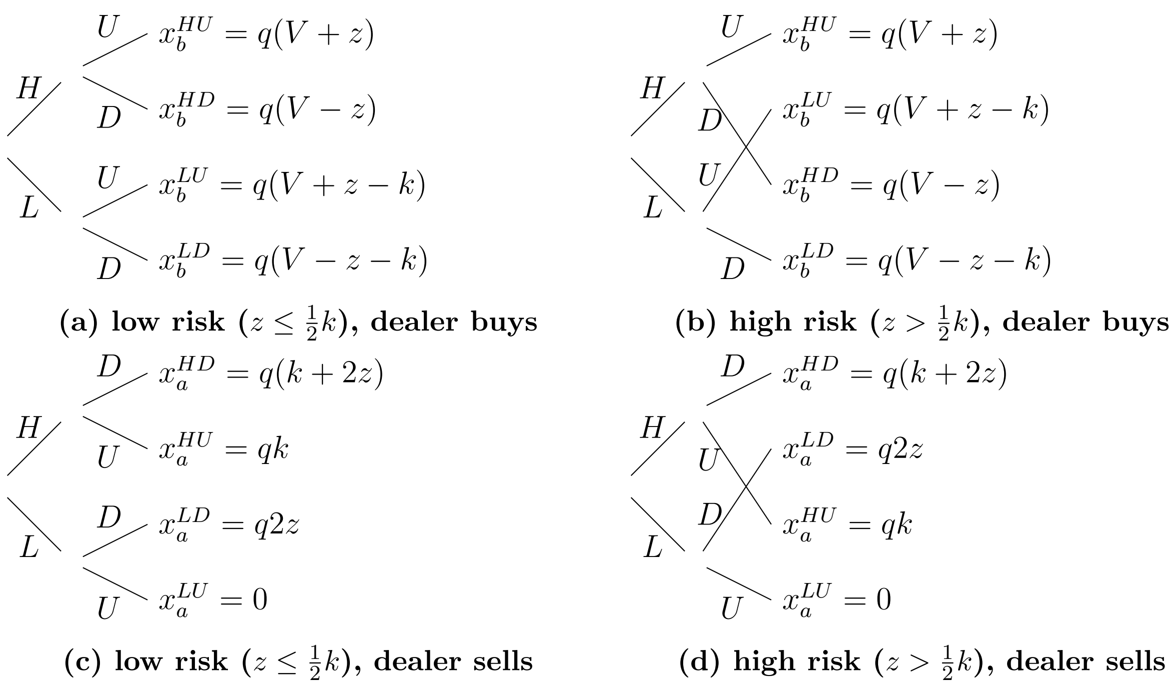
As before, the dealer's optimal contracting problem can be stated as

$$\max_{\{R_j\}} U_j(1, \{R_j\}), \text{ for } j \in \{b, a\} \quad \text{(P')}$$

subject to the incentive compatibility constraint

$$U_j(e = 1, \{R_j\}) \geq U_j(e = 0, \{R_j\}), \text{ for } j \in \{b, a\}. \quad (31)$$

[12] We have also considered an alternative description in which the dealer is only exposed to the risk of a change in fundamental value if Lacey is not found. This would be consistent with the interpretation that search effort increases not just the chance of finding good counterparties, but also the speed with which they are found. Results in this version of the model are similar.



(a) low risk ($z \leq \frac{1}{2}k$), dealer buys **(b) high risk ($z > \frac{1}{2}k$), dealer buys**

(c) low risk ($z \leq \frac{1}{2}k$), dealer sells **(d) high risk ($z > \frac{1}{2}k$), dealer sells**

Figure 6. Cash flows when fundamental value can changes
We illustrate the cash flows to a dealer as a function of whether the fundamental value of the asset moves up (U) or down (D), and whether the dealer finds Lacey (H) or not (L). There are four cases to consider: The dealer may buy from Earl, and benefit from an up-move in fundamental value, or the dealer may sell to Earl, and benefit from a down-move in fundamental value. Also, the effect of asset risk may dominate the effect of a successful search ($z > \frac{1}{2}k$), or it may not ($z \leq \frac{1}{2}k$).

After some manipulation, this can be written as

$$\Delta R_j \leq q\left(k - \frac{c}{\delta}\right), \tag{IC'}$$

where ΔR_j is defined as the expected difference in repayments in case Lacey is found versus the case in which Lacey is not found:

$$\Delta R_j := \frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right) - \frac{1}{2}\left(R_j^{LU} + R_j^{LD}\right). \tag{32}$$

In addition, there are limited liability constraints and the break-even constraint for financiers. We have:

$$R_j^i \leq x_j^i, \text{ for } j \in \{b, a\}, \text{ and } i \in \{HU, HD, LU, LD\}, \tag{LL'}$$

$$\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right) = \begin{cases} qb(q) - w & \text{if } j = b, \\ q(V + k + z) - qa(q) - w & \text{if } j = a. \end{cases} \tag{BE'}$$

To obtain more plausible, monotone contracts, we now also impose two additional monotonicity constraints, which restrict the space of possible contracts that we consider:

Assumption A.5 (Monotonicity)**.** *Net payoffs to the dealer must be non-decreasing in the underlying gross cash flows, and repayments to investors must be non-decreasing in the underlying gross cash flows:*

$$x_j^i - R_j^i \text{ is non-decreasing in } x_j^i, \tag{MCD}$$

$$R_j^i \text{ is non-decreasing in } x_j^i, \tag{MCF}$$

for $j \in \{b, a\}$*, and state* $i \in \{HU, HD, LU, LD\}$*.*

The first part of Assumption A.5 could be motivated as follows: If the dealer can secretly burn cash, the dealer would have an incentive to do so with a payoff function that is decreasing in cash flow. The contract should not give the incentive to do this. The second part of Assumption A.5 is as in Innes (1990) and can be motivated as follows: If the dealer can secretly inject cash, the dealer would have an incentive to do so when repayments to investors are decreasing in cash flow. The contract should not give the incentive to do this. Because of the monotonicity constraints, the cash flow ranking matters for the characterization of the optimal monotone contract.

The impact of Assumption A.5 depends on the ranking of the gross cash flows, which in turn depend on the relative size of z and k. We can distinguish two cases:

1. The asset has relatively low risk, when $z \leq \frac{1}{2}k$,
2. the asset has relatively high risk, when $z > \frac{1}{2}k$.

In the "high-risk case," what matters more for cash flows is whether the fundamental value moves up or down, and what matters less is whether Lacey is found. Conversely, in the "low-risk case," what matters more is whether the dealer does or does not find Lacey, and what matters less is whether the fundamental value moves up or down.

We make an additional assumption on parameters:

Assumption A.6. $\delta k < 2c$

We can re-write this assumption as

$$k - \frac{c}{\delta} < \frac{1}{2}k.$$

In this expression, the left-hand side relates to the NPV of effort, and the right-hand side relates to the critical cut-off for z, such that assets with a risk z that exceeds this number are "high-risk" in the above sense. The assumption ensures that the NPV of effort is relatively low, so that even for "low-risk" assets, the risk may be still be large in relation to the NPV generated by the provision of effort. This ensures that prices will be affected by relatively low values of the asset risk z.

The optimal contract solves (P') subject to the constraints (IC'), (LL'), (BE'), (MCD), and (MCF). As before, there are potentially many optimal contracts. As before, rather than discussing the set of optimal contracts directly, we focus on pledgeable income $\mathcal{P}(q)$, defined as the maximum amount of cash that can be raised from financiers. Using the break-even constraint (BE'), we can define pledgeable income as the solution to the following problem:

$$\mathcal{P}_j(q) = \max_{\{R_j\}} \frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right), \tag{33}$$

subject to the constraints (IC'), (LL'), (MCD), and (MCF)

Lemma A.1. *For any contract that solves* (33)*, the incentive compatibility* (IC') *constraint must bind.*

Proof. See the Appendix C. □

We can use this lemma to prove the following proposition, which states that the bid-ask spread is (weakly) increasing in asset risk.

Proposition A.5. *The incentive compatible ask and bid are given by*

$$\begin{aligned} a_{IC}(q) &= -\frac{w}{q} + V + \frac{c}{\delta} + \left(z - \left(k - \frac{c}{\delta}\right)\right)\mathbf{1}_{\{z \geq k - \frac{c}{\delta}\}} - \left(z - \frac{1}{2}k\right)\mathbf{1}_{\{z \geq \frac{1}{2}k\}} \\ b_{IC}(q) &= \frac{w}{q} + V - \frac{c}{\delta} - \left(z - \left(k - \frac{c}{\delta}\right)\right)\mathbf{1}_{\{z \geq k - \frac{c}{\delta}\}} + \left(z - \frac{1}{2}k\right)\mathbf{1}_{\{z \geq \frac{1}{2}k\}} \end{aligned}$$

Thus the equilibrium bid-ask spread in (weakly) increasing in asset risk z.

Proof. See the Appendix C. □

Figure 7 graphs the bid-ask spread as a function of "asset risk" z, as described by Proposition A.5.

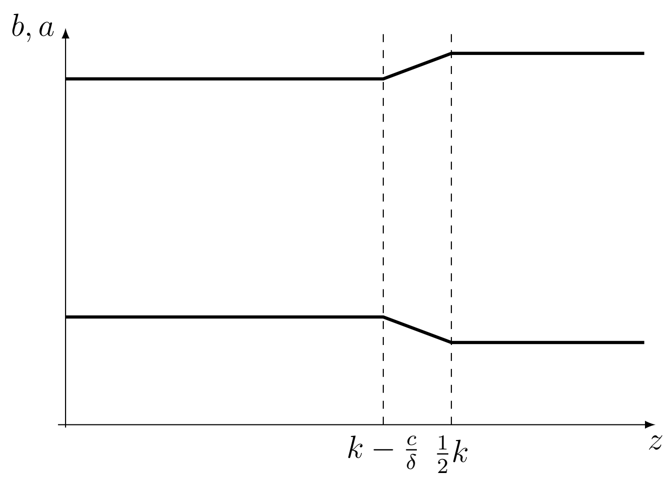


Figure 7. Bid-ask spread as a function of asset risk z

The bid-ask spread is weakly increasing in "asset risk" z, as described by the expression in Proposition A.5.

B Maximum leverage ratio and market liquidity

In the context of the model of Section 3, we consider what would happen if a regulator were to impose a maximum leverage ratio constraint. For simplicity, we only consider the case in which Earl sells to the dealer. In this case, the total assets of the dealer correspond to the asset bought from Earl. We assume that total assets are valued at the purchase price, which in this case is the bid price b, so that total assets would be worth qb.

We can interpret the repayments $\{R_H, R_L\}$ under the optimal contract as resulting from a debt and equity contract as described in Section 5 in the main text. The dealer will want to implement the optimal contract with the minimum possible amount of debt. Consider the resulting leverage ratio $\Lambda(q, w)$. We first show that this leverage ratio is increasing in q:

Lemma B.2. *Consider a competitive dealer. The dealer's leverage ratio $\Lambda(q, w)$ resulting from the optimal contract with minimum leverage is (weakly) increasing in q, and given by*

$$\Lambda(q,w) = \begin{cases} 0 & \text{if } q \leq q_{NL}(w) \\ \frac{V - \frac{\delta}{k} - \frac{w}{q}\frac{\delta k}{c}}{V-c} & \text{if } q_{NL}(w) < q \leq \bar{q}(w) \\ \frac{V-k}{\frac{w}{q} + V - \frac{c}{\delta}} & \text{if } \bar{q}(w) < q < q_{\max}(w) \end{cases} \tag{34}$$

where $q_{NL}(w) = \frac{w}{\frac{c}{\delta k}V - c}$, and $\bar{q}(w)$ and $q_{\max}(w)$ are as defined in (12) *and* (17).

Proof. See Appendix C. $\square$

Since the minimum leverage ratio associated with the optimal contract is increasing in q, we can see that for orders beyond some critical size $q_\Lambda(w)$, the leverage ratio constraint $\Lambda(q, w) \leq \Lambda_{\max}$ starts to bind.[13] Since orders with size $q > q_\Lambda(w)$ cannot be financed with the optimal contract, the competitive dealer has to offer a contract $\{D, \alpha\}$ (with $D \leq q(V - k)$) and solves the following problem

$$\max_{\{D,\alpha\}} \mathcal{P} = D + \alpha(qV - D) \tag{35}$$

$$\text{subject to} \quad \Delta_R \equiv \underbrace{D + \alpha(qV - D)}_{R_H} - [\underbrace{D + \alpha(q(V - k) - D)}_{R_L}] \leq q(k - \frac{c}{\delta}) \tag{36}$$

$$\Lambda \equiv \frac{D}{D + \alpha(qV - D) + w} \leq \Lambda_{\max} \tag{37}$$

where the incentive compatibility constraint (36) can further be simplified to $\alpha \leq 1 - \frac{c}{\delta k}$. The characterization of the optimal debt and equity contract and thus the pledgeable income under leverage ratio requirement is straightforward:

[13] This critical size is defined via $\Lambda\left(q_\Lambda(w), w\right) = \Lambda_{\max}$.

Proposition B.6. *Suppose the regulator imposes a maximum leverage ratio requirement $\Lambda_{\max}$. For trade size $q > q_\Lambda(w)$, where $q_\Lambda(w)$ is defined via $\Lambda\left(q_\Lambda(w), w\right) = \Lambda_{\max}$, the dealer optimally issues share α^* of outside equity and issues debt with promised repayment $D^*(\Lambda_{\max})$*

$$\alpha^* = \left(1 - \frac{c}{\delta k}\right); \quad D^*(\Lambda_{\max}) = \min\left\{\frac{w + (1 - \frac{c}{\delta k})qV}{\frac{1}{\Lambda_{\max}} - \frac{c}{\delta k}}, q(V - k)\right\}. \tag{38}$$

Therefore the pledgeable income under leverage requirement is

$$\mathcal{P}(q; \Lambda_{\max}) = \left(1 - \frac{c}{\delta k}\right) qV + \frac{c}{\delta k} D^*(\Lambda_{\max}). \tag{39}$$

Proof. See Appendix C □

Here, since we focus only on the bid, we can think of a reduction of the bid as being synonymous with a reduction of market liquidity. In that sense, the following comparative statics results shows that tightening the leverage requirement harms market liquidity of the asset and thus welfare:

Corollary 3. *For $q > q_\Lambda(w)$, tightening leverage requirement (lowering $\Lambda_{\max}$)*

1. *reduces liquidity (= the bid);*
2. *reduces liquidity (= the bid) especially for trades of larger size;*
3. *reduces market depth (the maximum possible size of an intermediated order), and hence welfare.*

Proof. See Appendix C. □

C Proofs

Proof of Proposition 1. See main text. $\square$

Proof of Proposition 2. Comparing to the single asset case, the two-market case entails a richer outcome space. There are four different possible outcomes, hence four contractible cash flows, and hence four possible repayments to investors. To preserve space, we use the notation $q = q^A + q^B$ in the proof.

First, consider the case in which Earl sells, and the dealer buys.

scenarios	contractible cash flow	repayment to investors
Counterparty found for both assets	$q^A V + q^B V = qV$	R_1
Counterparty found for asset B only	$q^A(V-k) + q^B V = qV - q^A k$	R_2
Counterparty found for asset A only	$q^A V + q^B(V-k) = qV - q^B k$	R_3
No counterparty found	$q^A(V-k) + q^B(V-k) = q(V-k)$	R_4

Similarly, the possible effort decisions are richer in the two-market case. The dealer can choose to exert effort to search for a counterparty, in the first market, in the second market, in both markets, or in neither market. To ensure that the dealer exerts effort to search for both assets, which is by assumption the efficient action, three incentive-compatibility constraints have to be satisfied:

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) + \delta(qV - q^B k - R_3) - q^A c \tag{IC1}$$

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) + \delta(qV - q^A k - R_2) - q^B c \tag{IC2}$$

$$\begin{aligned} q(V-c) - R_1 \geq\ & (1-\delta)^2(qV - R_1) + \delta(1-\delta)(qV - q^A k - R_2) + \\ & \delta(1-\delta)(qV - q^B k - R_3) + \delta^2(qV - qk - R_4) \end{aligned} \tag{IC3}$$

The three incentive constraints ensure that exerting search effort in both markets is better than, respectively, i) only exerting search effort in the market A; ii) only exerting search effort in market B; and iii) not exerting any search effort. Therefore, the financing contract that maximizes the pledgeable income is

$$\max_{R_1,R_2,R_3,R_4} \mathcal{P}(q^A, q^B) = R_1$$
$$\text{subject to} \quad (IC1), (IC2), (IC3) \text{ and } (LL)$$

It is immediate to see that it is optimal to set R_2, R_3 and R_4 as high as possible. Intuitively, in order to incentivize the dealer to exert effort to search in both asset, he should only be rewarded if he finds both counterparties. The incentive constraints then simplify to

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) - q^A c \Leftrightarrow R_1 \leq qV - \frac{q - q^A}{\delta} c \tag{IC1}$$

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) - q^B c \Leftrightarrow R_1 \leq qV - \frac{q - q^B}{\delta} c \tag{IC2}$$

$$q(V-c) - R_1 \geq (1-\delta)^2(qV - R_1) \Leftrightarrow R_1 \leq q\left(V - \frac{c}{1-(1-\delta)^2}\right) \tag{IC3}$$

Therefore, pledgeable income is

$$\mathcal{P}_b(q^A, q^B) = \min\left\{qV - \frac{q - q^A}{\delta}c, \quad qV - \frac{q - q^B}{\delta}c, \quad q\left(V - \frac{c}{1-(1-\delta)^2}\right)\right\} \tag{40}$$

as shown in (23) with $q = q^A + q^B$. Finally, it is immediate that $\mathcal{P}(q^A, q^B) > \mathcal{P}(q^A) + \mathcal{P}(q^B)$.

We omit the proof for $\mathcal{P}_a(q^A, q^B)$, which follows a similar structure. $\square$

Proof of Proposition 3. For orders $\{q^A, q^B\}$ where $q^A \leq q^B$, plug the expression of pledgeable income in (23) into (25) for the relevant range. After some re-arranging, we have

$$\begin{cases} q^B \leq \dfrac{w + q^A \ell}{\frac{c}{\delta} - \ell} & \text{if} \quad q^A < (1-\delta)q^B \\ q^B \leq \dfrac{w}{\frac{c}{\delta(2-\delta)} - \ell} - q^A & \text{if} \quad q^A \in [(1-\delta)q^B, q^B] \end{cases} \tag{41}$$

In the first region $q^A < (1-\delta)q^B$, as q^A increases from 0 and approaches $(1-\delta)q^B$, the upper bound of q^B approaches $\frac{w}{\frac{c}{\delta} - (2-\delta)\ell}$ and thus the maximum value of q^A approaches $\frac{(1-\delta)w}{\frac{c}{\delta} - (2-\delta)\ell}$. As q^A starts at $\frac{(1-\delta)w}{\frac{c}{\delta} - (2-\delta)\ell}$, it reaches the second region and the upper bound of $q^B = \frac{w}{\frac{c}{\delta} - (2-\delta)\ell}$. As q^A approaches q^B, the upper bound of q^B approaches $\dfrac{\frac{1}{2}w}{\frac{c}{\delta(2-\delta)} - \ell}$. Hence, it is also the maximum value of q^A in this range. That completes the characterization of $q^B_{max}(q^A, w)$ in (27). $\square$

Proof of Proposition 4. We refer to the difference between the pledgeable income of a cross-market dealer and the sum of pledgeable incomes of specializing dealer as the "excess balance-sheet capacity" of a cross-market dealer.

In equilibrium, bids are such that (cross-market) pledgeable income is exhausted by the winning dealer. If this were not the case, another dealer could outbid the winning dealer. This implies that bids must be at least as high as those made by specializing dealers exhausting their pledgeable income.

There are multiple equilibria, indexed by how the winning dealer splits excess balance-sheet capacity across markets. For any split of excess balance-sheet capacity, a deviating cross-market dealer cannot simultaneously outbid the winning dealer in both markets, since cross-market pledgeable income is exhausted. A deviating specializing dealer cannot outbid the winning dealer since the bids of the winning dealer are at least as high as those made by specializing dealers exhausting their pledgeable income.

To facilitate presentation, we focus on a unique equilibrium with the following properties: The excess balance-sheet capacity is split across markets proportionally according to trade sizes, except when one of the prices is already at the competitive limit of $V - c$. In this case, the excess balance-sheet capacity is allocated to increase the bid of the asset which is not already at the competitive limit.

We consider $q^A < q^B$, and proceed with the two cases $q^A < (1-\delta)q^B$ and $q^A \geq (1-\delta)q^B$ in sequence, to show that the competitive equilibrium bids and asks are as follows:

1. For $q^A < (1-\delta)q^B$,

outcomes \ w	$w \in [0, w_1)$	$w \in [w_1, w_2)$	$w \geq w_2$
b^A	$V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^B}{q^A}\frac{1}{q^A+q^B}$	$V-c$	$V-c$
b^B	$V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^A}{q^B}\frac{1}{q^A+q^B}$	$V - \frac{c}{\delta} + \frac{w}{q^B} + \frac{q^A}{q^B}c$	$V-c$
a^A	$V + \frac{c}{\delta}\frac{q^B}{q^A+q^B} - w\frac{q^B}{q^A}\frac{1}{q^A+q^B}$	$V+c$	$V+c$
a^B	$V + \frac{c}{\delta}\frac{q^B}{q^A+q^B} - w\frac{q^A}{q^B}\frac{1}{q^A+q^B}$	$V + \frac{c}{\delta} - \frac{w}{q^B} - \frac{q^A}{q^B}c$	$V+c$

2. For $q^A \geq (1-\delta)q^B$,

outcomes \ w	$w \in [0, w_1')$	$w \in [w_1', w_2')$	$w \geq w_2'$
b^A	$V - \frac{c}{\delta(2-\delta)} + w\frac{q^B}{q^A(q^A+q^B)}$	$V-c$	$V-c$
b^B	$V - \frac{c}{\delta(2-\delta)} + w\frac{q^A}{q^B(q^A+q^B)}$	$V - \frac{c}{\delta(2-\delta)} + \frac{w}{q^B} - c\frac{q^A(1-\delta)^2}{q^B\delta(2-\delta)}$	$V-c$
a^A	$V + \frac{c}{\delta(2-\delta)} - w\frac{q^B}{q^A(q^A+q^B)}$	$V+c$	$V+c$
a^B	$V + \frac{c}{\delta(2-\delta)} - w\frac{q^A}{q^B(q^A+q^B)}$	$V + \frac{c}{\delta(2-\delta)} - \frac{w}{q^B} + c\frac{q^A(1-\delta)^2}{q^B\delta(2-\delta)}$	$V+c$

for some thresholds w_1, w_2, w_1', w_2'. There always exists an equilibrium that features a cross-market dealer. For $w \geq c\frac{1-\delta}{\delta}q^B$, where $c\frac{1-\delta}{\delta}q^B > \max\{w_2, w_2'\}$, there exists another equilibrium with two specialized dealers that each set bids equal to $(V-c)$ and asks equal to $(V+c)$.

For $q^A < (1-\delta)q^B$: the pledgeable income is $\mathcal{P}(q^A, q^B) = q^A V + q^B\left(V - \frac{c}{\delta}\right)$. The condition for a cross-market dealer to outbid two specializing dealers is

$$w + q^A V + q^B\left(V - \frac{c}{\delta}\right) > q^A b^A(w, q^A) + q^B b^B(w, q^B) \tag{42}$$

where $b^i = \min\{\frac{w}{q^i} + (V - \frac{c}{\delta}), V - c\}$. The equilibrium bids hence depend on the level of dealer capital w.

1. Suppose w is such that specializing dealers who exhaust pledgeable income would bid less than $V-c$ for both assets. This requires that $w \in [0, c\frac{1-\delta}{\delta}q^A)$. Condition (42) would become $w < q^A\frac{c}{\delta}$, which is true in the proposed range of w. The excess balance-sheet capacity is $q^A\frac{c}{\delta} - w$. In the equilibrium in which the winning (cross-market) dealer shares the excess balance-sheet capacity in proportion to trade sizes across markets, the bids are

$$b^i = \frac{1}{q^i} \times \left(w + q^i\left(V - \frac{c}{\delta}\right) + \frac{q^i}{q^A+q^B}\left(q^A\frac{c}{\delta} - w\right)\right) \qquad \text{for } i = \{A, B\}$$

The equilibrium bids are therefore

$$b^A = V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^B}{q^A}\frac{1}{q^A+q^B}$$
$$b^B = V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^A}{q^B}\frac{1}{q^A+q^B},$$

but we do require that $b^A < V-c$, so that it must be the case that $w < w_1 \equiv c\frac{1-\delta}{\delta}q^A - c\frac{(q^A)^2}{q^B}$. For $w \in [w_1, c\frac{1-\delta}{\delta}q^A]$, b^A is already at the zero-profit bid $V-c$. The excess balance-sheet capacity becomes $q^A c$, and all of it is allocated to market B. Hence in this case, $b^B = \frac{1}{q^B} \times \left(w + \mathcal{P}(q^A, q^B) - q^A(V-c)\right) = \frac{w}{q^B} + V - \frac{c}{\delta} + \frac{q^A}{q^B}c$.

2. Suppose w is such that specializing dealers who exhausts pledgeable income would bid $V-c$ for asset A, but less than $V-c$ for asset B. This requires that $w \in \left[c\frac{1-\delta}{\delta}q^A, c\frac{1-\delta}{\delta}q^B\right)$, so the condition (42) becomes

$$w + q^A V + q^B\left(V - \frac{c}{\delta}\right) > q^A(V-c) + w + q^B\left(V - \frac{c}{\delta}\right)$$

which is true. The excess balance-sheet capacity is $q^A c$. Hence, the winning (cross-market) dealer will bid $b^A = (V-c)$ for asset A and use the remaining balance-sheet capacity for asset B, such that $b^B = \min\{V-c, \frac{w}{q^B} + \frac{q^A}{q^B}c + V - \frac{c}{\delta}\}$. Finally $b^B = \frac{w}{q^B} + \frac{q^A}{q^B}c + V - \frac{c}{\delta}$ when $w < w_2 \equiv c\frac{1-\delta}{\delta}q^B - q^A c$ and $b^B = V - c$ otherwise.

3. Suppose that w is such that specializing dealers who exhaust pledgeable income would bid $V-c$ for both assets. This requires that $w \geq c\frac{1-\delta}{\delta}q^B$. Condition (42) becomes

$$w + q^A V + q^B(V - \frac{c}{\delta}) > (q^A + q^B)(V-c),$$

or $w > c\frac{1-\delta}{\delta}q^B - q^A c$, which is true in the proposed range of w. The equilibrium bids therefore are $b^A = b^B = V - c$. Note that in this range of w, the equilibrium we describe can also be supported by two specializing dealers.

For $q^A \geq (1-\delta)q^B$: the pledgeable income is $\mathcal{P}(q^A, q^B) = (q^A + q^B)\left(V - \frac{c}{\delta(2-\delta)}\right)$. The condition for a cross-market dealer to outbid two specializing dealers is

$$w + (q^A + q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) > q^A b^A(w, q^A) + q^B b^B(w, q^B) \tag{43}$$

1. Suppose w is such that specializing dealers who exhaust pledgeable income would bid less that $V-c$ for both assets. This requires that $w \in \left[0, c\frac{1-\delta}{\delta}q^A\right)$. Condition (43) becomes

$$w + (q^A + q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) > 2w + (q^A + q^B)(V - \frac{c}{\delta})$$

or $w < (q^A + q^B)\frac{c}{\delta}\frac{1-\delta}{2-\delta}$, which is true in the proposed range of w. The excess balance-sheet capacity is $\left((q^A + q^B)\frac{c}{\delta}\frac{1-\delta}{2-\delta} - w\right)$. In the equilibrium in which the winning (cross-market) dealer shares the excess balance-sheet capacity in proportion to trade sizes across markets, the bids are

$$b^i = \frac{1}{q^i} \times \left(w + q^i\left(V - \frac{c}{\delta}\right) + \frac{q^i}{q^A + q^B}\left((q^A + q^B)\frac{c}{\delta}\frac{1-\delta}{2-\delta} - w\right)\right) \qquad \text{for } i = \{A, B\}$$

The equilibrium bids are

$$b^A = V - \frac{c}{\delta}\frac{1}{2-\delta} + w\frac{q^B}{q^A(q^A+q^B)}$$
$$b^B = V - \frac{c}{\delta}\frac{1}{2-\delta} + w\frac{q^A}{q^B(q^A+q^B)}$$

for $w < w_1' \equiv \frac{c(1-\delta)^2(q^A+q^B)q^A}{(2-\delta)\delta q^B}$. Define $w_2' \equiv \frac{c(1-\delta)^2(q^A+q^B)}{(2-\delta)\delta}$. For $w \in [w_1', w_2')$, b^A is already at the zero-profit bid $V-c$. The remaining balance-sheet capacity becomes $\frac{c}{\delta}\frac{1-\delta}{2-\delta}\left(q^B-(1-\delta)q^A\right)$, which is all allocated to market B. Hence, in this case, $b^B = \frac{w}{q^B} + V - \frac{c}{\delta}\frac{q^B+q^A(1-\delta)^2}{q^B(2-\delta)}$. And for $w \geq \left[w_2', c\frac{1-\delta}{\delta}q^A\right)$, $b^A = b^B = V-c$.

It is immediate to check that all the above ranges of w are non-empty.

2. Suppose w is such that specializing dealers who exhaust pledgeable income would bid $V-c$ for asset A, but less that $V-c$ for asset B. This requires that $w \in [c\frac{1-\delta}{\delta}q^A, c\frac{1-\delta}{\delta}q^B)$. Condition (43) becomes

$$w + (q^A+q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) > q^A(V-c) + w + q^B\left(V - \frac{c}{\delta}\right)$$

or $q^A < \frac{1}{1-\delta}q^B$. This is true because $q^A < q^B$.

The equilibrium bids are $b^A = b^B = V-c$.

3. Suppose w is such that specializing dealers who exhaust pledgeable income would bid $V-c$ for both assets. This requires $w \geq c\frac{1-\delta}{\delta}q^B$. Condition (43) becomes

$$w + (q^A+q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) > (q^A+q^B)(V-c)$$

or $w > (q^A+q^B)\frac{c}{\delta}\frac{(1-\delta)^2}{2-\delta}$. This is true, because $w > c\frac{1-\delta}{\delta}q_B$, and $q^A < q^B$. The equilibrium bids are $b^A = b^B = V-c$. Note that at this range of w, the equilibrium can also be supported by two specializing dealers.

Proof about asks prices is symmetric, and is ommitted for brevity. □

Proof of Corollary 2. We start with the negative price impact result. Using the results in Proposition 4, for $q^A < (1-\delta)q^B$ and $w < w_1$, $\frac{\partial b^A}{\partial q^A} > 0$ if and only if $w < \frac{c(q^A)^2}{\delta(2q^A+q^B)}$. Results concerning correlation and spillovers follow directly from the computation of the partial derivatives. □

Proof of Lemma A.1. We argue by contradiction. Suppose that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is maximized, but that (IC') is slack.

Now suppose that both of the two limited liability constraints in (LL') relating to R_j^{HU} and R_j^{HD} bind, so that neither of the two repayments can be increased. It can be easily seen that in this case, (IC') cannot be satisfied. We have reached a contradiction.

Now suppose that one of the two limited liability constraints in (LL') relating to R_j^{HU} or R_j^{HD} binds, but the other does not. These repayments are associated with cash flows x^{HU} and x^{HD}. If the limited liability constraint binds for the repayment associated with the higher cash flow, but does not bind for the repayment associated with the lower cash flow, then (MCD) cannot be satisfied. So this is a contradiction. If instead the limited liability constraint binds for the repayment associated with the lower cash flow, but does not bind for the repayment associated with the higher cash flow, then it must be possible to raise the repayment associated with the higher cash flow: This does not violate the corresponding limited liability constraint, nor (MCD), nor (MCF), nor (IC'). This implies that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is not maximized, and we have again reached a contradiction.

Finally, suppose that neither of the two limited liability constraints in (LL') relating to R_j^{HU} and R_j^{HD} bind. Then it must be possible to increase either one or the other or both, without violating (MCD), (MCF), or (IC'). This implies that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is not maximized, and we have again reached a contradiction. □

Proof of Proposition A.5. We first consider the case in which Earl wants to sell and the dealer buys the asset from him, and selling it on later. We then consider the case in which Earl wants to buy and the dealer borrows the asset and then sells it to him, buying it back later. For both cases, we need to consider the two subcases in which the asset is either relatively low-risk, $z \leq \frac{1}{2}k$, or relatively high-risk, $z > \frac{1}{2}k$, respectively.

Dealer buys; low-risk asset, $z \leq \frac{1}{2}k$. In this case, Assumption A.5 implies the following constraints:

$$\begin{array}{ccccl} R_b^{HU} \geq & R_b^{HD} \geq & R_b^{LU} \geq & R_b^{LD}(\geq 0), & \qquad (\text{MCF}_{b,l}) \\ x_b^{HU} - R_b^{HU} \geq & x_b^{HD} - R_b^{HD} \geq & x_b^{LU} - R_b^{LU} \geq & x_b^{LD} - R_b^{LD}(\geq 0), & \qquad (\text{MCD}_{b,l}) \end{array}$$

where (MCF$_{b,l}$) stands for the monotonicity constraint of the financiers, when the dealer buys a low-risk asset, and (MCD$_{b,l}$) stands for the monotonicity constraint of the dealer, when the dealer buys a low-risk asset.

By using Lemma A.1, the pledgeable income can be written as

$$\mathcal{P}_b(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(R_b^{LU} + R_b^{LD}\right).$$

First, we argue that at the optimum, we must have $R_b^{LD} = x_b^{LD}$. Suppose this were not the case. Then, since the objective is increasing in R_b^{LD}, it must be that a constraint is binding. The only constraint that could bind is $R^{LD} \leq R_b^{LU}$ from (MCF$_{b,l}$). But this implies that for all $i \in \{LD, LU, HD, HU\}$, we have $R_b^i < x_b^i$, due to the monotonicity of cash flows and (MCD$_{b,l}$).

This means that we could increase all R^i by the same small amount, and all constraints would still be satisfied. Therefore, we cannot be at an optimum, and we have reached a contradiction.

Now consider R_b^{LU}. It is optimal to set R_b^{LU} as high as possible. The two possible constraints that can bind here are either $R_b^{LU} = x_b^{LU}$ from (LL) or $R_b^{LU} = R_b^{HD} (< x_b^{LU})$ from (MCF$_{b,l}$).

Consider the first the case in which $R_b^{LU} = x_b^{LU}$. In this case, using the fact that (IC') binds gives us

$$\frac{1}{2}\left(R_b^{HU} + R_b^{HD}\right) = q\left(V - \frac{c}{\delta}\right). \tag{44}$$

The optimal values of R_b^{HU} and R_b^{HD} must furthermore satisfy (MCD$_{b,l}$) and (LL). Also, we require that $R_b^{HD} \geq R_b^{LU} = x_b^{LU}$ from (MCF$_{b,l}$). Conjecture that we set $R_b^{HD} = q\left(V - \frac{c}{\delta}\right)$, to relax this last constraint as much as possible given $R_b^{HD} \leq R_b^{HU}$ from (MCF$_{b,l}$) and equation (44). Note that $R_b^{HD} \leq x_b^{HD}$ from (LL) as long as $z \leq k - \frac{c}{\delta}$, so that the proposed repayments maximize pledgeable income subject to all the constraints.

Now consider the case in which $R_b^{LU} = R_b^{HD} (< x_b^{LU})$. First, in this case, we can use the fact that (IC') binds to give us $R_b^{HU} = q\left(V + (k-z) - \frac{2c}{\delta}\right)$. Second, from the expression for pledgeable income we can see that it is now optimal to set R_b^{HD} as high as possible. This would imply that $R_b^{HD} = R_b^{HU}$ from (MCF$_{b,l}$) would bind, so that $R_b^{LU} = R_b^{HD} = R_b^{HU} = q\left(V + (k-z) - \frac{2c}{\delta}\right)$. It remains to check that the constraint $R_b^{LU} \leq x_b^{LU} = q(V + z - k)$ is satisfied. It can be seen that this holds as long as $z \geq k - \frac{c}{\delta}$.

To summarize, we have shown the following: If $z \leq k - \frac{c}{\delta}$, the repayments

$$\{R_b^{LD}, R_b^{LU}, R_b^{HD}, R_b^{HU}\} = \left\{q(V-z-k), q(V+z-k), q\left(V - \frac{c}{\delta}\right), q\left(V - \frac{c}{\delta}\right)\right\}$$

describe the optimal contract. The pledgeable income is $\mathcal{P}_b(q) = q\left(V - \frac{c}{\delta}\right)$ and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V - \frac{c}{q}\right)$.

If $z \geq k - \frac{c}{\delta}$, the repayments

$$\{R_b^{LD}, R_b^{LU}, R_b^{HD}, R_b^{HU}\} = \left\{q(V-z-k), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + k - z - \frac{2c}{\delta}\right)\right\}$$

describe the optimal contract. The pledgeable income is $\mathcal{P}_b(q) = q\left(V + k - z - \frac{2c}{\delta}\right)$ and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V + k - z - \frac{2c}{q}\right)$.

Dealer buys; high-risk asset, $z > \frac{1}{2}k$ In this case, Assumption A.5 implies the following constraints:

$$\begin{array}{rrrrrr} R_b^{HU} \geq & R_b^{LU} \geq & R_b^{HD} \geq & R_b^{LD} \geq & 0 & (\mathrm{MCF}_{b,h}) \\ x_b^{HU} - R_b^{HU} \geq & x_b^{LU} - R_b^{LU} \geq & x_b^{HD} - R_b^{HD} \geq & x_b^{LD} - R_b^{LD} \geq & 0 & (\mathrm{MCD}_{b,h}) \end{array}$$

Again, using the fact that (IC') binds from Lemma A.1, we can write pledgeable income as

$$\mathcal{P}_b(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(R_b^{LU} + R_b^{LD}\right).$$

A similar argument to the one above shows that it is optimal to set $R_b^{LD} = x_b^{LD}$.

Now consider R_b^{LU}. It is optimal to set R_b^{LU} as high as possible. The three possible constraints that can bind here are either $R_b^{LU} = x_b^{LU}$ from (LL) or $R_b^{LU} = R_b^{HU}$, $R_b^{LU} = R_b^{HU} < x_b^{LU}$ from (MCF$_{b,h}$), or $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$ from (MCD$_{b,h}$).

Suppose first that $R_b^{LU} = x_b^{LU} = q(V + z - k)$. From (MCD$_{b,h}$), it then follows that it must be the case that $R_b^{HD} = x_b^{HD}$. Finally, from the fact that (IC') binds, we can see that $R_b^{HU} = q\left(V + z - \frac{2c}{\delta}\right)$. However, for this candidate R_b^{HU}, we have that $R_b^{HU} < R_b^{LU}$ due to Assumption A.6, so that (MCF$_{b,h}$) is not satisfied. We have reached a contradiction. It follows that $R_b^{LU} < x_b^{LU}$.

Suppose next, therefore, that $R_b^{LU} = R_b^{HU} < x_b^{LU}$. From the fact that (IC') binds, we can then deduce that $R_b^{HD} = q\left(V + k - z - \frac{2c}{\delta}\right)$. (We can see that $R_b^{HD} < x_b^{HD}$ due to Assumption A.6.) It is optimal to set $R_b^{LU} (= R_b^{HU})$ as high as possible; the constraint that will bind is $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$ from (MCD$_{b,h}$). Therefore, $R_b^{LU} = R_b^{HU} = q\left(V + z - \frac{2c}{\delta}\right)$.

Finally, start by supposing $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$. In the previous sub-case, we already worked out one solution that satisfied this condition. It remains to show that no other solution exists. Substitute the condition $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$ into the binding (IC') to obtain

$$\begin{aligned}
\frac{1}{2} R_b^{HU} &= q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(x_b^{LU} - x_b^{HD} + x_b^{LD}\right) \\
\Leftrightarrow R_b^{HU} &= q\left(V + z - \frac{2c}{\delta}\right)
\end{aligned}$$

It is optimal to set R_b^{LU} as high as possible. Since the candidate R_b^{HU} satisfied $R_b^{LU} < x_b^{LU}$, we note that the binding constraint on R_b^{LU} must be $R_b^{LU} = R_b^{HU}$ from (MCF$_{b,h}$). We have established that there is only one solution, as worked out in the previous paragraph.

Therefore, for $z > \frac{1}{2}k$, the repayments

$$\begin{aligned}
&\{R_b^{LD}, R_b^{HD}, R_b^{LU}, R_b^{HU}\} = \\
&\qquad \left\{q(V - z - k), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + z - \frac{2c}{\delta}\right), q\left(V + z - \frac{2c}{\delta}\right)\right\}
\end{aligned}$$

describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_b(q) = q\left(V + \frac{1}{2}k - \frac{2c}{\delta}\right)$ and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V + \frac{1}{2}k - \frac{2c}{\delta}\right)$.

Dealer sells; low-risk asset, $z \leq \frac{1}{2}k$ We note that here, it is necessarily the case that $R_a^{LU} = 0$ by (LL), because $x_a^{LU} = 0$. (This is also the case in the high risk asset case when the dealer sells.) By using Lemma A.1, this implies that the pledgeable income can be written as

$$\mathcal{P}_a(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2} R_a^{LD}.$$

Here, Assumption A.5 implies the following constraints:

$$\begin{array}{rrrrr} R_a^{HD} \geq & R_a^{HU} \geq & R_a^{LD} \geq & R_a^{LU} = 0, & (\text{MCF}_{a,l}) \\ x_a^{HD} - R_a^{HD} \geq & x_a^{HU} - R_a^{HU} \geq & x_a^{LD} - R_a^{LD} \geq & x_a^{LU} - R_a^{LU} = 0. & (\text{MCD}_{a,l}) \end{array}$$

Increasing R_a^{LD} increases $\mathcal{P}_a(q)$, and relaxes ($\text{MCD}_{a,l}$) and (IC'). Hence the optimal R_a^{LD} will satisfy either $R_a^{LU} = x_a^{LU}$ due to (LL) or $R_a^{HU} = R_a^{LD}$ due to ($\text{MCF}_{a,l}$). We consider these two sub-cases below in turn.

Suppose first that the optimal $R_a^{LU} = x_a^{LU}$. It then remains to pin down R_a^{HD} and R_a^{HU}. Consider the choice of R_a^{HU}. It has to satisfy $R_a^{HU} \geq R_a^{LD} = x_a^{LD}$ from ($\text{MCF}_{a,l}$) and $R_a^{HU} = 2q(k + z - \frac{c}{\delta}) - R_a^{HD}$ from (IC') (we conjecture and later verify that (LL) is slack for R_a^{HU}). The two constraints can be combined to state that $2q(k+z-\frac{c}{\delta}) - R_a^{HD} \geq x_a^{LD}$. This combined constraint is most relaxed when R_a^{HD} is as small as possible, which would imply a choice $R_a^{HD} = R_a^{HU}$, given that ($\text{MCF}_{a,l}$) has to be satisfied. Using the fact that (IC') binds, we then have $R_a^{HU} = R_a^{HD} = q(k + z - \frac{c}{\delta})$. The constraint $R_a^{HU} \geq x_a^{LD}$ from ($\text{MCF}_{a,l}$) then becomes $z \leq k - \frac{c}{\delta}$. By Assumption A.6, $k - \frac{c}{\delta} < \frac{1}{2}k$, so that $z \leq k - \frac{c}{\delta}$ also implies $z < \frac{1}{2}$.)

Finally, it is immediate to check that R_a^{HU} satisfies (LL) as $R_a^{HU} = q(k+z-\frac{c}{\delta}) < x_a^{HU} = qk$. To sum up, for $z \leq \min\{\frac{1}{2}k, k-\frac{c}{\delta}\} = k-\frac{c}{\delta}$ (by Assumption A.6), the repayments $\{R_a^{LU}, R_a^{LD}, R_a^{HU}, R_a^{HD}\} = \{0, qk, q(k + z - \frac{c}{\delta}), q(k + z - \frac{c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = q(k + z - \frac{c}{\delta})$ and the minimum ask for which effort can still be exerted is $a_{IC'}(q) = V + \frac{c}{\delta} - \frac{w}{q}$.

Next, suppose $R_a^{HU} = R_a^{LD} < x_a^{LD}$. The binding (IC') then implies that $R_a^{HD} = 2q(k - \frac{c}{\delta})$. The pledgeable income becomes $\mathcal{P}_a(q) = q(k - \frac{c}{\delta}) + \frac{1}{2}R_a^{HU}$ and is increasing in R_a^{HU}. Hence, it is optimal to set R_a^{HU} as high as possible until (MCF) binds, i.e., $R_a^{HU} = R_a^{HD} = 2q(k - \frac{c}{\delta})$. The assumption that $R_a^{HU} = R_a^{LD} < x_a^{LD}$ is satisfied if and only if $z > k - \frac{c}{\delta}$. Thus, for $z \in (k - \frac{c}{\delta}, \frac{1}{2}k]$, the repayments $\{R_a^{LU}, R_a^{LD}, R_a^{HU}, R_a^{HD}\} = \{0, 2q(k - \frac{c}{\delta}), 2q(k - \frac{c}{\delta}), 2q(k - \frac{c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = 2q(k - \frac{c}{\delta})$ and the minimum ask for which effort can still be exerted is $a(q) = V - k + z + \frac{2c}{\delta} - \frac{w}{q}$.

Dealer sells; high-risk asset, $z > \dfrac{1}{2}k$ Again, it is necessarily the case that $R_a^{LU} = 0$ by (LL), because $x_a^{LU} = 0$, and by using Lemma A.1, the pledgeable income can be written as

$$\mathcal{P}_a(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}R_a^{LD}.$$

Here, Assumption A.5 implies the following constraints:

$$\begin{array}{rrrrr} R_a^{HD} \geq & R_a^{LD} \geq & R_a^{HU} \geq & R_a^{LU} = 0, & (\text{MCF}_{a,h}) \\ x_a^{HD} - R_a^{HD} \geq & x_a^{LD} - R_a^{LD} \geq & x_a^{HU} - R_a^{HU} \geq & x_a^{LU} - R_a^{HU} = 0. & (\text{MCD}_{a,h}) \end{array}$$

Like before, setting a higher R_a^{LD} increases $\mathcal{P}_a(q)$ and relaxes (IC'). Hence, the optimal R_a^{LD} is either $R_a^{LD} = R_a^{HD}$ so that the relevant part of ($\text{MCF}_{a,h}$) binds, or $R_a^{LD} = x_a^{LD} - x_a^{HU} + R_a^{HU}$ so that the relevant part of ($\text{MCD}_{a,h}$) binds, or both.

Suppose first that $R_a^{LD} < R_a^{HD}$, so that the relevant part of (MCF$_{a,h}$) would not bind. Then it would have to be the case that $R_a^{LD} = x_A^{LD} - x_a^{HU} + R_a^{HU}$ so that (MCD$_{a,h}$) would bind. Together with the fact that (IC') has to bind, this would imply that $R_a^{LD} - R_a^{HU} = 2q\left(z - \frac{1}{2}k\right)$. Since (IC') binds, this means that $R_a^{HD} = q\left(2z + k - \frac{c}{\delta}\right)$. But R_a^{LD} is not yet pinned down. The maximisation of $\mathcal{P}_a$ requires that it is raised as far as possible given applicable constraints, which implies that it would have to be the case that $R_a^{LD} = R_a^{HD}$.

We now know that $R_a^{LD} = R_a^{HD}$. Then the fact that (IC') binds implies $R_a^{HU} = 2q\left(k - \frac{c}{\delta}\right)$. We can see that the constraint $R_a^{HU} \leq x_a^{HU}$ from (LL) is guaranteed to be satisfied because of Assumption A.6. Next, it is optimal to increase R_a^{LD} (and hence also R_a^{HD}), but $R_a^{HU} = 2q\left(k - \frac{c}{\delta}\right)$ is fixed. This implies that the $x_a^{LD} - R_a^{LD} = x_a^{HU} - R_a^{HU}$ from (MCD$_{a,h}$) must bind. In turn, this implies $R_a^{LD} = q\left(2z + k - \frac{2c}{\delta}\right)$.

Thus, for $z > \frac{1}{2}k$, the repayments $\{R_a^{LU}, R_a^{HU}, R_a^{LD}, R_a^{HD}\} = \{0, 2q(k - \frac{c}{\delta}), q(2z + k - \frac{2c}{\delta}), q(2z + k - \frac{2c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = q\left(\frac{3}{2}k - \frac{2c}{\delta} + z\right)$ and the minimum ask for which effort can still be exerted is $a_{IC'}(q) = V - \frac{k}{2} + \frac{2c}{\delta} - \frac{w}{q}$.

□

Proof of Lemma B.2. With the definitions of R_H, R_L, the constraint (IC) can be re-written as

$$\alpha \leq \left(1 - \frac{c}{\delta k}\right) \equiv \bar{\alpha}, \tag{45}$$

where $\bar{\alpha}$ is the maximum share of equity that can be issued to financiers, so that exertion of effort by the dealer is still incentive compatible.

A competitive dealer who bids $b_c(q, w) = \min\left\{\frac{w}{q} + \left(V - \frac{c}{\delta}\right) V - c\right\}$, must raise external finance $qb_c(q, w) - w$. The break-even constraint (BE) stipulates that $qb_c(q, w) - w = R_H$ so that the bid can be financed in this case as long as

$$R_H = \alpha(x^H - D) + D = \alpha x^H + (1 - \alpha)D \geq \max(qb_c(q, w) - w, 0) \tag{46}$$

where we note that here, $x^H = qV$.

We can distinguish three cases:

1. The minimum amount of debt D_{min} can be zero if there exist an $\alpha \leq \bar{\alpha}$ such that $\alpha qV + (1 - \alpha) \cdot \underbrace{D}_{=0} = \alpha qV = R_H$, where $R_H \geq qb_c - w$, that is, if

$$\bar{\alpha} qV \geq q(V - c) - w \tag{47}$$

or if

$$q \leq q_{NL}(w) = \frac{w}{\frac{c}{\delta k} V - c}. \tag{48}$$

2. For $q > q_{NL}(w)$, the minimum amount of debt $D_{\min}$ needed in the optimal contract is implicitly defined by

$$\bar{\alpha} qV + (1 - \bar{\alpha}) D_{\min} = q\left(V - c\right) - w \tag{49}$$

or

$$D_{\min} = q\left(V - \frac{\delta}{k}\right) - w\frac{\delta k}{c}. \tag{50}$$

Furthermore, debt should be risk-free, as assumed above, which requires

$$D_{\min} \leq x^L = q(V - k), \tag{51}$$

which is equivalent to requiring

$$q \leq \bar{q}(w). \tag{52}$$

3. Finally, for $q > \bar{q}(w)$, the maximum amount of (safe) debt is $D_{\min} = x^L = q(V - k)$, the maximum amount that can be raised is $R_H = \bar{\alpha}x^H + (1 - \bar{\alpha}D_{\min} = q\left(V - \frac{c}{\delta}\right)$. Since $b_c(q, w) = \frac{w}{q} + \left(V - \frac{c}{\delta}\right)$, this is exactly the amount that must be raised, and intermediation can take place as long as $q \leq q_{\max}(w)$ as defined above.

The leverage ratios in the lemma are calculated in the three cases with total assets as $D_{\min}/(qb_c(q, w))$. □

Proof of Proposition B.6. First notice that the incentive constraint (36) should be binding, as increasing α increases $\mathcal{P}$ and relaxes the leverage requirement. This pins down α^*. Similarly, the leverage constraint (37) should be binding, since increasing D increases $\mathcal{P}$ and relaxes the incentive constraint (36). This then pins down D^*. □

Proof of Corollary 3. The maximum incentive compatible bid under the maximum leverage requirement is $b_{IC}(q; \Lambda_{\max}) = \frac{w}{q} + \frac{\mathcal{P}(q;\Lambda_{\max})}{q}$. The first result, $\frac{\partial b_{IC}(q;\Lambda_{\max})}{\partial \Lambda_{\max}} > 0$, and second result, $\frac{\partial}{\partial q}\frac{\partial b_{IC}(q;\Lambda_{\max})}{\partial \Lambda_{\max}} > 0$, can be obtained by direct differentiation of the expression for $b_{IC}(q; \Lambda_{\max})$, using the results in Proposition B.6.

The third result comes from the definition of $q_{\max}$: The largest possible trade is the one for which the bid, $b_{IC}(q_{\max}; \Lambda_{\max})$, is just equal to Earl's reservation value of $V - \ell$, or for which $q_{\max} \cdot b_{IC}(q_{\max}; \Lambda_{\max}) = q_{\max}(V - \ell)$. Inserting the definitions of $b_{IC}(q_{\max}; \Lambda_{\max})$ and $\mathcal{P}(q; \Lambda_{\max})$, we find that

$$q_{\max} = \frac{w + \frac{c}{\delta k}D^*}{\frac{c}{\delta k} - \ell}$$

Hence sign $\left(\frac{\partial q_{\max}}{\partial \Lambda_{\max}}\right) = $ sign $\left(\frac{\partial D^*}{\partial \Lambda_{\max}}\right) > 0$. Reducing the maximum leverage ratio $\Lambda_{\max}$ decreases $q_{\max}$, and some larger orders are not intermediated. Hence welfare decreases. □