

# Dynamic Banking and the Value of Deposits\*

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## Abstract

We propose a dynamic theory of banking where the role of deposits is akin to that of productive capital in the classical  $q$ -theory of investment. As a cheap source of leverage, deposits typically create value for banks, but the marginal  $q$  of deposits can be negative. Deposit accounts commit banks to accept any inflows and outflows, so that banks cannot perfectly control leverage. Such uncertainty destroys value when banks have insufficient equity capital to buffer shocks. Our model lends itself to a re-evaluation of leverage regulations and offers new perspectives on banking in a low interest-rate environment.

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# 1 Introduction

During the unfolding COVID-19 pandemic U.S. banks have undergone unprecedented balance-sheet expansions as a result of massive inflows into deposit accounts. Most dramatically, deposits of US banks increased by \$865 billion just in April 2020 alone. From Q4 2019 to Q1 2020, JP-Morgan Chase experienced an increase of 18% of its deposit base, and the deposit liabilities of Citigroup and Bank of America increased by 11% and 10%, respectively.<sup>1</sup> Contrary to the conventional wisdom, abundant funding liquidity did not benefit bank valuation or stimulate lending. The banking sector is among the slowest sectors to recover from pandemic equity valuation lows.

We show that large deposit inflows are both an opportunity and a challenge for banks. Deposit account is a source of cheap funding. Depositors accept relatively low interest rates under banks' deposit market power (Drechsler, Savov, and Schnabl, 2017) and for the convenience of using deposits as means of payment.<sup>2</sup> But the consequence of allowing depositors to freely move funds in and out of their accounts is that banks cannot perfectly control the size of deposit base.

Facing equity issuance costs (Myers and Majluf, 1984), banks are endogenously averse to risk in its equity capital. Therefore, deposit-flow shocks present a challenge to bank risk management. By bringing in cheap funding, a positive (inflow) shock boosts the current earnings. However, it also injects risk in the future earnings and trajectories of equity capital because it is uncertain whether the new deposits will stay in the customers' accounts or not. In contrast, a negative (outflow) shock causes involuntary contraction of both risk and return on equity.

The equity  $K$  to deposit  $X$  ratio (denoted by " $k \equiv K/X$ ") emerges as the key state variable that drives the bank's decisions. The numerator  $K$  represents the bank's risk-taking capacity, while the denominator  $X$  measures the size of deposits as cheap sources of financing and scales the deposit-flow shocks. The ratio is endogenously bounded above by optimal dividend payout and below by costly equity issuances. The bank's endogenous risk aversion decreases in  $k$ . When  $k$  is high, the bank has a sufficient equity buffer and can take advantage of deposits as cheap financing

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<sup>1</sup>See "U.S. Banks are 'Swimming in Money' as deposits increase by 2 trillion dollars amid the coronavirus" by Hugh Son, CNBC June 21, 2020. Such deposit influx also happened in the financial crisis of 2007–2008.

<sup>2</sup>A recent literature incorporates the money premium into macroeconomic and banking models (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Nagel, 2016; Begeau, 2020)

(relative to bonds). When  $k$  is low, deposit inflows become problematic because the positive impact on current earnings is overwhelmed by the concern over the uncertainty that new deposits transmit into future earnings. When  $k$  is near the equity issuance (lower) bound, such concern is acute and the marginal value of deposits to bank shareholders (“marginal deposit  $q$ ”) turns sharply negative.

Moreover, deposit inflows may not induce more lending when  $k$  is low. The bank earns a net interest margin, which can be decomposed into the risk premium from lending and the deposit spread (the wedge between the risk-free bond rate and the lower deposit rate). Both sources of profits are risky. The optimal lending policy is characterized by a formula akin to the portfolio choice of Merton (1969), but with an endogenous  $k$ -dependent measure of risk aversion. Deposit inflows bring more cheap funding to lend but raise the endogenous risk aversion by lowering  $k$ . By increasing deposit risk exposure, deposit inflows reduce the bank’s capacity to take on more lending risk. Near the equity issuance (lower) bound for  $k$ , deposit inflows can even cause the bank to scale back lending and allocate more deposits into risk-free bonds.

We model the deposit stock as a stochastic process partially controlled by the bank through its deposit rate. Deposits are effectively long-duration debts as Drechsler, Savov, and Schnabl (2021) have observed, but in our model, the maturities are random. The inability to fully control the size of liabilities makes bank balance-sheet management conceptually very different from that of non-depository intermediaries and nonfinancial firms. In our model and as documented by Drechsler, Savov, and Schnabl (2017), the deposit base has random yet persistent flows. When  $k$  is high, the bank raises deposit rate in an effort to attract more deposits, just like nonfinancial firms investing in their capital stock. In effect, the optimal deposit-rate policy resembles the investment policy in Hayashi (1982). When  $k$  is low, risk concern dominates, so that the bank lowers its deposit rate in an effort to forestall any unintended balance-sheet expansion due to deposit inflow shocks.

Another realistic feature of our model is a lower bound for the deposit rate. A natural bound is zero, because depositors can always withdraw and hoard fiat money with a zero nominal return. This lower bound is increasingly binding in the current low-rate environment (Heider, Saidi, and Schepens, 2019). Although a lower bound is not required to generate endogenous risk aversion in our model, it further limits the bank’s ability to adjust the deposit flows, strengthening the

mechanism. Once the deposit rate hits the lower bound, the bank completely loses its ability to counteract deposit-inflow shocks through further reduction of the deposit rate.

Our model provides a unified theory of bank's deposit-taking, short-term borrowing, risky lending, dividend payout, and equity issuances. We draw a sharp distinction between deposits and short-term debt. With short-term debt, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted leverage. In contrast, deposits are long-term contracts without well-defined maturity. Deposits leave the bank only when depositors withdraw funds. When the equity capital-to-deposit ratio,  $k$ , is high, the bank issues short-term debt to obtain additional leverage for lending. If  $k$  declines, the bank deleverages by reducing short-term debt. And when  $k$  approaches the lower boundary of costly equity issuance, the bank switches from issuing short-term bonds to holding risk-free bonds, thereby de-risking the asset side of its balance sheet, given that the risk on the liability (deposit) side cannot be fully controlled.

To the extent that it is modeling deposit risk, the banking literature has done so only by assuming illiquid bank assets and examining costly liquidation due to deposit outflows in a coordination failure (bank run) (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). Our model departs from this framework: we assume that bank assets are liquid, but deposit risk still matters under equity issuance costs. Both inflow and outflow shocks can be problematic as they cause, respectively, involuntary expansion and contraction of earnings' exposure to future deposit risk.

Our dynamic model provides a complete characterization of the nonlinear dynamics. Under equity issuance costs, the marginal value of equity capital is generally greater than one. The marginal value of equity capital is equal to one only at the dividend payout (upper) boundary of  $k$ , where the bank is indifferent between retaining earnings or paying out dividends. At the peak of the stationary density of  $k$ , the marginal value of equity capital is only slightly above one, which means that most of the time the bank does not appear to be financially constrained. However, when  $k$  approaches the equity issuance (lower) boundary of  $k$ , the marginal value of equity capital shoots up dramatically. The strong concavity of the value function in  $k$  near the lower boundary causes a sharp increase in the bank's endogenous risk aversion. Near the lower boundary of  $k$ , deposit inflows can significantly raise the likelihood of a costly equity issuance as it flattens the probability

density of equity capital by injecting more uncertainty into the future earnings. The bank therefore wants to turn away deposits, but can only go as far as setting the deposit rate at the lower bound.

A distinguishing feature of banks is that their leverage cannot exceed a regulatory maximum. Leverage regulation makes deposit-taking more challenging. Due to the deposit-flow risk, a bank does not have full control over its balance-sheet size and composition. Unexpected deposit inflows increase leverage, so when the bank is undercapitalized, it has to incur the issuance costs and raise equity to avoid violating the regulatory restriction. Therefore, leverage regulation amplifies the bank's endogenous risk aversion that was caused by the equity issuance costs.

During the Covid-19 pandemic, U.S. banking regulators relaxed the supplementary leverage ratio (SLR) requirement. Our model shows that this policy move stimulates lending and deposit-taking. This regulatory relief is particularly effective in a low interest-rate environment, where the deposit rate is stuck at the lower bound so that banks are losing control of their leverage. However, the stimulative effect is short-lived. The relaxed leverage regulation implies less frequent equity issuances over the long run. Given that the bank incurs less issuance costs, it has a weaker incentive to boost earnings (through lending and deposit-taking) to compensate shareholders for paying the issuance costs. Tightening leverage requirements discourages lending and deposit-taking in the short-run as it makes equity issuance more imminent, but over the long run, the bank has to generate more earnings to compensate shareholders for more frequent costly equity issuances, reaching for yield by loading on more risks in both lending and deposit-taking. Hence, tightening leverage regulation, while successfully builds up bank capital by inducing more equity issuances, fails its original purpose of taming risk-taking per unit of equity over the long run.<sup>3</sup>

Total leverage regulation (e.g., the SLR requirement in the U.S.) and risk-based capital requirement play distinct roles in our model. Under the total leverage regulation, a deposit-inflow shock can trigger costly equity issuance through an involuntary increase of bank leverage beyond the regulatory maximum. In contrast, under risk-based capital requirement, a deposit inflow does

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<sup>3</sup>Kashyap, Stein, and Hanson (2010) argue that the impact of heightened leverage regulation on bank value should be temporary, because in a deterministic environment, the bank pays the equity issuance costs once and then settles on a lower leverage. We study a stochastic environment where under deposit and loan-return shocks, costly equity issuance is recurrent. The issuance costs are thus reflected in bank value even away from the equity issuance boundary. Tightening leverage regulation permanently reduces bank value by making costly equity issuance more frequent.

not trigger costly equity issuance as long as the bank invests the new deposits in risk-free bonds. Therefore, risk-based capital requirement is a more targeted measure to limit risk-taking, because its impact is isolated from banks' inability to perfectly control deposit flows.

Finally, our model also sheds light on the critical role of the prevailing interest rate level  $r$  in bank valuation and balance-sheet management. As in Drechsler, Savov, and Schnabl (2017), the bank earns the deposit spread,  $r - i$ . When  $k$  is high, the bank raises deposit rate  $i$  to attract more deposits. When  $k$  declines in the future (for example, following unexpected deposit inflows) the bank will have more room to reduce  $i$  before hitting the deposit rate lower bound. Therefore, when  $r$  is high, the bank has more flexibility in raising deposit rate in the high- $k$  region without squeezing the deposit spread too much. The distance between  $r$  and deposit rate lower bound essentially determines the degree of flexibility to control deposit flows through adjusting the deposit rate. In a low interest rate environment, the bank has less flexibility, so that the deposit marginal  $q$  declines. Moreover, with a narrower deposit spread,  $r - i$ , the franchise (continuation) value is lower, so that the bank becomes more aggressive in its shareholder payout. This speaks to the massive bank stock buybacks in the last decade of low interest rates.

**Literature.** The bank allows depositors to move funds freely in and out of their accounts. Therefore, the maturity of deposit contracts is not chosen by the bank. It depends on depositors' payment needs that are uncertain. In a dynamic setting, the deposit stock retires stochastically over time. Drechsler, Savov, and Schnabl (2021) emphasize the long duration of deposits as the bank has to carry the deposits as long as its depositors do not withdraw. We also model deposits as long-duration liabilities but our approach differs by introducing the randomness in deposit flow.<sup>4</sup>

The key to our results is the bank's lack of control of its deposit liabilities. The randomness in leverage translates into uncertainty in the future trajectories of equity capital. Equity issuance costs make the bank averse to equity capital risk.<sup>5</sup> This mechanism delivers a rich set of empirical pat-

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<sup>4</sup>In static settings, the literature explores the implications of payment risk on banks' liquidity holdings and incentive to lend (Freixas, Parigi, and Rochet, 2000; Donaldson, Piacentino, and Thakor, 2018; Parlour, Rajan, and Walden, 2020). Empirically, banks face large payment flow shocks (Furfine, 2000; Bech and Garratt, 2003; Afonso and Shin, 2011; Denbee, Julliard, Li, and Yuan, 2018; Choudhary and Limodio, 2017; Copeland, Duffie, and Yang, 2021).

<sup>5</sup>While our model introduces the costs of issuing equity (Myers and Majluf, 1984), the link between equity and

terns, such as bank capital and valuation (Mehran and Thakor, 2011; Minton, Stulz, and Taboada, 2019), bank capital and risk-taking (Ben-David, Palvia, and Stulz, 2020), deposit funding relative to total liabilities (Drechsler, Savov, and Schnabl, 2017), equity issuance and payout cyclicality (Adrian, Boyarchenko, and Shin, 2015; Black, Floros, and Sengupta, 2016; Baron, 2020), comovement in loan growth and deposit rate (Ben-David, Palvia, and Spatt, 2017), and occasionally binding capital requirement (Gropp and Heider, 2010; Begenau, Bigio, Majerovitz, and Vieyra, 2019). The relevance of equity issuance costs in practice is demonstrated by banks seeking ways to avoid raising new equity in distress, for example, through the use of contingent capital (Pennacchi, 2010; Bolton and Samama, 2014; Glasserman and Nouri, 2016; Pennacchi and Tchisty, 2018, 2019). Our contribution is in studying jointly equity issuance costs and deposit-flow risk.

Dynamic banking models often differentiate deposits and short-term bonds in their interest expenses and operation costs (Hugonnier and Morellec, 2017; Van den Heuvel, 2018; Begenau, 2020). In these models, banks do not face uncertainty in the size of deposit stock. Bianchi and Bigio (2014), De Nicolò, Gamba, and Lucchetta (2014), Bigio and Sannikov (2019), and Vandeweyer (2019) model deposits as one-period debts and deposit-flow shocks as intra-period shocks, so banks can freely adjust deposits every period without facing the problem of losing control of leverage.

The macro-finance literature recognizes deposits as means of payment (Piazzesi and Schneider, 2016; Drechsler, Savov, and Schnabl, 2018; Begenau and Landvoigt, 2018) but model deposits as short-term debts with yields reduced by a money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2020). Brunnermeier and Sannikov (2016) is a notable exception. They model deposits as infinite-maturity nominal liabilities and study the Fisherian deflationary spiral.

The traditional banking models focus on bank runs when it comes to banks' commitment to allow depositors to withdraw funds without prior notice (Diamond and Dybvig, 1983; Allen

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risk-taking capacity is more general. For example, it arises from agency friction (He and Krishnamurthy, 2012, 2013). Equity issuance costs are key ingredients in banking models (Bolton and Freixas, 2000; Bianchi and Bigio, 2014; Brunnermeier and Sannikov, 2014; Allen, Carletti, and Marquez, 2015; Nguyen, 2015; Phelan, 2016; Klimenko, Pfeil, Rochet, and Nicolo, 2016; Hugonnier and Morellec, 2017; Begenau, Bigio, Majerovitz, and Vieyra, 2019) and dynamic models of nonfinancial firms (Gomes, 2001; Riddick and Whited, 2009; Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; DeAngelo, DeAngelo, and Whited, 2011; Hugonnier, Malamud, and Morellec, 2015; Décamps, Gryglewicz, Morellec, and Villeneuve, 2017).

and Gale, 2004b; Goldstein and Pauzner, 2005). A key model ingredient is the illiquidity of bank assets, which causes the coordination failure among the depositors. Deposit outflow triggers inefficient liquidation of assets, but deposit inflow is not a concern. To distinguish our model from the literature, we allow the bank to freely adjust its assets so coordination failure does not happen. The deposit risk matters because the deposit shocks feed into the trajectory of bank equity capital, and managing such risks is important under equity issuance costs. Even deposit inflow can be problematic due to the uncertainty of whether the new deposits will stay or flow out in the future.

## 2 Model

We model the decisions of a single bank that maximizes risk-neutral shareholders' value.<sup>6</sup>

**Risky Assets.** We use  $A_t$  to denote the value of the bank's holdings of loans and other risky assets at time  $t$ . Let  $r$  denote the risk-free rate. The value of risky assets evolves as follows:

$$dA_t = A_t (r + \alpha_A) dt + A_t \sigma_A d\mathcal{W}_t^A. \quad (1)$$

The parameter  $\alpha_A$  reflects the return from the bank's expertise.<sup>7</sup> The second term in (1) describes the shock to the asset value (e.g., unexpected loan charge-offs), where  $\sigma_A$  is the diffusion-volatility parameter and  $\mathcal{W}^A$  is a standard Brownian motion.<sup>8</sup> The bank may adjust  $A_t$  at any time  $t$ .<sup>9</sup>

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<sup>6</sup>Risk-neutrality can be reinterpreted as modelling under the risk-neutral measure by taking as exogenous a pricing kernel (stochastic discount factor) that depends on the aggregate dynamics of the broader economy. Then the risk-free rate,  $r$ , is the expected return under the risk-neutral measure of all financial assets that are traded by bank shareholders.

<sup>7</sup>The bank may have expertise in monitoring (Diamond, 1984), loan screening (Ramakrishnan and Thakor, 1984), relationship lending (Boot and Thakor, 2000), restructuring (Bolton and Freixas, 2000), asset management and diversification (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014, 2016), collateralization (Rampini and Viswanathan, 2018), and serving local credit markets (Gertler and Kiyotaki, 2010).

<sup>8</sup>We add jump risk in Appendix C as Parlour, Stanton, and Walden (2012); Hugonnier and Morellec (2017)

<sup>9</sup>The bank may adjust the loan amount  $A_t$  by selling loans. Technological progress on the reduction of information asymmetries facilitates loan trading. The design of contract between loan buyers and originators alleviates the moral hazard (reduced monitoring incentive) on the part of loan originators (Pennacchi, 1988; Gorton and Pennacchi, 1995).



**Bonds.** The bank can trade standard risk-free bonds and it is costless to do so. Let  $B_t$  denote the value of bonds that the bank issues at  $t$  and will mature at  $t + dt$ . When  $B_t > 0$ , the bank issues bonds (e.g., commercial papers) and incurs interest expenses of  $B_t r dt$  over time interval  $dt$ . When  $B_t < 0$ , the bank holds bonds issued by other entities (e.g., the government).

**Deposits.** At the core of our model is the law of motion of deposits. The deposit stock at time  $t$ , which we denote by  $X_t$ , evolves as follows:

$$dX_t = -X_t (\delta_X dt - \sigma_X d\mathcal{W}_t^X) + X_t n(i_t) dt, \quad (2)$$

where  $\mathcal{W}_t^X$  is a standard Brownian motion. Let  $\phi dt$  denote the instantaneous covariance between  $d\mathcal{W}_t^X$  and  $d\mathcal{W}_t^A$ .<sup>10</sup> The flow that the bank cannot control is given by  $-X_t (\delta_X dt - \sigma_X d\mathcal{W}_t^X)$ . We interpret such flow as driven by payment activities.<sup>11</sup> When the depositors pay other banks' depositors, outflow happens,  $(\delta_X dt - \sigma_X d\mathcal{W}_t^X) > 0$ . When the depositors receive cash or electronic payments from other banks' depositors, the bank receives inflow,  $(\delta_X dt - \sigma_X d\mathcal{W}_t^X) < 0$ .<sup>12</sup>

The bank chooses the deposit rate,  $i_t$ , to adjust the flow via  $n(i_t) dt$ . Lowering the deposit rate reduces the deposit flow, i.e.,  $n'(i_t) < 0$ , but such downward adjustment has a limit as  $i_t \geq 0$ . This lower bound is motivated by the fact that depositors can always withdraw dollar bills and earn a zero return, which is an empirically relevant friction (Heider, Saidi, and Schepens, 2019) and is also emphasized by Drechsler, Savov, and Schnabl (2020) in the context of banking and inflation.

The deposit rate  $i_t$  can be below  $r$ , and the deposit demand function,  $n(i_t)$ , depends on the bank's market power (Drechsler, Savov, and Schnabl, 2017) and the convenience yield that agents derive from holding deposits as means of payment (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Nagel, 2016; Piazzesi and Schneider, 2016; Li, 2018). For deposits to function as means of payment, depositors must be able to move funds in and out of

<sup>10</sup>This specification of slow-moving deposits captures the well-documented inertia in depositors' decisions to switch banks (Kim, Klinger, and Vale, 2003) and broadly banks' deposit market power (Drechsler, Savov, and Schnabl, 2017).

<sup>11</sup>Freixas, Parigi, and Rochet (2000), Bianchi and Bigio (2014), Donaldson, Piacentino, and Thakor (2018), and Parlour, Rajan, and Walden (2020) study deposits as both banks' sources of funds and depositors' means of payment.

<sup>12</sup>The values of  $\delta_X$  and  $\sigma_X$  depend on the bank's position in payment networks. The payment flow risk is significant in data (Afonso and Shin, 2011; Denbee, Julliard, Li, and Yuan, 2018; Copeland, Duffie, and Yang, 2021).

their accounts freely, and this exposes the bank to unhedgeable deposit shock in (2).

Following Hugonnier and Morellec (2017) and Drechsler, Savov, and Schnabl (2021), we assume that the bank pays a flow cost  $C(n(i_t), X_t) dt$ , which captures the expenses of maintaining the existing deposit franchise and serving new customers ( $\frac{\partial C(n(i_t), X_t)}{\partial n(i_t)} > 0$  and  $\frac{\partial C(n(i_t), X_t)}{\partial X_t} > 0$ ).

In our model, deposits are essentially long-term debts with stochastic and partially controllable maturity, as shown in (2).<sup>13</sup> Our treatment of deposits stands in contrast with the macro-finance literature and dynamic banking literature that generally treats deposits simply as short-term debts. We share with Drechsler, Savov, and Schnabl (2021) the view that the right to withdrawal does not necessarily translate into a low duration of deposits as the deposit base is often sticky.

**Payout and Costly Equity Issuance.** The following identity summarizes the balance sheet:

$$K_t + X_t = A_t - B_t, \quad (3)$$

where  $K_t$  is the bank's equity capital. The long-term funding in the form of equity capital and deposits finances the bank's investment in risky assets (net off the funds from bond issuances).

The bank can pay out dividends that reduce  $K_t$ . We use  $U_t$  to denote the cumulative dividends, so the amount of (non-negative) incremental payout is  $dU_t$ . The bank can issue equity. Let  $F_t$  denote the bank's cumulative equity financing up to time  $t$ . The law of motion of  $K_t$  is given by

$$dK_t = A_t [(r + \alpha_A) dt + \sigma_A d\mathcal{W}_t^A] - B_t r dt - X_t i_t dt - C(n(i_t), X_t) dt - dU_t + dF_t. \quad (4)$$

The first three terms on the right side record the return on risky assets, bond interest expenses if  $B_t > 0$  or interest income if  $B_t < 0$ , and deposit interest expenses. The fourth term is the cost of running the deposit franchise. The last two terms are payout and equity issuance, respectively.

In reality, banks face significant external financing costs due to asymmetric information,

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<sup>13</sup>Related, long-term debts are often modelled as perpetual debts with a constant amortization rate (Leland, 1998; He and Xiong, 2012; Diamond and He, 2014; He and Milbradt, 2016; DeMarzo and He, 2021). He and Manela (2016) model the withdrawal dynamics of depositors under the illiquidity of bank assets. In our model, bank assets are liquid. There exists a broad empirical literature on the measurement of duration of bank assets and liabilities (Begenau, Piazzesi, and Schneider, 2015; English, Van den Heuvel, and Zakrajšek, 2018; Begenau and Stafford, 2021).

incentive issues, and transaction costs. A large empirical literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction to the announcement of a new equity issue.<sup>14</sup> Let  $H_t$  denote the (undiscounted) cumulative costs of equity issuance up to time  $t$ . The bank maximizes the equityholders' value. The objective function is given by

$$V_0 = \max_{\{A,B,i,U,F\}} \mathbb{E} \left[ \int_{t=0}^{\tau} e^{-\rho t} (dU_t - dF_t - dH_t) \right]. \quad (5)$$

We assume that  $\rho > r$ , a common assumption in dynamic corporate finance and macro-finance models, e.g., DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and Brunnermeier and Sannikov (2014) among others.<sup>15</sup> Let  $\tau$  denote the stochastic stopping time of bank closure. Regulators shut down the bank when it violates the regulatory requirements specified below.

**Capital Requirement.** Following Nguyen (2015), Davydiuk (2017), Van den Heuvel (2018), and Begenau (2020), we introduce the capital requirement as follows:

$$\frac{A_t}{K_t} \leq \xi_K. \quad (6)$$

In accordance with Basel III capital standards, banks maintains a minimal ratio of capital to risk-weighted assets of 7%.<sup>16</sup> We set  $\xi_K$  equal to  $1/0.07 = 14.3$ .<sup>17</sup>

<sup>14</sup>Explicitly modeling informational asymmetry would result in a substantially more involved analysis. Lucas and McDonald (1990) provide a tractable analysis under assumption that the informational asymmetry lasts one period. Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs), the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was  $-3\%$  and the loss as a percentage of the new issue size was as high as  $-31\%$  (Eckbo, Masulis, and Norli, 2007).

<sup>15</sup>Impatience induces payout; otherwise, the bank never pays out dividends and always accumulates financial slack (equity) to avoid the refinancing costs. This impatience can be microfounded by an exogenous Poisson exit rate  $\rho - r$ .

<sup>16</sup>See Thakor (2014) for a review of the debate on bank capital and its regulations.

<sup>17</sup>Davydiuk (2017) and Begenau (2020) set  $\xi_K$  to be the sample average of the ratio of Tier 1 equity to risky assets for the reason that banks typically maintain a buffer to prevent regulatory corrective action. In our model, the buffer arises endogenously, so we set  $\xi_K$  to the regulatory threshold. In theoretical studies on banking regulations, De Nicolò, Gamba, and Lucchetta (2014) calibrate the capital requirements to 4% and 12%, Hugonnier and Morellec (2017) calibrate the thresholds to 4%, 7%, 9%, and 20% to investigate the effects of the proposal by Admati and Hellwig (2013), and Phelan (2016) calibrates the threshold to 7.7% and 10.6% in a macroeconomic model.

**Supplementary Leverage Ratio (SLR).** Banks in the U.S. face an SLR requirement since January 1, 2018. It supplements the capital requirement that can be vulnerable to manipulation (Plosser and Santos, 2014). The SLR requirement targets the ratio of total assets (or liabilities) to equity capital. When the bank issues bonds, i.e.,  $B > 0$ , the leverage ratio requirement restricts  $A/K$ , just as the capital requirement does:

$$\frac{A}{K} = \frac{K + X + B}{K} \leq \xi_L; \quad (7)$$

when  $B < 0$ , the SLR requirement is given by

$$\frac{A - B}{K} = \frac{K + X}{K} \leq \xi_L. \quad (8)$$

The U.S. bank holding companies that have been identified as global systemically important banks must maintain an SLR of greater than 5% (i.e.,  $\xi_L = 20$ ), and failing to do so triggers restrictions on the capital distributions to shareholders and discretionary bonus payments to the management.

**Discussion: the Role of Deposit Risk.** The traditional banking models emphasize the illiquidity of bank assets, and the deposit risk manifests itself in a coordination failure and inefficient liquidation of assets (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). To distinguish our model from this literature, we assume that the bank's risky asset,  $A_t$ , is freely adjustable in every instant (i.e., liquid) and the bank can issue bonds so that the bank can always meet deposit withdrawal. Therefore, a bank run does not happen in our model. Here the deposit risk is motivated by the uncertainty in payment flows. As shown in (4), the drift of equity capital ( $\mathbb{E}_t [dK_t]$ ) is a function of the deposit stock,  $X_t$ . Through the randomness in  $X_t$ ,  $\mathbb{E}_t [dK_t]$  becomes a stochastic process. Without the deposit shock, the drift of  $K_t$  would be perfectly controlled by the bank through  $A_t$ ,  $B_t$ , and  $i_t$ , and the bank is only exposed to the risk in lending (i.e.,  $d\mathcal{W}_t^A$ ). Under the deposit shock, the bank faces both shocks to the realized equity growth (i.e.,  $d\mathcal{W}_t^A$ ) and shocks to the expected equity growth (i.e.,  $d\mathcal{W}_t^X$ ). In the next section, we show that under the equity issuance costs, the bank becomes effectively risk-averse, so jointly managing the two types of risks is important.

## 3 Dynamic Banking

### 3.1 Bank Optimization

We derive the optimality conditions for the bank's control variables and the Hamilton-Jacobi-Bellman (HJB) equation for the value function. In the next subsections, we parameterize  $C(n(i_t), X_t)$  and  $n(i_t)$  to provide intuitive characterizations of the bank's optimal policies.

**State and Control Variables.** The bank solves a dynamic optimization problem with two state variables, deposit stock  $X_t$  and equity capital  $K_t$ . Let  $V_t$  denote the shareholders' value at time  $t$ . The bank chooses its loan portfolio size  $A_t$ , its position in bonds  $B_t$ , the deposit rate  $i_t$ , the payout of dividends  $dU_t$ , and the value of newly issued equity  $dF_t$  to maximize the shareholders' value. The value function is a function of the state variables, i.e.,  $V_t = V(X_t, K_t)$ . To solve the bank's optimal decisions and value function, we need the laws of motion of state variables (i.e., (2) and (4)) that show how the control variables affect their evolution. The deposit stock and equity capital are slow-moving state variables that constitute the long-term funds of the bank. Given  $X_t$  and  $K_t$ , the bank's choices of  $A_t$  and  $B_t$  resemble a portfolio problem (Merton, 1969). Let  $\pi_t^A$  denote the portfolio weight on loans, i.e.,  $\pi_t^A (X_t + K_t) = A_t$ , so the weight on bonds is  $(\pi_t^A - 1)$  as implied by the balance-sheet identity (3). We now rewrite the law of motion for  $K_t$  as

$$dK_t = (X_t + K_t) [(r + \pi_t^A \alpha_A) dt + \pi_t^A \sigma_A dW_t^A] - X_t i_t dt - C(n(i_t), X_t) dt - dU_t + dF_t. \quad (9)$$

Given the Markov nature of the bank's problem, we suppress the time subscripts for  $X$ ,  $K$ , and control variables going forward to simplify the notations wherever it does not cause confusion.

The regulatory requirements translate into constraints on the bank's control variables and state variables. If the bank issues bonds (i.e.,  $B > 0$  or  $\pi^A > 1$ ), the capital requirement (6) and SLR requirement (7) are both restrictions on  $A/K$  so the bank faces  $A/K \leq \min\{\xi_K, \xi_L\}$  or

$$\pi^A \leq \min\{\xi_K, \xi_L\} \left( \frac{K}{X + K} \right). \quad (10)$$

If the bank holds bonds (i.e.,  $B \leq 0$  or  $\pi^A \leq 1$ ), the capital requirement (6) is still a restriction on the control variable  $\pi^A$ ,

$$\pi^A \leq \xi_K \left( \frac{K}{X + K} \right), \quad (11)$$

while the SLR requirement, now given by (8) instead of (7), stipulates a boundary in the space of state variables  $X$  and  $K$ ,

$$\frac{X + K}{K} \leq \xi_L. \quad (12)$$

Our numerical solution will show that the bank holds bonds for risk management when its equity capital  $K$  is low relative to its deposit liabilities  $X$ . Therefore, given  $X$ , the bank has to pay the issuance costs and raise equity when  $K$  declines significantly following negative shocks and the SLR requirement (12) binds. In reality, equity issuance may happen before the constraint binds because, once a bank is close to violating the constraint, regulators intervene and often restrict managerial compensation or payout to shareholders. The newly introduced SLR requirement is a boundary condition on the state variables and is an effective a tool to trigger bank recapitalization. In contrast, the traditional capital requirement restricts the control variable  $\pi^A$  (risk-taking).

**The HJB Equation and Boundaries.** When the bank does not pay out dividends ( $dU = 0$ ) or issue equity ( $dF = 0$  and  $dH = 0$ ), the HJB equation for the value function is

$$\begin{aligned} \rho V(X, K) = \max_{\{\pi^A, i\}} & V_X(X, K) X [-\delta_X + n(i)] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \\ & + V_K(X, K) (X + K) (r + \pi^A \alpha_A) + \frac{1}{2} V_{KK}(X, K) (X + K)^2 (\pi^A \sigma_A)^2 \\ & - V_K(X, K) [Xi + C(n(i), X)] + V_{XK}(X, K) X (X + K) \pi^A \sigma_A \sigma_X \phi \end{aligned} \quad (13)$$

The optimality conditions on dividend payout and equity issuance specify the boundaries of  $(X, K)$ , denoted by  $(\bar{X}, \bar{K})$ , the payout boundary and  $(\underline{X}, \underline{K})$ , the equity issuance boundary.

The bank pays out dividends only if the payout value overcomes the decrease of continuation

value, i.e.,  $dU \geq V(\bar{X}, \bar{K}) - V(\bar{X}, \bar{K} - dU)$  or in the differential form,

$$V_K(\bar{X}, \bar{K}) \leq 1. \quad (14)$$

The optimality of payout also requires the following super-contact condition (Dumas, 1991):

$$V_{KK}(\bar{X}, \bar{K}) = 0. \quad (15)$$

The bank raises equity and pays the issuance costs only when the increase of existing shareholders' value after issuance overweighs the new equity investment,  $dF$ , and issuance costs,  $dH$

$$V(\underline{X}, \underline{K} + dF) - V(\underline{X}, \underline{K}) \geq dF + dH, \quad (16)$$

We assume that the issuance costs depend on both the issuance amount and the size of the bank, i.e.,  $dH = \phi_1 dF + \phi_0 \underline{X}$ . We use the deposit base to measure the size of the bank, because, as we will show shortly, the bank's problem has a homogeneity property that significantly simplifies the analysis and allows for an intuitive presentation of our results. Finally, the optimality of  $dF$  also requires the following smooth-pasting condition

$$V_K(X, K) = 1 + \psi_1. \quad (17)$$

Equation (17) states that the marginal value of bank equity is equal to the marginal cost of issuance.

Equations (12) and (14)–(17) define the boundaries of  $(X, K)$  given the value function. The HJB equation (13) solves the value function given the boundary conditions. The solution structure is akin to the dynamic models of corporate liquidity and risk management under equity issuance costs (e.g., Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Décamps, Gryglewicz, Morellec, and Villeneuve, 2017). Note that when characterizing the boundaries of  $(X, K)$ , we do not consider bank closure. In our model, the bank does not default on its debts because under (continuous) diffusive shocks, the bank can and will preserve the positive continuation value for shareholders by immediately adjusting its balance sheet in response

to shocks so that  $\tau = +\infty$  in (5). One implication is that deposits and bonds are risk-free, which is in line with the fact that banking regulators often intervene before banks default on their debts.<sup>18</sup>

**Homogeneity.** We specify the cost of maintaining deposits and managing deposit flows as

$$C(n(i), X) = c(n(i)) X, \quad (18)$$

where  $c(\cdot)$  is an increasing and strictly convex function. Under this functional form and the previous specification of  $dH$ , the bank's optimal choices of  $\pi^A$  and  $i$  become univariate functions of the equity capital-to-deposit ratio,

$$k \equiv \frac{K}{X}, \quad (19)$$

and the bank's value function becomes  $V(X, K) = v(k)X$ . We demonstrate these results as follows. First, given  $V(X, K) = v(k)X$ , we obtain the following derivatives

$$\begin{aligned} V_K(X, K) &= v'(k), \quad V_X(X, K) = v(k) - v'(k)k \\ V_{KK}(X, K) &= v''(k) \frac{1}{X}, \quad V_{XX}(X, K) = v''(k) \frac{k^2}{X}, \quad V_{XK}(X, K) = -v''(k) \frac{k}{X}. \end{aligned} \quad (20)$$

Substituting these expressions into the HJB equation (13) and dividing both sides by  $X$ , we obtain

$$\begin{aligned} \rho v(k) &= \max_{\pi^A, i} [v(k) - v'(k)k] [-\delta_X + n(i)] + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\ &\quad + v'(k) (1+k) (r + \pi^A \alpha_A) + \frac{1}{2} v''(k) (1+k)^2 (\pi^A \sigma_A)^2 \\ &\quad - v'(k) [i + c(n(i))] - v''(k) k (1+k) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (21)$$

Therefore, the  $X$ -scaled HJB equation (21) is an ordinary differential equation (ODE) for the  $X$ -scaled value function,  $v(k)$ . From this equation, we can solve  $\pi^A$  and  $i$  as univariate functions of  $k$ . In the next subsections, we will discuss the implications of these optimal choices in details.

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<sup>18</sup>For example, on November 21, 2008, the FDIC implemented the Temporary Liquidity Guarantee Program that guaranteed newly issued senior unsecured debts and non-interest-bearing transaction accounts at FDIC-insured banks.



The constraints (10) and (11) on  $\pi^A$  translate to

$$\pi^A \leq \min \{ \xi_K, \xi_L \} \left( \frac{k}{1+k} \right) \text{ if } \pi^A > 1, \quad (22)$$

and

$$\pi^A \leq \xi_K \left( \frac{k}{1+k} \right) \text{ if } \pi^A \leq 1, \quad (23)$$

respectively. And the SLR requirement (12) implies a lower boundary of  $k$  when  $\pi^A \leq 1$ :

$$k \geq \underline{k} \equiv \frac{1}{1 - \xi_L^{-1}} - 1 \text{ if } \pi^A \leq 1. \quad (24)$$

When  $k$  is low, our numerical solution features  $B < 0$  (or equivalently,  $\pi^A < 1$ ), so  $\underline{k}$  in (24) is a lower (equity issuance) boundary of  $k$ . Let  $m \equiv dF/X$  denote the (scaled) equity issuance. The equity issuance boundary conditions (16) and (17) are simplified as follows:

$$v(\underline{k} + m) - v(\underline{k}) = \psi_0 + (1 + \psi_1)m, \quad (25)$$

and

$$v'(\underline{k} + m) = 1 + \psi_1. \quad (26)$$

Let  $\bar{k}$  denote the upper (dividend payout) boundary of  $k$ . The payout boundary conditions (14) and (15) can be simplified as follows:

$$v'(\bar{k}) = 1, \quad (27)$$

and

$$v''(\bar{k}) = 0. \quad (28)$$

Our numerical solution of  $v(k)$  is concave, so (27) and (26) imply that the bank pays out dividends when  $k$  is high and raises equity when  $k$  is low, i.e.,  $\bar{k} > \underline{k}$ . Given  $\underline{k}$  in (24), the boundary conditions (25)–(28) and second-order ODE (21) solve the  $X$ -scaled value function,  $v(k)$ , the optimal issuance amount  $m$ , and the upper (dividend payout) boundary  $\bar{k}$ . Note that the amount

of dividend payout is determined as follows: At  $\bar{k}$ , any positive shocks to  $K$  (numerator of  $k$ ) or negative shocks to  $X$  (denominator of  $k$ ) trigger payout, and the payout amount (i.e., the reduction in  $K$ ) is the amount needed to bring down  $k$  to  $\bar{k}$ . In other words,  $\bar{k}$  is a reflecting boundary of  $k$ .

In Appendix A, we provide a richer setup where, as in Drechsler, Savov, and Schnabl (2018), the bank holds reserves and is subject to a regulatory reserve requirement. In this richer setup, our results on the value of deposits and the optimal strategies of payout, equity issuance, risk-taking, and deposit rate still hold.<sup>19</sup> The only difference is that the reserve requirement generates another lower bound for  $k$ . Therefore, the bank raises equity to meet either the SLR requirement binds (i.e., at  $\underline{k}$  given by (24)) or the reserve requirement binds. After the financial crisis, the liquidity coverage ratio requirement replaces the role of reserve requirement with a more broadly defined set of assets that can be easily traded intraday to settle interbank payments and other liquidity needs.<sup>20</sup>

### 3.2 The Main Mechanism: Equity Risk and Return

Under the equity issuance costs, the bank is effectively averse to risk in equity capital because, when negative shocks deplete equity capital, the bank has to incur issuance costs and raise equity. To analyze the risk-return trade-off, we use the balance-sheet identity (3) to substitute out bond financing,  $B_t$ , in the law of motion (4) of equity capital, and use (18), i.e.,  $C(n(i_t), X_t) = c(n(i_t)) X_t$ , to simplify the law of motion, so, in the interior region (where  $dU_t = 0$  and  $dF_t = 0$ ),

$$\frac{dK_t}{K_t} = rdt + \frac{A_t}{K_t} (\alpha_A dt + \sigma_A d\mathcal{W}_t^A) + \frac{X_t}{K_t} \underbrace{[r - i_t - c(n(i_t))]}_{\text{net deposit spread}} dt. \quad (29)$$

The first and second terms on the right side are standard in portfolio problems (Merton, 1973). The bank's net worth (equity capital) grows at a base rate  $r$  through the first term, while the excess return  $\alpha_A$  from risky investments  $A_t$  comes with an additional risk ( $\sigma_A$ ) per dollar invested. The last term shows how deposit-taking contributes to return on equity. The bank may set a deposit rate  $i_t$  below  $r$ , earning a positive interest spread. And, the net deposit spread,  $r - i_t - c(n(i_t))$ ,

<sup>19</sup>Our solution of optimal reserve holdings resembles the classic money demand (Baumol, 1952; Tobin, 1956).

<sup>20</sup>Liquidity requirement in our model triggers costly equity issuance and thus its role is different from that in models that emphasize the illiquidity of bank assets (Diamond and Kashyap, 2016; Carletti, Goldstein, and Leonello, 2019).

reflects the cost of running the deposit franchise (Drechsler, Savov, and Schnabl, 2017).

Profits from deposit-taking are not risk-free. The expected growth rate of equity capital,  $\mathbb{E}_t[dK_t/K_t]$ , is a function of the deposit stock  $X_t$  that evolves randomly (see (2)). The deposit shocks transmit into equity dynamics, with the net deposit spread as a multiplier. When the equity capital-to-deposit ratio,  $k_t = K_t/X_t$ , is low, the variation of  $X_t$  is large relative to  $K_t$ , so that the deposit risk has a significant impact on equity dynamics. When  $k_t$  is high, the impact of deposit risk is muted. Therefore,  $k_t$  is the key state variable that drives the dynamic balance-sheet management. With a higher deposit base,  $X_t$ , deposit inflow shocks force the bank to earn more through the net deposit spread and to bear more risk in the expected growth rate of equity. From bank shareholders' perspective, whether such involuntary expansion in both return and risk is desirable depends on  $k$ . When  $k$  is low, deposit inflows may have an overall negative impact on bank shareholders' value and force the bank to become more cautious (even scale back risky lending).

Net interest margin—the spread between loan rate and deposit rate—is often used as a profitability measure. It can be decomposed into the asset-side and liability-side (deposit) components (Egan, Lewellen, and Sunderam, 2017). To earn the excess asset return  $\alpha_A$ , the bank has to increase its exposure to the asset shock,  $d\mathcal{W}_t^A$ , which affects realized equity growth. Similarly, while earning a net deposit spread the bank also loads on the deposit shock,  $d\mathcal{W}_t^X$ , which affects expected equity growth. Under equity issuance costs, the bank balances the two sources of profits and risks.

### 3.3 Optimal Risky Investment

From the  $X$ -scaled HJB equation (21), we can solve  $\pi^A$ . Using  $\frac{A}{K} = \frac{\pi^A(X+K)}{K} = \pi^A \left(\frac{1+k}{k}\right)$ , we obtain the following formula for the risky asset-to-capital ratio (within the regulatory constraints):

$$\frac{A}{K} = \frac{\alpha_A}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \quad (30)$$

In (30),  $\gamma(k)$  is a measure of endogenous risk aversion based on the value function:

$$\gamma(k) \equiv -\frac{v''(k)k}{v'(k)}, \quad (31)$$

This solution resembles Merton's portfolio choice including both the mean-variance term and the hedging-demand term. In the numerator, a higher excess return,  $\alpha_A$ , increases lending. The bank's risk-taking is state-dependent through  $\gamma(k)$ . Even though the bank evaluates the equity-holders' payoffs with a risk-neutral objective in (5), it is endogenously risk-averse, i.e.,  $\gamma(k) > 0$ , due to the equity issuance costs. When  $\gamma(k)$  is low, the bank chooses a high loan-to-capital ratio; when  $\gamma(k)$  is high, the bank reduces its risk exposure. Our numeric solution show  $\gamma'(k) < 0$ .

The bank's incentive to lend is also strengthened when deposits are natural hedges when the asset-side shock,  $dW^A$ , and the liability-side (deposit) shock,  $dW^L$  are positively correlated ( $\phi > 0$ ). The risk of deposit flow is essentially the bank's background risk from the perspective of portfolio management. When  $\phi > 0$ , it captures the synergy between lending and deposit-taking that has been studied extensively in the literature (e.g., Calomiris and Kahn, 1991; Berlin and Mester, 1999; Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006; Hanson, Shleifer, Stein, and Vishny, 2015). This hedging mechanism also echoes the finding of Drechsler, Savov, and Schnabl (2021) that financing lending with deposits helps banks to hedge risk.<sup>21</sup>

### 3.4 Optimal Deposit Rate

When the bank increases the deposit rate by 1, it obtains new deposits with the marginal value equal to  $V_X(X, K) X n'(i)$ , but it also reduces the return on equity capital through higher interest payments on the existing deposits, which is valued at  $V_K(X, K) X$ , and through the marginal cost of maintaining a larger deposit franchise,  $V_K(X, K) X c'(n(i)) n'(i)$ . The optimal deposit rate is implicitly defined by the condition that the marginal benefit is equal to the marginal cost:

$$V_X(X, K) n'(i) X = V_K(X, K) [X + X c'(n(i)) n'(i)] . \quad (32)$$

Rearranging the equation, we obtain:

$$c'(n(i)) = \frac{V_X(X, K)}{V_K(X, K)} - \frac{1}{n'(i)} = \frac{v(k) - v'(k)k}{v'(k)} - \frac{1}{n'(i)} . \quad (33)$$

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<sup>21</sup>The finding of Drechsler, Savov, and Schnabl (2021) focuses on interest-risk risk.

Because  $c(\cdot)$  is strictly convex and  $n'(i) > 0$ , the optimality condition (33) implies that the optimal deposit rate increases in the ratio of marginal value of deposits to marginal value of equity capital  $\frac{V_X(X, K)}{V_K(X, K)}$ . Intuitively, when deposits are more valuable relative to equity capital, the bank is willing to raise  $i_t$ , sacrificing return on equity for deposit-taking. Moreover, when deposit flow is more responsive to deposit rate, i.e.,  $n'(i)$  is high, the bank sets a higher  $i_t$ .

Since deposits are at the core of our model, we sharpen the intuitions about the optimal deposit rate by adopting the following functional forms. First, we specify  $n(i)$  as a linear function:

$$n(i) = \omega i, \quad (34)$$

where, as shown in (2),  $\omega$  is the semi-elasticity of deposits stock  $X$  with respect to  $i$ . Next, we specify the cost of attracting new deposits in a simple quadratic form

$$c(n(i)) = \frac{\theta}{2} n(i)^2. \quad (35)$$

These functional forms lead to a Hayashi style optimal policy for the deposit rate. In Hayashi (1982), firms make investments in productive capital, while, in our model, the bank attracts depositors by raising the deposit rate, building up its customer capital. Using (33), we obtain

$$i = \frac{\frac{V_X(X, K)}{V_K(X, K)} - \frac{1}{\omega}}{\theta \omega} = \frac{\left( \frac{v(k) - v'(k)k}{v'(k)} \right) - \frac{1}{\omega}}{\theta \omega}. \quad (36)$$

The difference between our optimal deposit-rate policy and Hayashi's investment policy is two-fold. First, it is not a single Tobin's  $q$  that dictates the optimal decision but rather the ratio of marginal deposit  $q$ ,  $V_X(X, K)$ , to marginal equity  $q$ ,  $V_K(X, K)$  that drives the optimal deposit rate.<sup>22</sup> Second, through the ratio  $\frac{V_X(X, K)}{V_K(X, K)}$ , our optimal deposit rate is state-dependent.

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<sup>22</sup>Related, Bolton, Chen, and Wang (2011) find the ratio of marginal value of productive capital to the marginal value of cash drives a firm's investment under adjustment costs. Kargar, Passadore, and Silva (2020) find the ratio of marginal value of a subset of assets to the marginal value of wealth drives portfolio decisions under transaction costs.

An interesting feature of the optimal deposit rate is that it hits the zero lower bound when

$$\frac{V_X(X, K)}{V_K(X, K)} = \frac{v(k) - v'(k)k}{v'(k)} \leq \frac{1}{\omega}. \quad (37)$$

Once the deposit rate reaches zero, the bank cannot further decrease the deposit rate to reduce deposits. Later we show that this restriction makes deposits undesirable, especially when the bank is undercapitalized, and thus, is concerned of a high leverage from large deposits that amplifies the impact of negative shocks on equity and increases the likelihood of costly equity issuance.

When the deposit demand is more elastic, i.e.,  $\omega$  is high, the bank has to pay a higher deposit rate, as shown in (36). However, given the value function, it is less likely for the condition (37) to hold, because a high demand elasticity allows the bank to control the deposit flow more effectively and thereby to avoid hitting the zero lower bound. This result suggests that the deposit-rate lower bound is more acute a problem for larger banks with greater deposit market power or stickier deposit base (i.e., smaller  $\omega$ ). Smaller banks with less deposit market power are less concerned of the deposit-rate lower bound, but they have to pay higher interest rates to attract depositors.

## 4 Quantitative Analysis

### 4.1 Functional Form and Parameter Choices

For the functional forms of  $n(\cdot)$  and  $c(\cdot)$ , we use (34) and (35) respectively.<sup>23</sup> In Table 4 we report our calibration and parameter choices. We set the unit of time to year and  $r$  to 1% in line with the average Fed funds rate in the last decade. Shareholders' discount rate  $\rho$  is set to 4.5% in line with the commonly used value in dynamic corporate finance models.<sup>24</sup> We set  $\alpha_A$  to 0.2% so that the model generates an average return on assets (ROA) of 1.05%, close to the average ROA of US banks in the last decade (source: FRED). Note that when  $k$  is large, the bank only holds risky assets

<sup>23</sup>We also experiment with an alternative specification of quadratic  $n(i)$  that allows the deposit flow to be increasingly sensitive to deposit rate as  $i$  approaches zero. The results are very similar and are available upon request.

<sup>24</sup>One example is Bolton, Chen, and Wang (2011). This is also consistent with the dynamic contracting literature (DeMarzo and Fishman, 2007; Biais, Mariotti, Plantin, and Rochet, 2007).

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. The unit of time is year.

Parameters	Symbol	Value
risk-free rate	$r$	1%
discount rate	$\rho$	4.5%
bank excess return	$\alpha_A$	0.2%
asset return volatility	$\sigma_A$	10%
deposit flow (mean)	$\delta_X$	0
deposit flow (volatility)	$\sigma_X$	5%
deposit maintenance cost	$\theta$	0.5
deposit demand semi-elasticity	$\omega$	5.3
corr. between deposit and asset shocks	$\phi$	0.8
equity issuance fixed cost	$\psi_0$	0.1%
equity issuance propositional cost	$\psi_1$	5.0%
SLR requirement parameter	$\xi_L$	20
capital requirement parameter	$\xi_K$	14.3

(and the asset value is  $A_t$ ), but when  $k$  is small, the bank also holds risk-free assets ( $B < 0$ ) and the asset value is  $A_t - B_t$ . Therefore, the ROA is state-dependent. To calculate the average ROA and other averages later, we use the stationary distribution of  $k$ . We set the asset return volatility,  $\sigma_A$ , to 10% as in Sundaresan and Wang (2014) and Hugonnier and Morellec (2017).<sup>25</sup>

For deposit dynamics, we set  $\delta_X$  to 0% and  $\sigma_X$  to 5% following Bianchi and Bigio (2014). We further set  $\omega$  to 5.3, the semi-elasticity of deposits to the deposit rate from Drechsler, Savov, and Schnabl (2017). The correlation between asset-side and liability-side (deposit) shocks,  $\phi$ , directly affects  $A/K$  in (30) and is set to 0.8 so that the (stationary) probability of a binding capital requirement is in line with the evidence (Begenau, Bigio, Majerovitz, and Vieyra, 2019). As for the cost of maintaining deposit franchise, we set the maintenance cost parameter,  $\theta$ , to 0.5. With this value, the model generates an average deposit-to-total liabilities ratio equal to 96% in line with the evidence (Drechsler, Savov, and Schnabl, 2017). We set the proportional issuance cost parameter,  $\psi_1$ , to 5% (Boyson, Fahlenbrach, and Stulz, 2016). The fixed cost parameter,  $\psi_0$ , is set to 0.1%,

<sup>25</sup>Sundaresan and Wang (2014) in turn refer to the calculation of Moody's KMV Investor Service.

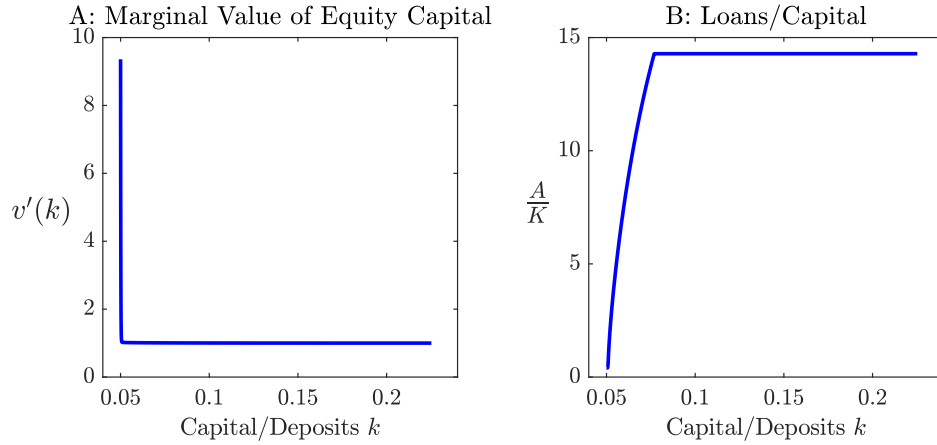


Figure 1: Marginal Value of Equity Capital and Bank Risk-Taking.

so the model generates an issuance-to-equity ratio of 1% roughly in line with the evidence (Baron, 2020).<sup>26</sup> The regulatory parameters were discussed in Section 2.

## 4.2 Marginal Value of Equity Capital and Risk-Taking

The marginal value of equity capital,  $V_K(K, M) = v'(k)$ , should be equal to one without financial frictions because the bank is indifferent between paying out one dollar and retaining one dollar of earnings. In other words, precautionary savings do not add value without financial frictions. Under the equity issuance costs, the marginal value of equity capital can be above one, and the wedge between  $v'(k)$  and one widens as the bank approaches the boundary of equity issuance. Panel A of Figure 1 plots the marginal value of equity capital,  $v'(k)$ , against the equity capital-to-deposit ratio,  $k$ . At the equity issuance boundary of  $k$ ,  $\underline{k}$ , a value of  $v'(k)$  close to nine means that one dollar of equity is worth nine dollars because of the imminence of costly equity issuance.<sup>27</sup>

The interior region ends at the endogenous payout boundary  $\bar{k}$ . At that point the the marginal value of equity capital is equal to one and bank has a sufficient amount of retained earnings, so that it is optimal to pay out dividends to shareholders as they discount cash flows at a higher rate  $\rho$

<sup>26</sup>The 1% is calculated across simulated issuance events. Baron (2020) documents a cross-sectional average of 0.5% (as of 2005) and, in the sample, 50% banks did not issue equity, so we double the number as our calibration target.

<sup>27</sup>The proportional cost is only 5%, but due to the fixed cost, the marginal value of equity is much higher than 1.05.



than  $r$ . Note that near the payout boundary,  $\bar{k}$ , the marginal value of equity capital is close to one and relatively insensitive to variations in  $k$  because, at that point, the likelihood of a large loss of equity or a large deposit inflow that dramatically decrease  $k$  to the equity issuance boundary  $\underline{k}$  is low. In other words, distress in the form of costly equity issuing is a distant scenario near  $\bar{k}$ .

Throughout the whole region of  $k$ , the marginal value of equity capital stays positive, which implies that when the bank accumulates more equity capital, *ceteris paribus*, the bank shareholders' value increases. This is in line with the empirical findings of Mehran and Thakor (2011) and Minton, Stulz, and Taboada (2019) that bank value is positively associated with bank capital. In the next subsection, we examine the marginal contribution of deposits to bank shareholders' value and discuss further the implications of our model on empirical analysis of bank valuation (Atkeson, d'Avernas, Eisfeldt, and Weill, 2019). Moreover, our model predicts that the bank pays dividends when equity capital is high relative to its deposit liabilities and raises equity when equity capital is low. The procyclical payout and countercyclical equity issuance are consistent with the evidence on bank equity management (Adrian, Boyarchenko, and Shin, 2015; Baron, 2020).

As shown in the solution of optimal loan-to-capital ratio,  $A_t/K_t$ , given by (30), the marginal value of equity capital directly drives the bank's risk-taking behavior through  $\gamma(k)$ , the bank's endogenous relative risk aversion defined in (31). Panel A of Figure 1 suggests that  $\gamma(k)$  decreases in  $k$ , because as  $k$  increases, the concavity of bank value in equity capital subdues quickly and, as  $k$  approaches  $\bar{k}$  (the payout boundary), bank value is almost linear in  $k$  with  $v'(k)$  close to one. Indeed, in Panel B of Figure 1, we show that the loan-to-capital ratio increases in  $k$ . The bank cannot exceed the capital requirement (i.e.,  $A/K \leq \xi_K = 14.3$ ), but it can expand its balance sheet up to that limit. Our model predicts that risk-taking is procyclical, in line with the evidence that distressed banks decrease observable measures of riskiness (Ben-David, Palvia, and Stulz, 2020). In Appendix C, we include jump risk in the asset return. The jump risk makes the increase of  $A/K$  in  $k$  much smoother as it motivates the bank to be more cautious even as  $k$  increases.

Figure 1 reports the marginal value of equity capital and optimal loan-to-capital ratio given any value of  $k$ . To understand the long-run behavior of this model (i.e., how much time the bank spends in different regions of  $k$ ), we examine the stationary density of  $k$ . Panel A of Figure 2 plots

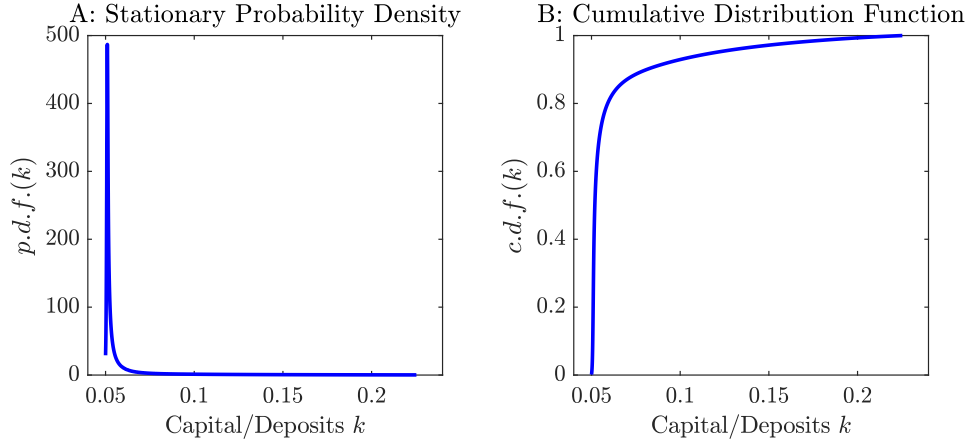


Figure 2: Stationary Probability Density and Cumulative Distribution Function of  $k$ .

the stationary probability density of  $k$  and Panel B plots the corresponding cumulative distribution function (c.d.f.). While the probability mass is concentrated in the area where  $k$  is near the lower boundary  $\underline{k}$ , the marginal value of equity capital is only slightly above one (1.02) where the density function peaks. However, even if for the majority of time the bank does not seem to be financially constrained, the shadow value of equity rises dramatically when equity is depleted relative to the bank's deposit liabilities and  $k$  approaches  $\underline{k}$ , the boundary of costly equity issuance, as shown in Panel A of Figure 1. These results illustrate the sharp contrast between normal times, when the bank is comfortably meeting its total leverage requirement, and crisis times, when it is in danger of violating the leverage requirement and triggering equity issuance.

With the stationary distribution of the key state variable  $k$ , we now report the model predictions on the distribution of marginal value of equity capital and loan-to-capital ratio. In Panel A of Figure 3, we plot the marginal value of equity capital against the stationary c.d.f. of  $k$  (note  $c.d.f.(\underline{k}) = 0$  and  $c.d.f.(\bar{k}) = 1$ ). A interval on the horizontal axis represents the fraction of time that the bank spends in the corresponding region of  $v'(k)$  on the vertical axis. For example, the bank spends 25% of the time with its marginal value of equity between 1.019 and 1.022. The bank spends less than 5% of the time in the region where it is in danger of violating the leverage requirement with  $v'(k)$  above 1.08. In other words, crisis states are rare but they cast a long shadow

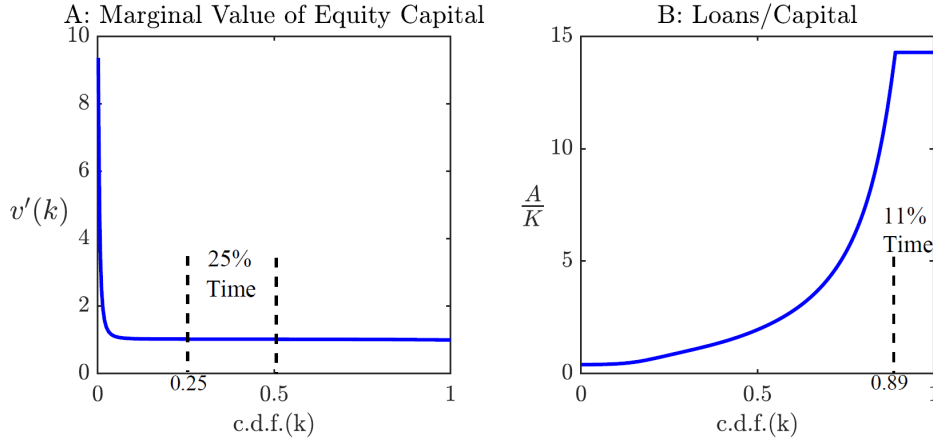


Figure 3: Long Run Distribution of Marginal Value of Equity Capital and Loan-Capital Ratio.

over the bank’s management of its balance sheet. As the bank becomes better capitalized relative to its deposit liabilities (as  $k$  increases), the marginal value of equity declines dramatically, so that the bank value is concave in equity and the bank is endogenously risk averse.

In Panel B of Figure 3, we plot the optimal loan-to-capital ratio,  $A_t/K_t$ , against the stationary c.d.f. of  $k$ . We show that capital requirement binds about 11% of the time (the horizontal part of the curve on the right end). Capital requirement becomes relevant when the bank is well-capitalized and the risk-taking incentive is strong. Such procyclicality suggests that capital requirement can act as a macroprudential tool as suggested by Gersbach and Rochet (2017). In contrast, the SLR requirement motivates the bank to replenish equity capital in bad times when its equity capital is low relative to its deposit liabilities (see (24)). While capital requirement and SLR requirement play distinct roles in our model, they both contribute to a form of parity between risk and capital with the former restricting risk-taking given equity capital and the latter triggering capital raising.

### 4.3 Deposit Marginal $q$

Bank value depends on equity capital,  $K$ , and deposit stock,  $X$ . Panel A of Figure 4 plots the marginal value of deposits (“deposit marginal  $q$ ”),  $V_X(X, K) = v(k) - v'(k)k$ . When the bank has ample capital relative to deposits, i.e., when  $k$  is large, deposit marginal  $q$  is positive. However, it

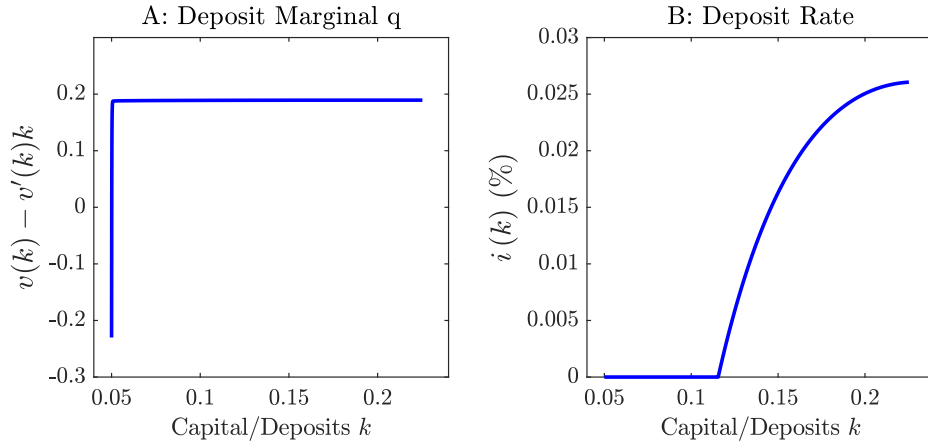


Figure 4: Deposit Marginal  $q$  and Deposit Rate.

turns sharply negative when  $k$  nears the lower boundary of costly equity issuance.

Deposits create value by allowing the bank to finance risky lending with relatively cheap sources of funds. Therefore, deposit stock serves as a form of productive capital for the bank. Intuitively, when the bank becomes better capitalized, it raises deposit rate to attract more deposits for more risky lending. Panel B of Figure 4 shows that the deposit rate increases in  $k$  as the loan-to-equity ratio does in Panel B of Figure 1. The positive comovement of loan growth and deposit rate increase is consistent with the finding of Ben-David, Palvia, and Spatt (2017).

A key finding is that deposit marginal  $q$  declines sharply and can turn negative when the bank's equity capital is low relative to its deposit liabilities.<sup>28</sup> The reason is that when  $k$  is near the equity issuance boundary,  $\underline{k}$ , deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage that amplifies the impact of shocks on equity capital and makes the costly equity issuance more likely. The bank may want to delever, turning away deposits by lowering the deposit rate. However, as shown by Panel B of Figure 4, doing so has a limit, that is the zero lower bound of deposit rate. In practice, banks are reluctant to impose negative deposit rate on depositors. Consistent with our zero lower bound on the deposit rate, Heider, Saidi, and

<sup>28</sup>In Appendix C, we show that the jump risk in bank asset return makes the dynamics of deposit marginal  $q$  and deposit rate smoother near the equity issuance (lower) boundary of  $k$ .

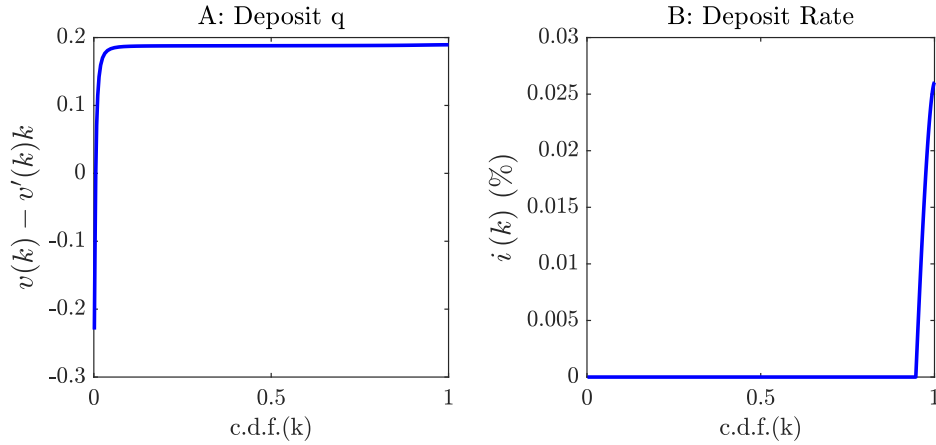


Figure 5: Long Run Distribution of Deposit Marginal  $q$  and Deposit Rate.

Schepens (2019) find that the distribution of deposit rates of euro-area banks is truncated at zero.<sup>29</sup>

In Figure 5, we plot deposit marginal  $q$  and optimal deposit rate against the stationary c.d.f. of  $k$ . Deposit marginal  $q$  is positive and larger than 0.185 in 81% of the time, but near the lower boundary of costly equity issuance (i.e.,  $c.d.f.(k) = 0$ ), it can drop to  $-0.23$ . The deposit rate hovers around the lower bound at zero, showing that the bank is very conservative in deposit-taking. The deposits attracted by high rate now is helpful in financing lending (i.e., earning  $\alpha_A$ ) but can become burdensome when negative shocks deplete bank equity capital and  $k$  declines. However, for a bank with sufficiently strong balance sheet, i.e., a higher value of the capital-to-deposit ratio  $k$ , the bank is willing to offer more attractive deposit rate to attract depositors.

Deposits are very different from short-term debt. For short-term debt, the bank can continuously and freely adjust its debt level, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who hold accounts at other banks. As long as depositors are willing to hold deposits, the bank cannot turn away the existing depositors. Moreover, the bank must accept any deposit inflow unconditionally, for example, when a depositor receives a payment or deposits cash. Therefore, after hitting the zero lower bound, the bank can no longer de-

<sup>29</sup>Moreover, when the ECB lowers the policy rate, more deposit rates bunch at zero.

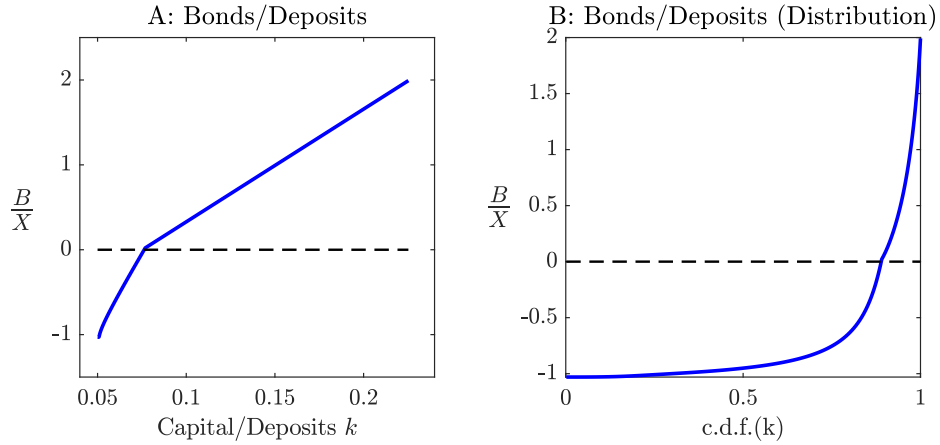


Figure 6: Bank Debt Structure

crease its deposit rate further to reduce deposit inflow and thus loses control of its leverage. When the bank is sufficiently close to incur costly equity issuance (i.e.,  $k$  is close to  $\underline{k}$ ), the marginal value of deposits turns sharply negative.

Figure 6 analyzes the the bank's debt structure. Panel A plots the ratio of short-term debts to deposits,  $B/X$ , against  $k$  and Panel B plots this ratio against the stationary c.d.f. of  $k$  to how much time the bank spends in different regions of  $B/X$ . When capital is abundant relative to deposits, the bank raises funds from short-term debts for risky lending, i.e.,  $B_t > 0$  when  $k$  is high. As  $k$  increases, the bank becomes increasingly reliant on short-term debt as the source of financing instead of deposits. The substitution from deposits to short-term debts reflects the bank's concern over the lack of control over deposit liabilities and the bank's preference for more controllable short-term debts in spite of higher debt costs. In our solution, the deposit rate is below the cost of short-term debt or the risk-free rate  $r$  (1%) (see Panel B of Figure 4). This result captures the bank's incentive to avoid deposit risk. Deposit risk management is a unique feature of our model and is distinct from the standard loan risk management (dictated by the Merton-style formula (30)).

Panel A of Figure 6 also shows that when the bank's equity capital is low relative to its deposit liabilities, the bank holds risk-free bonds to reduce the overall riskiness of its asset portfolio, i.e.,  $B_t < 0$  when  $k$  is low. To avoid incurring the equity issuance costs, the bank manages its

exposure to both asset-return risk and deposit risk. When  $k$  declines, the optimal deposit rate approaches the lower bound. Once the deposit rate hits the lower bound, the bank loses control of its deposit liabilities and can no longer manage deposit risk. Therefore, the bank focuses on reducing its exposure to asset-return risk, and doing so requires holding safe assets (i.e.,  $B_t < 0$ ).

Our model reveals a new channel of safe asset demand. Undercapitalized banks demand safe assets because deposits serve as means of payments and the uncertainty in payment flows translates into deposit risk. Copeland, Duffie, and Yang (2021) provide recent evidence on such safe-asset demand of banks. Our model shows that the demand is particularly strong in crises when banks are undercapitalized and close to violating the SLR requirement. The government is in a unique position to supply safe assets. In a general equilibrium setting where the interest rate  $r$  is endogenous, banks' demand for safe assets is likely to push down  $r$ , reducing the government's financing cost. The government can take advantage of a lower borrowing cost, issuing more debts to meet the banks' demand and using the proceeds to stimulate the economy.

Empirically, we often observe banks holding safe assets and simultaneously issuing short-term debts. In Appendix A, we follow Drechsler, Savov, and Schnabl (2018) to incorporate the bank's need to hold reserves and other liquid assets under payment settlement frictions (Furfine, 2000; Bech and Garratt, 2003; Ashcraft, McAndrews, and Skeie, 2011; Bianchi and Bigio, 2014). This additional feature distinguishes safe assets and the bank's own short-term debts.

## 5 Leverage Regulation

The supplementary leverage ratio (SLR) is the U.S. implementation of the Basel III Tier 1 leverage ratio. The SLR, which does not distinguish between assets based on risk, is conceived as a backstop to risk-weighted capital requirements. In our model, the SLR plays the critical role of pinning down the (lower) boundary of equity issuance for the state variable,  $k$ . In contrast, capital requirement imposes a restriction on the control variable,  $\pi^A$ , the loan-risk exposure, as previously discussed.

In response to the crisis provoked by the Covid-19 pandemic, U.S. banking regulators relaxed the SLR requirement. Jerome Powell, the Federal Reserve Chairman, emphasized that the SLR is

straining banks' ability to handle large deposit inflows. “*Many, many bank regulators around the world have given leverage ratio relief,*” Powell said at a news conference following an FOMC meeting. “*What it’s doing is allowing [banks] to grow their balance sheet in a way that serves their customers.*”<sup>30</sup> To shed light on this decision, we examine the effects of relaxing the SLR requirement on bank balance-sheet management and valuation.

Relaxing the SLR increases the deposit marginal  $q$ , helping the bank to absorb deposit influx like the one we saw during the Covid-19 pandemic. However, if the deposit influx sustains, the deposit marginal  $q$  can fall below the level prior to the relaxed SLR.

Relaxing the SLR also stimulates lending immediately by reducing the likelihood of costly equity issuance in the near term, but contrary to the conventional wisdom, it leads to less risk-taking over the long run. Tightening the SLR has the opposite effect: It discourages the bank from risk-taking immediately by making costly equity issuance more imminent, but over the long run, the bank becomes more aggressive in taking risk. The key is that equity issuance costs generate a reach-for-yield incentive. To compensate its shareholders for the occasionally incurred issuance costs, the bank needs to maintain its return on equity over the long run. Relaxing (tightening) the SLR decreases (increases) the frequency of costly equity issuances and thus causes the bank to boost (reduce) return on equity via risk-taking. Next we provide more details in these results.

## 5.1 Lending and Risk-Taking

The bank must raise equity and incur issuance costs in order to stay in compliance with the leverage requirement, as shown in (24). In our model, the cost of financial distress or undercapitalization is in the form of equity issuance costs instead of bankruptcy costs. Given that the bank only faces small diffusive shocks, it can avoid insolvency by adjusting its balance sheet continuously, but when  $k = K/X$  hits the lower boundary  $\underline{k}$  – for example after an unexpected deposit inflow that increases  $X$  – the bank must raise equity. This is a realistic approach as in practice, bank insolvency is relatively rare, and recapitalization is often triggered by regulatory intervention.

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<sup>30</sup>See “Fed’s Powell makes case why Congress should relax bank capital rule” by Hannah Lang, American Banker July 29, 2020. The SLR requirement also affected the functioning of Treasury market (He, Nagel, and Song, 2021).



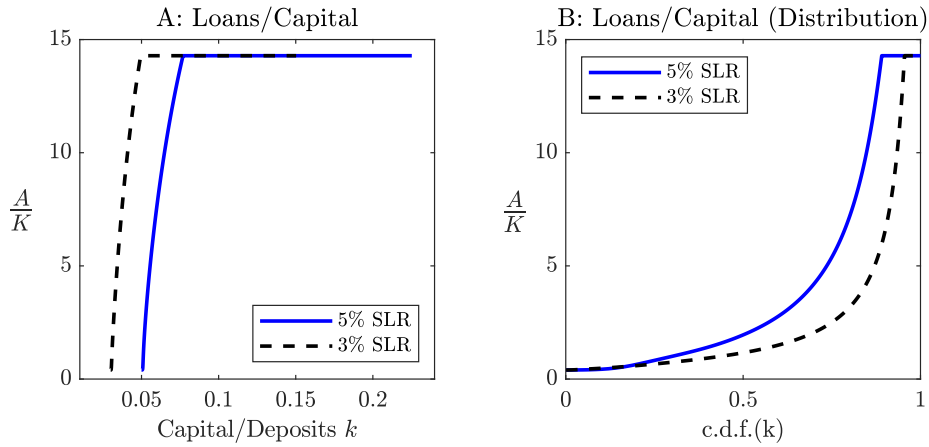


Figure 7: The Impact of Relaxing the SLR Requirement on Bank Lending

Relaxing the SLR lowers  $\underline{k}$  so that given the value of  $k$ , i.e., the current balance-sheet status, costly equity issuance becomes a more distant event. A reduced likelihood of paying the equity issuance costs makes the bank less risk-averse and thereby stimulates lending as shown in Panel A of Figure 7 where we compare the loan-to-equity capital ratio,  $A/K$ , under the SLR requirement equal to 5% (the baseline value) and 3% (the dashed line). Given  $k$ ,  $A/K$  is higher when the SLR requirement is lower. In both cases,  $A/K$  peaks at the level given by the risk-based capital requirement (6). Note that it is not the SLR that causes risk aversion. Even without it, the bank still has to raise equity when  $k$  falls to zero. It is costly but optimal to do so since the continuation value is positive. The SLR simply pushes the equity issuance boundary  $\underline{k}$  above zero.

Many are concerned that relaxing leverage regulations will cause the bank to take on more risks over the long run.<sup>31</sup> Consistent with this intuition, Panel A of Figure 7 shows that the payout and equity issuance boundaries both shift leftward after the regulatory change: Relaxing the SLR requirement makes the bank less risk-averse and maintain less equity (relative to deposit liabilities). However, this does not necessarily imply a higher risk exposure per unit of equity capital over the long run as shown in Panel B. Drawing the distinction between Panels A and B is important for

<sup>31</sup>When discussing the relaxation of SLR requirement, Fed chairman Powell emphasized that “This will not be a permanent change in capital standards.” (see “Fed’s Powell makes case why Congress should relax bank capital rule” by Hannah Lang, American Banker July 29, 2020).

understanding the result. Panel A shows the impact of relaxing the SLR requirement given  $k$ , which summarizes the current state of balance-sheet conditions of the bank. The move from the solid line to the dashed line mimics the *immediate effect* of regulatory change. In contrast, Panel B shows the *long-run effect*. The plot of  $A/K$  against the stationary c.d.f. of the state variable  $k$  shows how much time the bank spends (horizontal axis) at different values of  $A/K$  (vertical axis). Quite contrary to conventional wisdom, relaxing the SLR actually leads to a smaller risk exposure per unit of equity capital over the long run as the dashed line is below the solid line in Panel B.

Every time the bank raises equity it pays the issuance costs. Therefore, over the long run the bank must generate sufficient earnings to offset these costs. Relaxing the SLR requirement reduces the frequency of costly equity issuance, so the amount of earnings that the bank needs to generate declines. Therefore, the bank becomes less aggressive in earning the excess asset return,  $\alpha_A$ .

By the same logic, tightening leverage regulations can actually lead to more aggressive risk-taking over the long run, as it means more frequent equity issuance. The bank has to engage in more risk-taking per unit of equity to generate earnings (return on equity) that offset issuance costs. Equity issuance costs generate a reach-for-yield incentive. Thus, tightening the SLR achieves the purpose of incentivizing the bank to maintain more equity over the long run but fails to tame risk-taking per unit of equity. The mechanism captures the real-world bankers' focus on return on equity and is similar to the channel of financial instability in Li (2019).<sup>32</sup>

In 2021, the U.S. banking regulators restored the SLR requirement to the pre-pandemic level. Through the lens of our model, the policy change incentivizes banks to expand risky lending over the long run but to scale back risky lending in the short run. From the perspective of impulse response, our model predicts that banks' incentive to lend declines immediately and then rises over time and settles eventually at a higher level. Therefore, the impact of this policy change on economic recovery depends on how fast the demand for bank credit rebounds. If the credit demand recovers slowly, the response of banks' credit supply may very well match the demand. However, if the credit demand rebounds sharply, for example, triggered by a speedy reopening

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<sup>32</sup>Li (2019) presents a model of financial instability induced by government debt where the supply of government-issued money-like securities (e.g., Treasury bills) squeezes banks' profits from issuing money-like securities, so banks become more aggressive in risk-taking to sustain earnings that can offset the costs of issuing equity over the long run.

of the economy under the availability of effective vaccines against Covid-19, then the response of banks' credit supply may lag the demand and slow down the overall economic recovery.

**Discussion: SLR and capital requirement.** When  $k$  is low and  $B < 0$ , the SLR requirement sets the lower (equity issuance) bound on the state variable  $k$  (see (8)). When  $k$  is high and  $B > 0$ , the SLR requirement becomes a restriction on the control variable, loan-to-equity capital ratio (see (7)), just as the risk-based capital requirement does (see (6)). Under the current parameter values, the capital requirement binds before the SLR when  $k$  is high, so in our model, the two regulations play distinct roles: The SLR pins down the lowest amount of equity capital relative to deposits, i.e., the lower bound of  $k$ , and the capital requirement restricts the risk exposure per unit of equity capital. This seems to suggest that risk-based capital requirements are more direct in taming risk-taking than the SLR. However, this conclusion relies on an important assumption that the riskiness of loans, given by the parameter  $\sigma_A$ , is time-invariant. When loan risk is countercyclical, risk-based capital requirements amplify the procyclicality of bank risk-taking (Repollo and Suarez, 2012). Moreover, risk weights are vulnerable to manipulation (Plosser and Santos, 2014). Because our model is designed to focus on deposit risk and the bank's imperfect control of balance-sheet size and composition, we do not include the possibility of equilibrium bank failures in our model and the associated externalities that motivate both the leverage and risk-based capital requirements. Therefore, our analysis does not aim to provide a comprehensive evaluation of banking regulations.

## 5.2 Deposit Marginal q and Deposit Rate

One key motivation for relaxing the SLR during the Covid-19 pandemic is allowing banks to accommodate the unprecedented deposit inflows without concerns over violating regulatory constraints. In Panel A of Figure 8, we plot the marginal value of deposits,  $V_X(X, K) = v(k) - v'(k)k$ , before (solid line) and after (dashed line) the SLR requirement is reduced. To see the model predictions, pick any value of  $k$  on the solid line and consider the vertical movement to the dashed line. This mimics the immediate response of a bank to the regulatory change given its balance-sheet condition (i.e., the value of  $k$ ). The regulatory change achieves its intended purpose of stimulating

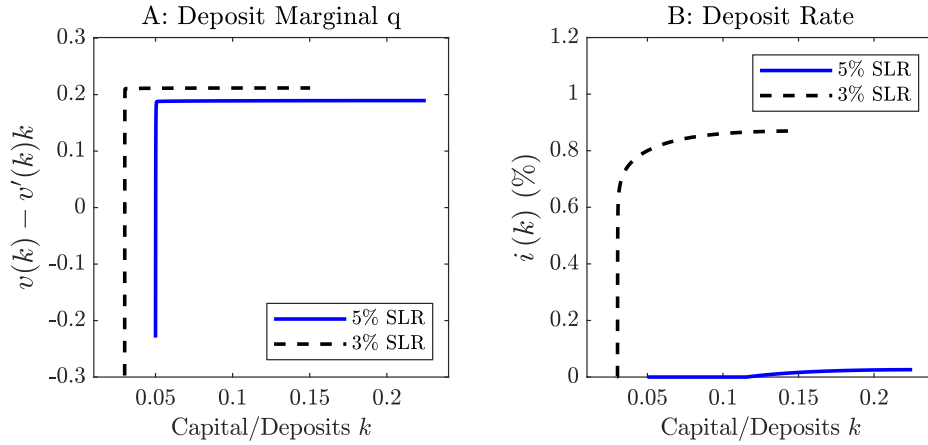


Figure 8: The Impact of Relaxing the SLR Requirement on Deposit Taking

deposit-taking as the deposit marginal  $q$  jumps up. The jump is most significant at the low values of  $k$  where the deposit margin  $q$  turns negative before the regulatory change.

If the deposit influx continues after the regulatory change (for example, due to new rounds of stimulus payments to households) and raises the bank's deposit liabilities,  $X$ , faster than the growth of its equity capital,  $K$ , via retained earnings, the bank moves along the dashed line to the left in Panel A of Figure 9 and its deposit marginal  $q$  declines. Note that after the SLR is relaxed, deposit marginal  $q$  is even more negative near the new and lower equity issuance boundary, because the equity capital is now lower relative to deposits at the new issuance boundary so that the effects of deposit inflows on  $k$  ( $= K/X$ ) are greater. Once the deposit influx pushes deposit marginal  $q$  into the negative territory, further relaxing the SLR becomes necessary to stimulate deposit-taking.

We plot the optimal deposit rate in Panel B of Figure 8. After the SLR requirement is reduced, the bank increases rate to attract deposits because the deposit marginal  $q$  is higher. As a result, the region of  $k$  where the deposit-rate lower bound binds shrinks significantly. By the same logic, tightening leverage regulation has the unintended consequence of making the lower bound more binding. The bank controls its deposit stock through the deposit rate. When the deposit-rate lower bound is more binding, the bank has less control over the size and composition of its balance sheet. This unintended consequence of leverage regulation is a unique prediction of our model.

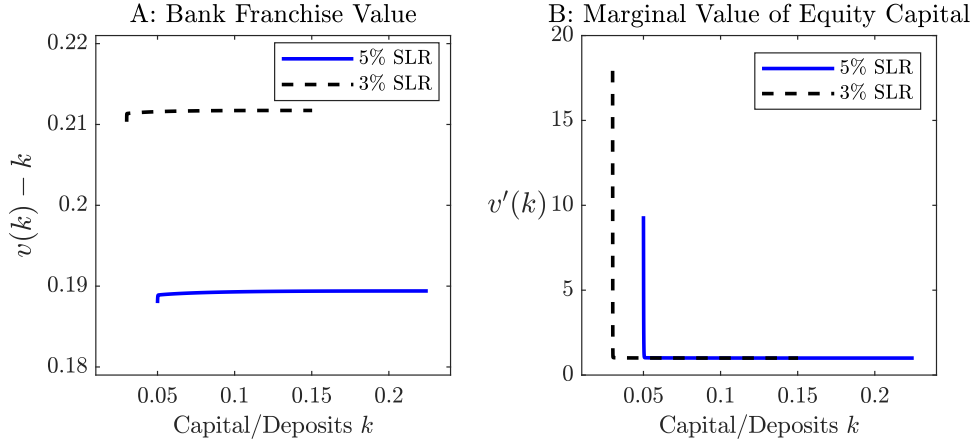


Figure 9: The Impact of Relaxing the SLR Requirement on Bank Valuation

### 5.3 Bank Franchise Value

Finally, we examine the impact of the SLR requirement on bank shareholders' value. Panel A of Figure 9 shows a clear increase of bank franchise value (scaled by deposit stock),  $(V(X, K) - K)/X = v(k) - k$ , when the SLR requirement is reduced. A higher shareholder value implies that the bank is more eager to protect its continuation value, explaining why the marginal value of equity is higher near the equity issuance boundary, as shown in Panel B of Figure 9.

Tightening leverage requirements results in a sizeable loss of bank shareholder value across all values of  $k$ . Kashyap, Stein, and Hanson (2010) point out that the impact of tightening leverage requirements on bank shareholders' value is temporary because shareholders pay the equity issuance (dilution) costs once and then the bank will settle on a higher level of equity capital. This argument holds in a deterministic environment. In our model, uncertainty is the key. Either negative shocks to earnings due to loan losses ( $dW^A < 0$ ) or positive shocks to the stock of deposit liabilities ( $dW^X > 0$ ) can reduce  $k = K/X$  and trigger costly equity issuance when  $k$  hits  $\underline{k}$ . Therefore, in a risky environment, the impact of leverage requirements on bank shareholder value is no longer a one-time cost of raising equity. The cost is now recurring, and through shareholders' rational expectations, is reflected in bank valuations even when  $k$  is away from  $\underline{k}$ . Moreover, to reduce the likelihood of incurring the equity issuance cost, the bank has to retain a higher level

of equity capital when the leverage requirement is tightened, which is also costly to shareholders because dividend payouts are delayed. Overall, our result contributes to the ongoing debate on the cost of equity capital regulations for banks (Admati, DeMarzo, Hellwig, and Pfleiderer, 2013).

## 6 Banking in a Low Interest Rate Environment

When the bank finances lending with deposits, it expects to earn a net interest margin (NIM), i.e., the spread between the expected loan return,  $r + \alpha_A$ , and the deposit rate,  $i$ . Earning the NIM requires the bank to take on the asset-return risk and deposit flow risk, and risk management is crucial under the equity issuance costs. We decompose the net interest margin into two components,  $\alpha_A$  (lending expertise) and  $r - i$  (deposit spread).<sup>33</sup> Our model emphasizes the deposit spread. In this section, we show that the bank suffers in a low interest rate environment, because as  $r$  declines, it squeezes the NIM and makes the deposit-rate lower bound a more binding constraint. In our model, the NIM is not only a measure of profitability as the classic banking theories predict, but, more importantly, the NIM reflects the bank's flexibility in managing its deposit liabilities.

The bank increases the deposit rate when it is well-capitalized (i.e.,  $k$  is high). Given the deposit rate lower bound, the higher the bank can set its deposit rate in the high- $k$  region, the more flexibility it has in reducing its deposit rate when  $k$  declines. However, raising the deposit rate increases interest expenses and hurts earnings. Therefore, the bank faces a trade-off. It can sacrifice its earnings in the high- $k$  region to gain flexibility of adjusting the deposit rate in the low- $k$  region. When the risk-free rate  $r$  is high, the bank can set a high deposit rate and still earn a positive deposit spread  $r - i$ . When the risk-free rate  $r$  is low, the bank has less room to manipulate the deposit rate without squeezing the deposit spread too much.

Therefore, the flexibility to adjust deposit rate and to control deposit flows depends on the distance between  $r$  and zero, the deposit-rate lower bound. When  $r$  is high, the bank has more flexibility in setting its deposit rate and thus is more in control of the size of its deposit liabilities.

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<sup>33</sup>The deposit spread reflects the bank's deposit market power (Drechsler, Savov, and Schnabl, 2017) and the extent to which depositors value the convenience of deposit accounts for payment activities. Motivated by the role of deposits as means of payment, the deposit spread is also called money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2020).

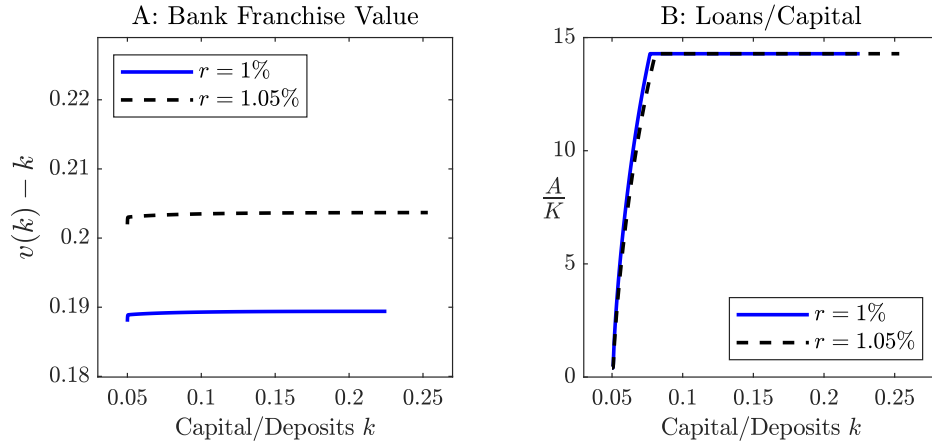


Figure 10: Interest Rate Level, Bank Valuation, and Risk-Taking

In contrast, the bank in a low rate environment faces a greater challenge of managing its deposit liabilities. This mechanism is consistent with the empirical findings. For example, Heider, Saidi, and Schepens (2019) find that the distribution of deposit rates of euro-area banks is truncated at zero and more deposit rates bunch at zero once the ECB lowers the policy rate.

Panel A of Figure 10 compares the bank franchise value under different risk-free rates and shows that a higher  $r$  leads to a higher bank franchise value. In Panel B, we show that when  $r$  increases, the bank reduces its risk exposure per unit of equity capital.<sup>34</sup> The increase of franchise value under a higher  $r$  results from more flexibility to adjust deposit rate rather than more aggressive risk-taking to earn the loan spread,  $\alpha_A$ . Moreover, as shown in both Panel A and B, when  $r$  increases, the bank sets the optimal payout boundary,  $\bar{k}$ , at a higher value (i.e., the right ends of the curves extend). This result shows that under a higher  $r$ , a higher franchise value incentivizes the bank to retain more equity capital as a risk buffer. By the same logic, in a low rate environment, the bank's incentive to maintain equity capital is weaker and it pays out dividend at a lower  $\bar{k}$ .

Panel A of Figure 11 shows that when  $r$  is higher, the deposit marginal  $q$  is higher at all levels of  $k$ . Deposits become more valuable when the bank can better control the deposit flows by adjusting deposit rate. In Panel B of Figure 11, we plot the deposit rate. When  $r$  is higher, the bank

<sup>34</sup>When  $r$  increases, the expected return from risky lending,  $r + \alpha_A$  in (1), also increases. When we adjust the risk-free rate, we keep the loan spread constant in line with the evidence in Drechsler, Savov, and Schnabl (2021).

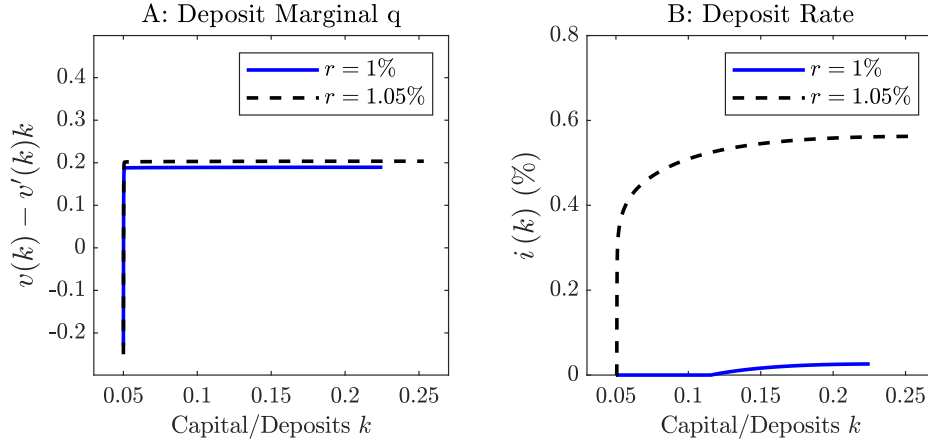


Figure 11: Interest Rate Level, Deposit marginal  $q$ , and Deposit-Taking

is more aggressive in raising deposit rate in the high- $k$  region to preserve more flexibility for rate reduction when  $k$  declines due to negative earning shocks ( $dW_t^A < 0$ ) or positive deposit shocks ( $dW_t^X > 0$ ). Under a higher  $r$ , the deposit rate lower bound becomes less binding.

Our model provides a rationale that links bank profitability and franchise value to the level of interest rate. The mechanism is related to the channel of deposit market power in Drechsler, Savov, and Schnabl (2017). In their paper, a higher risk-free rate makes cash, the deposit substitute, becomes more expensive to hold, and this allows banks to raise deposit spreads,  $r - i$ , without losing deposits to cash. Our specification of deposit flow (2) captures deposit market power through the stickiness of deposit stock. When the bank adjusts deposit rate, the flow happens by the order of  $dt$ . Different from Drechsler, Savov, and Schnabl (2017), we highlight the risk in deposit flow and the fact that a higher risk-free rate offers the bank more flexibility to manage such risk.

As shown in Panel B of Figure 11, a lower  $r$  implies less flexibility to set deposit rate, and more importantly, a greater region of the state variable  $k$  where the deposit rate lower bound binds and the bank completely loses control of its deposit stock. The banking literature has largely focused on the positive effect of low interest rate on risk-taking, which we revisits in our setting (Panel B of Figure 10). Our paper puts more emphasis on the management of deposit risk. Moreover, our model predicts that in a low interest rate environment, the bank is more eager to pay out



to shareholders (i.e., a lower right bound of  $k, \bar{k}$ , when  $r$  is lower). This is consistent with the massive share repurchases done by banks in the last decade of a low interest rate environment.

## 7 Conclusion

Deposit-taking is a double-edged sword. It provides relatively cheap funds but also exposes the bank to deposit-flow risk. The bank's inability to fully control its liabilities makes bank balance-sheet management conceptually very different from that of non-depository intermediaries and non-financial firms. Under equity issuance costs, the deposit marginal  $q$  turns sharply negative for undercapitalized banks, meaning that deposit inflows hurt bank shareholders. Our model stands in contrast with the existing banking literature that has mainly been concerned with deposit outflows and bank runs under the illiquidity of banks assets. Our model delivers a rich set of predictions on bank lending, payout to shareholders, equity issuance, and the choice of leverage through deposit-taking and short-term borrowing, and it sheds light on recent developments surrounding the SLR requirement and the challenges of running a bank in a low rate environment.

Our paper also contributes to the recent literature on safe assets (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2016; Maggiori, 2017; Bolton, Santos, and Scheinkman, 2018; He, Krishnamurthy, and Milbradt, 2019; Brunnermeier, Merkel, and Sannikov, 2020). The bank loses control of its leverage once it hits the deposit rate lower bound, so that it holds risk-free bonds in an effort to reduce the risk exposure of equity capital. The government is in a unique position to supply safe assets. In a general equilibrium setting where the interest rate  $r$  is endogenous, banks' demand is likely to push down  $r$ . The government can take advantage of a lower borrowing cost, issuing debts and using the proceeds to stimulate the economy. Supplying safe assets is also essential for sustaining  $r$  at a sufficiently high level so that banks have enough flexibility in adjusting their deposit rate between zero and  $r$ . Finally, government debts as deposit substitutes can absorb part of the money demand and thereby liberate banks from unwanted deposits and leverage.<sup>35</sup>

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<sup>35</sup>The monetary service of government liabilities is an old theme (Patinkin, 1965; Friedman, 1969). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jørgensen (2012), Greenwood, Hanson, and Stein (2015), Bolton and Huang (2016), and Nagel (2016).

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## A An Extended Model with Liquidity Requirement

In this appendix, the bank must hold assets that are more liquid than loans (Drechsler, Savov, and Schnabl, 2018). These assets can be reserves or other high-quality liquid assets (HQLA). The bank chooses liquidity holdings, denoted by  $R_t$ , which earns an interest rate  $\iota$  ( $< r$ ). The bank may to pay a carry cost for having a more liquid asset portfolio. The law of motion of equity capital is

$$dK_t = A_t \left[ (r + \alpha_A) dt + \sigma_A d\mathcal{W}_t^A \right] - B_t r dt - X_t i_t dt - C(n(i_t), X_t) dt - dU_t + dF_t + R_t \iota dt - S(R_t, X_t, A_t) dt. \quad (\text{A.1})$$

The last term,  $S(R_t, X_t, A_t)$ , captures loss due to illiquidity of asset portfolio. This specification is isomorphic to the following microfoundation: a Poisson-arriving withdrawal of a large amount of deposits can only be met by liquidity holdings and selling a large amount of loans in exchange for liquidity incurs a fire-sale cost (Moreira and Savov, 2017; Drechsler, Savov, and Schnabl, 2018). Accordingly, we assume  $S_R(R_t, X_t, A_t) < 0$ ,  $S_X(R_t, X_t, A_t) > 0$ , and  $S_A(R_t, X_t, A_t) > 0$ .

The bank has to meet the regulatory requirement of liquidity holdings:

$$R_t \geq \xi_R X_t. \quad (\text{A.2})$$

This constraint can be motivated by reserve requirement or more recent requirement on liquidity coverage ratio (Basel Committee on Banking Supervision, 2013). Risk-free bonds ( $B_t < 0$ ) are not part of liquidity holdings. Here we draw the distinction between liquid and illiquid safe assets in line with the evidence that these assets offer different yields (Krishnamurthy, 2002; Nagel, 2016). Let  $\pi_t^R$  denote the weight of  $(X_t + K_t)$  on  $R_t$ . We rewrite the law of motion for  $K_t$  in (A.1) as

$$dK_t = (X_t + K_t) \left[ r + \pi_t^A \alpha_A - \pi_t^R (r - \iota) \right] dt + (X_t + K_t) \pi_t^A \sigma_A d\mathcal{W}_t^A - X_t i_t dt - C(n(i_t), X_t) dt - S(\pi_t^R (X_t + K_t), X_t, \pi_t^A (X_t + K_t)) - dU_t + dF_t. \quad (\text{A.3})$$

Accordingly, the HJB equation in the interior region where  $dU_t = 0$  and  $dF_t = 0$  is

$$\begin{aligned} \rho V(X, K) = & \max_{\{\pi^A, \pi^R, i\}} V_X(X, K) X [-\delta_X + n(i)] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \\ & + V_K(X, K) (X + K) [r + \pi^A \alpha_A - \pi^R (r - \iota)] + \frac{1}{2} V_{KK}(X, K) (X + K)^2 (\pi^A \sigma_A)^2 \\ & - V_K(X, K) [S(\pi^R (X + K), X, \pi^A (X + K)) + Xi + C(n(i), X)] \\ & + V_{XK}(X, K) X (X + K) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (\text{A.4})$$

**Risk-taking.** The first-order condition for  $\pi^A$  gives the following solution:

$$\pi^A = \min \left\{ \frac{\alpha_A + \epsilon(X, K) \sigma_A \sigma_X \phi - S_A(R, X, A)}{\gamma(X, K) \sigma_A^2 \left(\frac{X+K}{K}\right)}, \frac{K}{\xi_K(X+K)} \right\}. \quad (\text{A.5})$$

While setting up  $\pi^A = A/(X+K)$  as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e.,  $A/K = \pi^A(X+K)/K$ :

$$\frac{A}{K} = \min \left\{ \frac{\alpha_A + \epsilon(X, K) \sigma_A \sigma_X \phi - S_A(R, X, A)}{\gamma(X, K) \sigma_A^2}, \frac{1}{\xi_K} \right\}. \quad (\text{A.6})$$

In comparison with (30), the only difference is that the numerator is deducted by  $S_A(R, X, A)$ .

**Liquidity Holdings.** When the liquidity requirement (A.2) does not bind, the optimality condition for  $\pi^R$  equates the marginal cost of holding reserves, i.e., accepting the below- $r$  rate of return  $\iota$ , and the marginal benefit of holding reserves to reduce the payment settlement cost:

$$r - \iota = -S_R(\pi^R(X+K), X, \pi^A(X+K)). \quad (\text{A.7})$$

The reserve requirement can be rewritten as the following restriction on  $\pi^R$ :

$$\pi^R \geq \frac{\xi_R X}{(X+K)}. \quad (\text{A.8})$$

Next, we specify the functional form of  $S(R, X, A)$  that satisfies the properties that  $S(R, X, A)$  decreases in  $R$  and increases in  $X$  and  $A$ :

$$S(R, X, A) = \frac{1}{2} \frac{(\chi_1 X + \chi_2 A)^2}{R}. \quad (\text{A.9})$$

The numerator is convex in  $X$  and  $A$  while the denominator is linear in  $R$ . Therefore, to maintain the same level of  $S(R, X, A)$ , the bank will have to hold increasingly more liquidity as it expands its balance sheet (i.e., increases  $X$  and  $A$ ). This captures the decreasing marginal return to liquidity holdings that have been microfounded in various ways (Moreira and Savov, 2017).

Under this functional form of  $S(R, X, A)$ , we obtain

$$S_R(R, X, A) = -\frac{1}{2} \left( \frac{\chi_1 X + \chi_2 A}{R} \right)^2. \quad (\text{A.10})$$

Therefore, the optimality condition for  $\pi_t^R$  implies that  $r - \iota = \frac{1}{2} \left( \frac{\chi_1 X + \chi_2 A}{R} \right)^2$ , so rearranging the

equation we obtain the following reserve holding policy

$$R = \frac{\chi_1 X + \chi_2 A}{\sqrt{2(r - \iota)}} \quad (\text{A.11})$$

This is in the spirit of Baumol (1952) and Tobin (1956) who show that liquidity demand is equal to the product of transaction costs (mapping to  $\chi_1$  and  $\chi_2$ ) and transaction needs (mapping to  $X$  and  $A$ ) divided by the square root of two times the carry cost.

As in the main text, the bank's value function is  $v(k) X$ , where

$$k = \frac{K}{X}. \quad (\text{A.12})$$

And, as in the main text, we simplify the expressions of the effective risk aversion in (31)

$$\gamma(k) = \frac{-V_{KK}(X, K) K}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \quad (\text{A.13})$$

and the elasticity of marginal value of capital to deposits

$$\epsilon(k) = \frac{V_{XK}(X, K) X}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \quad (\text{A.14})$$

which happens to be equal to  $\gamma(k)$ . Next, we simplify loan-to-capital ratio. First, note that from (A.11), we obtain the marginal illiquidity cost of loans:

$$S_A(R, X, A) = \chi_2 \left( \frac{\chi_1 X + \chi_2 A}{R} \right) = \chi_2 \sqrt{2(r - \iota)}, \quad (\text{A.15})$$

Using (A.15) and  $\epsilon(k) = \gamma(k)$ , we simplify the optimal loan-to-capital ratio:

$$\frac{A}{K} = \min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r - \iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\}, \quad (\text{A.16})$$

The only difference from (30) is that the numerator declines by the marginal illiquidity cost  $\chi_2 \sqrt{2(r - \iota)}$ . To make lending profitable, we impose the parameter restriction:  $\alpha_A > \chi_2 \sqrt{2(r - \iota)}$ .

Using these expressions, we can rewrite the HJB equation (A.4) as

$$\begin{aligned} \rho v(k) = & \max_{\pi^A, \pi^R, i} [v(k) - v'(k) k] (-\delta_X + \omega i) + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\ & + v'(k) (1+k) [r + \pi^A \alpha_A - \pi^R (r - \iota)] - v'(k) \left[ \frac{1}{2} \left( \frac{\chi_1}{1+k} + \frac{\chi_2 \pi^A}{\pi^R} \right)^2 \pi^R (1+k) \right. \\ & \left. + i + \theta_0 + \frac{\theta_1}{2} (\omega i)^2 \right] + \frac{1}{2} v''(k) (1+k)^2 (\pi^A \sigma_A)^2 - v''(k) k (1+k) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (\text{A.17})$$

To show that (A.17) is an ODE for  $v(k)$ , we need to show that the control variables only depend on  $k$  and the level and derivatives of  $v(k)$ . First, from (A.16), we obtain:

$$\pi^A = \left(\frac{A}{K}\right) \left(\frac{K}{K+X}\right) = \min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r-\iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\} \left(\frac{k}{1+k}\right). \quad (\text{A.18})$$

Rearranging (A.11), we can solve  $\pi^R$  as a linear function of  $\pi^A$  and the state variable  $k$ :

$$\pi^R = \frac{\chi_2}{\sqrt{2(r-\iota)}} \pi^A + \frac{\chi_1}{(1+k) \sqrt{2(r-\iota)}}, \quad (\text{A.19})$$

so it also only depends on  $k$  and the level and derivatives of  $v(k)$ . The deposit rate, still given by (36) in the main text, only depends on  $V_X(X, K) = v(k) - v'(k)k$  and  $V_K(X, K) = v'(k)$ .

After substituting the optimal control variables into the HJB equation, we obtained an ordinary equation with the same boundary conditions discussed in the main text. The determination of endogenous upper bound of  $k$  also follows the main text. The only difference is in the determination of endogenous lower bound of  $k$ , i.e., the equity issuance boundary.

Let  $\underline{k}_S$  denote the lower bound in (24) implied by the supplementary leverage ratio (SLR) requirement. The liquidity requirement implies another lower bound  $\underline{k}_L$ . Substituting (A.19) into the reserve requirement (A.8), we have

$$\frac{\chi_2}{\sqrt{2(r-\iota)}} \pi^A + \frac{\chi_1}{(1+k) \sqrt{2(r-\iota)}} \geq \frac{\xi_R}{(1+k)}, \quad (\text{A.20})$$

Using (A.18) to substitute out  $\pi^A$  and rearranging the equation, we have

$$\min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r-\iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\} k \geq \frac{\xi_R \sqrt{2(r-\iota)} - \chi_1}{\chi_2}. \quad (\text{A.21})$$

In our numeric solution, the right side increases in  $k$  (as  $\gamma(k)$  increases in  $k$ ). Therefore, (A.21) imposes a lower bound of  $k$ , denoted by  $\underline{k}_L$ . Therefore, we have

$$\underline{k} = \max \{0, \underline{k}_S, \underline{k}_L\}. \quad (\text{A.22})$$

To sum up, introducing the bank's needs to hold reserves or HQLA leads to three changes

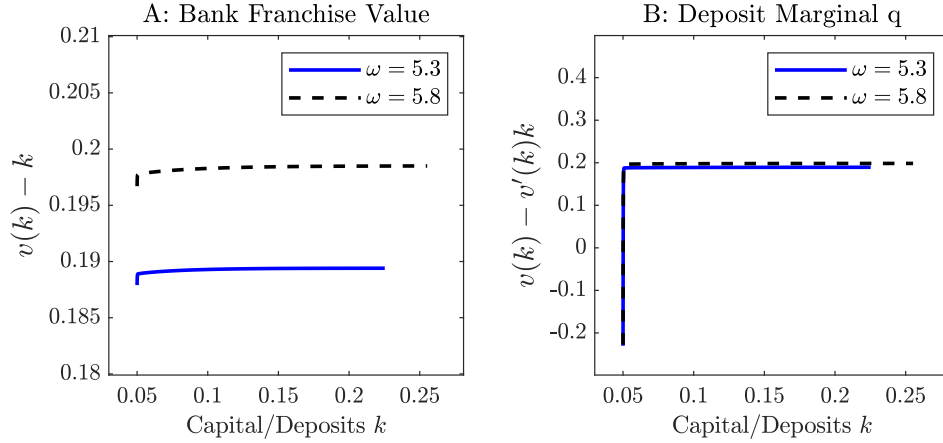


Figure B.1: Deposit Demand Elasticity, Bank Valuation, and Deposit  $q$

in the solution. First, the new control variable, optimal liquidity-holding policy, is given by the Baumol-Tobin style money demand (A.11). Second, in the optimal risk-taking policy (A.16),  $\alpha_A$  is subtracted by the marginal illiquidity cost of loans. Third, the equity issuance boundary is defined by (A.22) nesting considerations of liquidation, SLR requirement, and liquidity requirement.

## B Deposit Market Power and Bank Franchise Value

The deposit demand elasticity,  $\omega$ , (the coefficient for  $n(i) = \omega i$  appearing in (2)) determines how responsive the deposit flow is to the variation of deposit rate. The higher the value of demand elasticity the easier it is for the bank to manage its deposit liabilities. Panel A of Figure B.1 shows that bank franchise value increases in  $\omega$ . In Panel B, we plot the marginal  $q$  of deposits, which also increases in  $\omega$ . The optimal deposit rate depends on the marginal value of deposits and marginal value of equity. In Panel A of Figure B.2, we show that the deposit rate  $i(k)$  is much higher under a higher value of  $\omega$ . This is consistent with the mechanism that a higher deposit marginal  $q$  tends to drive up the deposit rate. In Panel B of Figure B.2, we plot the loan-to-equity capital ratio,  $A/K$ . Under a higher deposit demand elasticity, the bank reduces risky lending because the higher deposit rate drives up the cost of financing. In spite of earning less from the spread between the loan return and the deposit rate, bank value still increases because deposit risk management is more effective when the deposit flow is more responsive to changes in deposit rate.



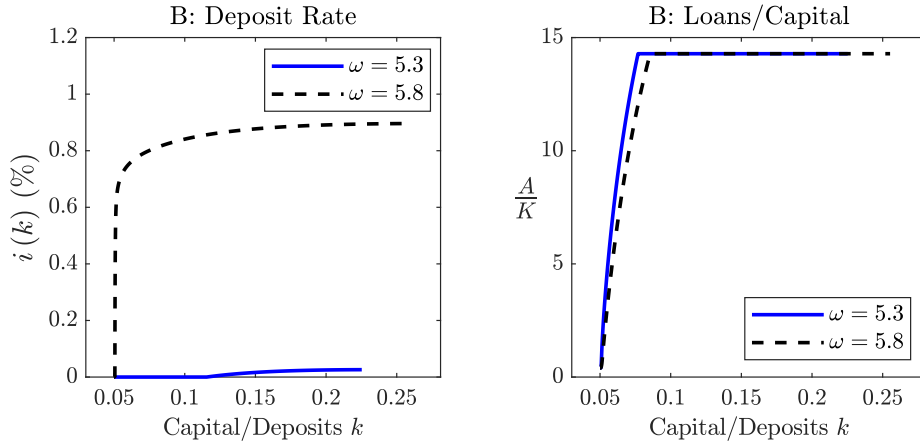


Figure B.2: Deposit Demand Elasticity, Deposit-Taking, and Lending

A higher deposit demand elasticity is often associated with a more competitive deposit market. Consistent with the findings of Drechsler, Savov, and Schnabl (2017), our model generates a higher deposit rate when  $\omega$  is higher. The traditional mechanism in the banking literature emphasizes the deposit demand side – when depositors are more price-sensitive, the bank has to set a higher interest rate to attract depositors. This mechanism leads to the conclusion that competition erodes bank franchise value (Keeley, 1990).<sup>36</sup> Our model predicts the opposite. When deposit demand elasticity increases, bank franchise value increases. In our model, the increase in financing cost that results from a more elastic demand does have a negative impact on bank value, but such impact is dominated by the positive impact of the bank having more control over its deposit liabilities. Our focus is on the deposit supply side – when depositors are more price-sensitive, the bank can regulate deposit flows more effectively through deposit rate.

So far, our analysis seems to suggest that stronger deposit market power, represented by a more elastic deposit demand, amplifies the challenge of deposit management because the deposit base becomes less responsive to deposit rate. However, there is another aspect of deposit market power. Depositors at a bank with a large deposit market share are more likely to send payments to and receive payments from depositors within the same bank. Therefore, the bank is less concerned about the uncertainty in payment-driven deposit flows (i.e.,  $\sigma_X$  declines).

<sup>36</sup>We refer the readers to the vast literature on how competition affects bank value (Petersen and Rajan, 1995; Jayaratne and Strahan, 1996; Allen and Gale, 2004a; Boyd and De Nicoló, 2005; Bertrand, Schoar, and Thesmar, 2007; Erel, 2011; Scharfstein and Sunderam, 2016; Drechsler, Savov, and Schnabl, 2017; Liebersohn, 2017).

Our paper contributes to the literature on deposit market power (Drechsler, Savov, and Schnabl, 2017) by using two parameters, the deposit demand elasticity  $\omega$  and the size of deposit-flow uncertainty  $\sigma_X$ , to capture their distinct effects on bank value. The level of the risk-free rate  $r$  is also key to the impact of deposit market power on bank value. To the extent that the bank can exploit its deposit market power, it does so by earning the deposit spread  $r - i$ . In a low interest rate ( $r$ ) environment, the bank has limited freedom in adjusting the spread given that  $i$  as discussed in Section 6. In contrast, a high  $r$  allows the bank to exploit its deposit market power more by earning a larger deposit spread,  $r - i$ , and having more flexibility in adjusting the deposit flow through the deposit rate  $i$ . We provide the analysis on the impact of  $r$  in Section 6.

## C Jump Risk

Jump risk in asset returns has been emphasized as a way to distinguish the risk-return profile of banking relative to other forms of financial intermediation (e.g., Parlour, Stanton, and Walden, 2012; Hugonnier and Morellec, 2017). We extend our model to incorporate jump risk. The time subscript  $t-$  denotes the pre-jump value of variables. The risky investment  $A_{t-}$  evolves as follows

$$dA_t = A_{t-} (r + \alpha_A) dt + A_{t-} \sigma_A d\mathcal{W}_t^A - A_{t-} (1 - Z_t) \mathcal{J}_t^A, \quad (\text{C.23})$$

where  $\mathcal{J}_t^A$  is a time-homogeneous Poisson counting process with the arrival rate  $\lambda_A$  and the recovery rate  $Z_t$  is uniformly distributed between  $(1/\xi_L, 1]$  (which implies that the SLR requirement rules out insolvency as intended). With (1) replaced by (C.23), the law of motion of  $K_t$  is

$$\begin{aligned} dK_t = & (X_{t-} + K_{t-}) [(r + \pi_{t-}^A \alpha_A) dt + \pi_{t-}^A \sigma_A d\mathcal{W}_t^A - \pi_{t-}^A (1 - Z_t) \mathcal{J}_t^A] \\ & - X_{t-} i_{t-} dt - C(n(i_{t-}), X_{t-}) dt - dU_t + dF_t. \end{aligned} \quad (\text{C.24})$$

The law of motion of  $X_t$  is the same as the baseline model (see (2)).

The value function takes the form of  $v(k)X$  (time subscripts suppressed) and  $v(k)$  satisfies

$$\begin{aligned} \rho v(k) = & \max_{\pi^A, i} [v(k) - v'(k)k] [-\delta_X + n(i)] + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\ & + v'(k) (1+k) (r + \pi^A \alpha_A) + \frac{1}{2} v''(k) (1+k)^2 (\pi^A \sigma_A)^2 \\ & - v'(k) [i + c(n(i))] - v''(k) k (1+k) \pi^A \sigma_A \sigma_X \phi + \lambda_A \mathbb{E} [v(\tilde{k}) - v(k)], \end{aligned} \quad (\text{C.25})$$

where  $\tilde{k}$  denotes the post-jump value of  $k$ :

$$\tilde{k} = \frac{\tilde{K}}{X} = \frac{K - (X + K)\pi^A(1 - Z)}{X} = k - (1 + k)\pi^A(1 - Z). \quad (\text{C.26})$$

If  $\tilde{k} < \underline{k}$ , the bank raises external equity financing to stay in compliance with the SLR requirement:

$$v(\tilde{k}) = v(\underline{k} + m) - \psi_0 - (1 + \psi_1) \left( m + \underline{k} - \tilde{k} \right), \quad (\text{C.27})$$

where the optimal  $m$  (i.e., the amount of equity raised in excess of the SLR requirement) satisfies

$$v'(\underline{k} + m) = 1 + \psi_1. \quad (\text{C.28})$$

As in the baseline model,  $\underline{k}$  is equal to 0.05 under the SLR requirement  $\xi_L = 20$ .<sup>37</sup>

In our numerical solution, we adjust the value of  $\alpha_A$  upward by  $\lambda_A \mathbb{E}[1 - Z_t]$  to compensate the decline of expected return on risky investment due to the jump risk. Therefore, what changes is the distribution of return but not the expected return. The jump risk essentially extends the left tail. When the bank chooses  $\pi_A$ , it takes into consideration the loading on the jump risk and the potential negative consequence of costly equity issuance triggered by a low realization of  $Z$ .

Under the extra precaution, the bank takes on more risk in a less dramatic fashion than the baseline model when  $k$  increases. As shown in Panel C of Figure C.3, the ratio of risky investment to equity capital rises more smoothly with  $k$ . Moreover, as in the baseline model, the bank seeks a higher leverage when  $k$  increases by issuing bonds (i.e., increasing  $B_t$ ), but the jump risk incentivizes the bank to build up leverage more slowly. In Panel D of Figure C.3, the ratio of bond liabilities to deposits increases more smoothly in  $k$  than the baseline solution. Finally, in Figure C.3, the curves end at a higher level of  $k$  than the baseline solutions. The jump risk incentivizes the bank to set a higher payout boundary of  $k$  and preserve a higher level of equity capital.

As discussed in Section 3.2, the bank profits from risky investment and deposit-taking. The expected excess return on risky investment,  $\alpha_A$ , comes with a skewed distribution (generated by both the Brownian shock,  $d\mathcal{W}_t^A$ , and Poisson shock,  $d\mathcal{J}_t^A$ ). In contrast, to earn the net deposit spread (defined in Section 3.2), the bank only loads on the Brownian shock,  $d\mathcal{W}_t^X$ . Therefore, deposit-taking as a source of profits becomes more important under the jump risk in asset return. Panel A of Figure C.3 shows that the deposit marginal  $q$  is higher than the baseline solution, and

<sup>37</sup>Given  $\underline{k}$ , the boundary conditions (25), (26) (same as (C.28)), (27), and (28) and second-order ODE (C.25) solve the  $X$ -scaled value function,  $v(k)$ , the optimal issuance amount  $m$ , and the upper (dividend payout) boundary  $\bar{k}$ .

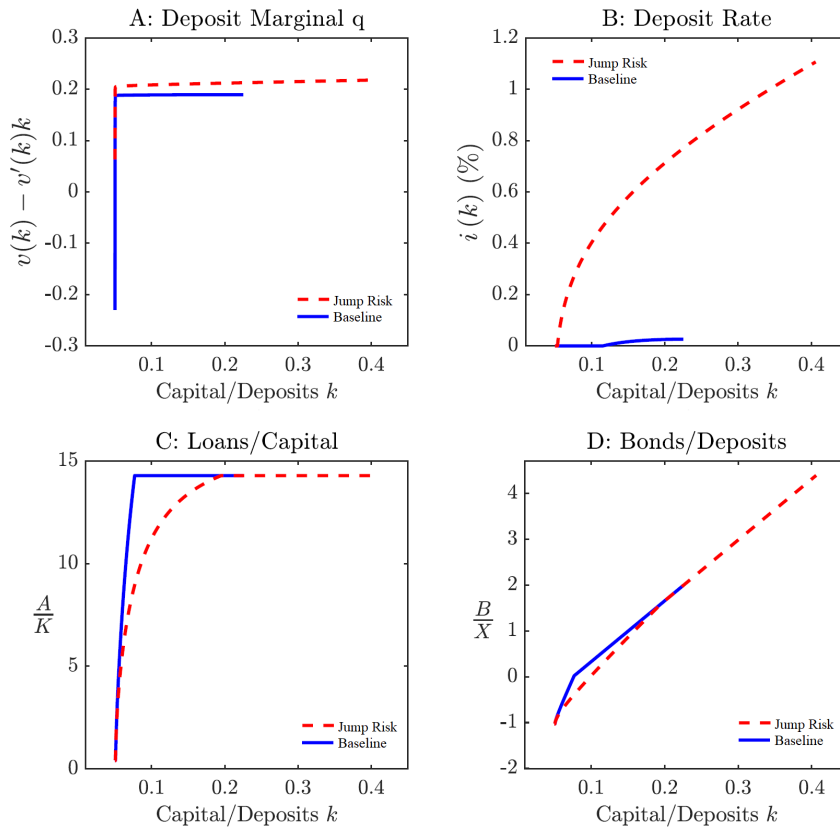


Figure C.3: The Impact of Jump Risk in Loan Returns

Panel B shows that the bank is willing to pay a higher interest rate to attract deposits.