Firm Characteristics and Stock Price Levels: a Long-Term Discount Rate Perspective

Yixin Chen and Ron Kaniel*

December 11, 2021

We study how firm characteristics are correlated with stock price *levels* by measuring the long-term discount rates (defined as the internal rate of return) of anomaly portfolios over a long horizon. Utilizing a simple novel non-parametric estimation methodology, which proxies ex-ante equity payout expectations with ex-post realizations, we reveal that the patterns of long-term discount rates are out-of-line with the average short-term holding period returns for multiple prominent anomalies. The set of stylized facts uncovered correspondingly shed new light on the mechanisms underlying various asset-pricing anomalies. Moreover, they indicate that long-term discount rates better characterize firms' equity financing cost than short-term expected returns; with a representative example, we demonstrate how structural models that posit a tight connection between the two could imply counterfactual patterns in price levels.

JEL Classification: G12

Key Words: Stock Price Level, Discount Rate, Asset Pricing Anomaly

*Chen(yixin.chen@simon.rochester.edu) and Kaniel(ron.kaniel@simon.rochester.edu) are from the Simon Business School at University of Rochester. We thank the seminar participants at University of Rochester.

1 Introduction

Since the seminal work of Fama and French (1992, 1993), an extensive literature has been established to study asset pricing factors/anomalies[1], focusing on the relation between firm characteristics and expected short-term stock returns. The expected short-term return of a stock captures how the price tends to move shortly after portfolio formation, but fairly little is understood regarding what determines the *level* of stock prices. We aim to study this subject in our paper.

The standard protocol to investigate the relation between firm characteristics and equity financing cost is to sort stocks into portfolios along a certain dimension, rebalance them frequently, and then compare the average holding period return from one dynamically-rebalanced decile to another. However, despite its wide adoption, this procedure has significant limitations, because it only measures the short-term changes in stock prices one period following portfolio formation. Yet, the notion of equity financing cost should inherently concern the long run, as it is supposed to reflect the level of stock prices relative to the amount of all the cash flows that firms are expected to pay shareholders over a long horizon stretching into the distant future. Motivated by this observation, we develop a novel non-parametric methodology to estimate the long-term cash flow discount rates (defined below) of equity claims, and show that over-interpreting the performances of the dynamically-rebalanced portfolios can lead to misperceptions of firms' equity financing costs due to a disconnect between the short run and the long run – namely, there can be substantial differences between the cross-sectional patterns of the average short-term returns and the long-term cash flow discount rates, so that the former only represent the tendency of the immediate changes in stock prices, while the latter properly measure stock price levels and firms' overall equity financing costs.

To fix ideas, consider the well-known idiosyncratic volatility anomaly documented by Ang et al. (2006), whereby stocks with high idiosyncratic volatility tend to generate low returns shortly after portfolio formation. But we reveal that those stocks also face higher long-term discount rates, so that, contrary to conventional wisdom, even though they deliver lower returns in the short run, they are also cheaper in price levels. We will further show below that the idiosyncratic volatility anomaly is not the only example; in fact, such a disconnect between the short run and the long run is not uncommon among prominent anomalies.

Our objective is to compare equity financing costs across different firms, and thus the valuation levels of their equity claims. However, prices of these claims are not directly comparable, because their cash flows might have different patterns. To resolve this issue, we draw an analogy with

[1] In this paper, we use the term "anomaly" or "factor" to simply refer to the spread in the expected stock return along a certain firm characteristic. We do not take a stand on whether such a spread is caused by rational or behavioral forces. We thus use these two terms interchangeably.

the fixed-income literature: similar to quoting bond prices with the yield–to-maturity metric, we normalize stock prices by quoting them with the internal rate of return (IRR) over the long run:

$$P_0 = \sum_{t=1}^{\infty} \frac{\mathbb{E}_0(D_t)}{(1+y)^t} \approx \sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{(1+y)^t} + \frac{\mathbb{E}_0(P_T)}{(1+y)^T},$$

where P_0 (P_T) is the stock price observed at time 0 (T); $\mathbb{E}_0(D_t)$ is the expected total net payout of the equity claim occurring in time t; T represents a long horizon; and y is the internal rate of return.

Analogous to the yield-to-maturity measure of a bond claim, the long-horizon IRR is *not* a return that can be earned by investors, but a metric that quotes the price level of an equity claim. In other words, the long-horizon IRR simply normalizes the stock price by all expected future payouts, so that it can be directly compared across different equity claims.

While equity discount rates can be different for different installments within the cash flow stream, the IRR is a non-linear weighted average across all payouts over different horizons and captures the overall equity financing cost of the firm. To reflect the fact that the IRR metric summarizes firm's cost of equity capital and to contrast it with the short-term holding period returns generated by dynamic trading strategies, we refer to this metric as the "long-term discount rate".

Acknowledging that the long-term discount rate can be time-varying, in this paper we choose to focus on its time average; in parallel to the anomaly literature, we study how firm characteristics are correlated with the long-term discount rate in the cross section of stocks.

The challenge of estimating the long-term discount rate from the data is that cash flow expectations are not directly observable. The existing literature attempts to circumvent this issue by using either analysts' subjective forecasts as proxies for market expectations or by building structural models to predict cash flows. However, both approaches have limitations, as analysts' forecasts are known to be inaccurate and have limited coverage[2] and models can be mis-specified. To address these issues, we propose a simple non-parametric methodology that approximates cash flow expectations and estimates the long-term discount rate with realized corporate payouts and stock prices.

Our estimation starts by replacing the ex-ante expectations of the cash flows with their ex-post

[2]It is well-documented that analysts' forecasts are overly optimistic and exhibit heterogenous biases for different stocks in the cross section. See, for example, Das, Levine, and Sivaramakrishnan (1998), Clement (1999), Lim (2001), Hong, Kubik, and Solomon (2000), Bradshaw, Richardson, and Sloan (2001), Hong and Kubik (2003), etc. Moreover, long-term growth forecasts, which are important for recovering the long-term discount rates, are only available for a small set of large firms.

realizations and inferring the ex-post discount rate $\tilde{y}$:

$$P_0 = \sum_{t=1}^{T} \frac{D_t}{(1+\tilde{y})^t} + \frac{P_T}{(1+\tilde{y})^T}.$$

We then utilize the time series and measure $\tilde{y}$ repeatedly with portfolios formed at different points in time.[3] Naturally, the sample mean of these ex-post rates serve as an estimate of the true ex-ante rate y.

Alas, such an estimate is biased. For a given price, the true discount rate is a non-linear function of the cash flow expectations (i.e. $y = f\left(\left\{\mathbb{E}_0\left(D_t\right)\right\}_t, \mathbb{E}_0\left(P_T\right), P_0\right)$, where $f(\cdot)$ is the non-linear IRR function that uncovers y from current price and future cash flow expectations); and to precisely recover the discount rate, one needs to take the expectations of the cash flows inside the non-linear IRR function. However, by taking the average after inferring the rates from the realized cash flow paths, we essentially are taking the expectation outside of the IRR function (i.e. $\mathbb{E}_0(\tilde{y}) = \mathbb{E}_0\left[f\left(\left\{D_t\right\}_t, P_T, P_0\right)\right]$). The order of the expectation is changed, and as a result, a Jensen's term arises and biases the estimated rate. To address this issue, as the final step of the estimation procedure, we non-parametrically estimate the Jensen's term and correct for such a bias with Taylor approximations. Using simulations, we verify that our methodology indeed produces accurate estimates for our purposes.

In addition to the long-term discount rate, we also define and estimate the "relative discount factor" (or *RDF*) with a similar strategy:

$$RDF \equiv \frac{P_0}{\sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{\left(1+r_{f,0,t}\right)^t} + \frac{\mathbb{E}_0(P_T)}{\left(1+r_{f,0,T}\right)^T}},$$

where $r_{f,0,t}$ is the t-year zero-coupon risk-free rate observed at time 0 from the term structure of interest rates.

The relative discount factor is a ratio between two prices: the actual price of the stock, and its counterfactual price if cash flows were priced with the risk-free rates. Therefore, the relative discount factor measures the difference between the equity financing cost of a firm and the borrowing cost of the US government when issuing a default-free security with the same expected cash flows. As a complement to the long-term discount rate, the relative discount factor captures the same economics, but it also incorporates the impact of duration on firm's equity financing cost.

We focus on a number of prominent anomalies; utilizing our methodology, we show how the

[3]We apply our methodology to portfolios instead of individual stocks because we intend to study the discount rate over the long run and individual stocks might only exist for a short period in the sample.

pattern of the association between firm characteristics and the long-term discount rates can be substantially different from that of the average short-term holding period returns. For example, among the Fama-French five factors, instead of displaying a monotonic pattern across characteristics deciles, the long-term discount rates of the portfolios sorted along gross profitability and investment show a strong U-shape. In another interesting case, as briefly mentioned earlier, while Ang et al. (2006) document that high idiosyncratic volatility stocks tend to generate lower returns than low idiosyncratic volatility stocks in the short term; once we consider the long term, the pattern is inverted with the high idiosyncratic volatility stocks featuring significantly higher discount rates than the low idiosyncratic volatility stocks. Moreover, the estimation of the long-term discount rate also uncovers firm characteristics that are important for equity pricing but were previously overlooked from studies centered around short-term stock returns. For example, while average short-term holding period returns are almost flat across portfolios sorted by credit rating, as soon as we turn to the long term, we reveal that high rating firms face much lower long-term discount rates than low rating firms. Therefore, the equity market is much more integrated with the credit market than one previously would conclude by only focusing on the short-term holding period returns.

Our finding of the disconnect between the short term and the long term has important implications for a large class of structural asset pricing models that aim to reconcile anomalies with rational forces. Using Kogan and Papanikolaou (2013) as an example, we illustrate how this class of models interprets the spreads in the short-term expected returns as manifestations of the differences in firms' overall long-term discount rates or stock price levels, and why this can be problematic. By replicating their simulations, we first verify that the mechanism of their paper indeed generates realistic short-term premia for multiple anomalies. However, once we apply our methodology to the simulated environment, we reveal that the long-term discount rates implied by their model always take the same shape as the short-term expected returns, which is apparently counterfactual for several anomalies such as idiosyncratic volatility, gross profitability and investment. Therefore, this exercise highlights how drawing inferences about firms' equity financing costs from average short-term stock returns can be misleading when the patterns of the two diverge, with these models being hardwired to produce counterfactual patterns of the long-term discount rates.

Overall, from analyzing a host of anomalies, our empirical findings illustrate two important insights. First, the patterns of the long-term discount rates in the cross section can be very different from that of the average short-term holding period returns. It is not always the case that the spread in the long-term discount rates is even in the same direction as the short-term expected returns. Often times, the spread is inverted, or the shape in the long-term discount rates is non-monotonic. In these cases, simply assuming the short-term expected returns follow mean-reverting processes, as in Van Binsbergen and Opp (2019) for example, cannot reconcile the difference between the patterns in the long-term discount rates and the short-term expected returns. Second, even though the average short-term holding period return is an important metric that informs the profitability of a dynamically-rebalanced trading strategy, it can be misleading in representing the long-term equity financing cost of a firm. We argue that, compared with the short-term expected returns, the long-term discount rate or discount factor serve as more appropriate measures of firm's equity cost of capital.

Related Literature

Our paper belongs to a growing literature that studies stock price levels. Notable examples in the literature include Cohen, Polk, and Vuolteenaho (2009), Van Binsbergen and Opp (2019), Cho and Polk (2019). Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019) propose testing asset pricing models with price levels instead of short-term returns, and argue that the CAPM cannot be rejected in price level tests. Van Binsbergen and Opp (2019) build a structural model to estimate the cost to the real economy due to asset price distortions with counterfactual analysis. Compared to these earlier work, we present the novel finding that there can be substantial differences between the patterns of the long-term discount rates and that of the expected short-term returns, which cannot be generated by simply assuming mean-reversion in factor premia as in Van Binsbergen and Opp (2019). Moreover, our paper adopts a novel model-free approach. In a way, our methodology is the cross-sectional counterpart of Shiller (1981). Shiller (1981) studies the time variations of stock prices by comparing the price levels of the stock market with the ex-post realized dividends. Our focus is on the cross section; we estimate the long-term discount rates of anomaly portfolios with realized cash flows, and document how stock price levels in the cross section are correlated with various firm characteristics. As a result, we produce multiple new stylized facts that could advance our understanding about the determinants of stock price levels.

Our paper is also related to a large literature in finance and accounting that aims to estimate cost of equity capital, including Botosan (1997), Fama and French (1997), Fama and French (1999), Gebhardt, Lee, and Swaminathan (2001), Easton (2004), Easton and Monahan (2005), Ohlson and Juettner-Nauroth (2005) Hou, Van Dijk, and Zhang (2012), Lu (2016), Levi and Welch (2017) etc. The long-term discount rate studied in this paper can be regarded as the average cost of capital of a firm's equity claim. However, our paper differs from existing literature in a number of ways. First and foremost, we stress the distinction between the long-term cost of equity capital and the average short-term holding period return immediately after portfolio formation. In other words, we emphasize that the factor premia of dynamically-rebalanced trading strategies do not necessarily reflect the long-term equity financing costs of firms. Also, the existing implied cost of capital literature uses analyst subjective forecasts or model predictions as proxies for market expectations of future cash flows. Such approaches suffer from issues of cross-sectionally heterogenous analyst bias or model mis-specification. We circumvent these issues by working with ex-post cash flow realizations and leveraging repeated observations to make inference about the properties of ex-ante cost of capital. In addition, the existing literature often computes cost of capital with cash flow projections over a short horizon of three to five years. Our methodology, on the other hand, takes advantage of the benefit of hindsight and utilizes long realized payout streams that stretch out by as far as 15 years.

Lastly, our paper belongs to the cross-sectional asset pricing literature. We contribute to this literature by offering a new perspective. Structural models in this literature often regard the spread in the short-term expected returns of firms as a manifestation of the differences in the their equity cost of capital. Notable examples include Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Zhang (2005), etc. However, this assumed nexus between short-term expected returns and long-term discount rates has not been thoroughly studied and validated. In other words, the models make statements on long-term discount rates, yet researchers evaluate these models by conducting tests centered around short-term expected returns. We show empirically that this common approach is not always reliable. The cross-sectional patterns found in one do not always conform to the other. Moreover, our paper is closely related to a recent growing strand in this literature that studies the evolution of factor premia over a long horizon. For example, Keloharju, Linnainmaa, and Nyberg (2019) find that anomalies based on high-frequency signals are usually transient and decay fast after portfolio formation. In a contemporaneous paper, Baba-Yara, Boons, and Tamoni (2020) show that the evolutions of factor prema are not tied to the dynamics of firm characteristics. We differ from these papers by revealing that the long-term discount rates in the cross section can take substantially different shapes as the average short-term stock returns, so that simply appealing to mean-reversion in factor premia is insufficient to reconcile the wedge between the two.

The remaining of the paper is organized as the following. Section 2 explains our empirical methodology in detail and presents the relevant derivations. Section 3 reports the empirical findings. Section 4 conducts a case study with the Kogan and Papanikolaou (2013) model to illustrate the implications of our findings for asset pricing theories. Section 5 concludes.

2 Empirical Method

2.1 Data

We obtain stock returns from CRSP. We follow Boudoukh et al. (2007) to calculate the total net payout of common stock for each firm every year using Compustat data. Total net payout adds up dividend payments and share repurchases, and subtracts stock issuance. Our sample is from 1970 to 2018. We start the sample from 1970 because this is when share repurchase and issuance data first became available in Compustat. The term structure of zero-coupon risk-free rates are constructed from the data downloaded from the federal reserve website[4]. All rates and cash flows are in nominal terms.

When a firm exits the market, we treat its delisting market cap as its last payout. Following Shumway (1997) and Shumway and Warther (1999), when the delisting return is missing from CRSP, we assign a delisting return of -35%(-55%) for NYSE and AMEX stocks (for Nasdaq stocks) if the delisting code is 500 or between 520 and 584, and zero otherwise.

Following Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019), we construct a three-dimensional dataset to keep track of the performances and characteristics of the anomaly portfolios. Each observation is identified with a triplet (c,t,i), where c denotes the cohort of the portfolio, i.e. the year of portfolio formation; t is the year of observation and i is the portfolio ID. For example, $D^i_{c,t}$ refers to the total net payout in year t by portfolio i formed in year c. We assume all payouts occur at fiscal year ends. As in Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019), all portfolios are not rebalanced. We keep track of the portfolios for 15 years, and construct a new cohort of value-weighted portfolios by the end of every year (month) for each annual (monthly) frequency anomaly. By the end of the 15-year period, we liquidate the portfolios and treat the liquidating market values of the portfolios as their last payouts. Table 1 reports the list of well-known anomalies that we study in this paper, which includes the Fama-French 5 factors, momentum, idiosyncratic volatility, long-term reversal, etc.

2.2 Estimating the Long-Term Discount Rate

We define the long-term discount rate of an equity claim as the internal rate of return implied by its expected net payouts. An equity claim can, in theory, have an infinite horizon. For our empirical exercises, we follow the literature (Cohen, Polk, and Vuolteenaho (2009), Cho and Polk (2019), etc.) and truncate the horizon at 15 years, and treat the liquidating value of the portfolio by the end of the 15-year period as its last payout. Therefore, the long-term discount rate of an equity claim,

[4] See https://www.federalreserve.gov/data/nominal-yield-curve.htm.

y, is defined by the following equality:

$$
\begin{aligned}
P_0 &= \sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{(1+y)^t} + \frac{\mathbb{E}_0(P_T)}{(1+y)^T}, \\
&\Rightarrow y \equiv f\left(\{\mathbb{E}_0(D_t)\}_t, \mathbb{E}_0(P_T), P_0\right) \qquad (1)
\end{aligned}
$$

where T is the horizon of the cash flows, which is specified as 15 years[5]; P_T is the liquidating value of the equity claim at T; D_t is the total net payout of the stock, which incorporates dividends, share repurchases and share issuance; and $f(\cdot)$ is the internal rate of return function that uncovers y from cash flows and prices.

The challenge of estimating the long-term discount rate y from Equation (1) is that the expected payouts and liquidating value of a stock is not directly observable.[6] The existing literature attempts to circumvent this problem by proxying market expectations with either analysts' subjective forecasts[7] or predictions from parametric models[8]. However, both approaches have limitations, because they may suffer from issues of cross-sectionally heterogenous analyst bias or model mis-specification.

We propose a new methodology to estimate y non-parametrically by replacing the ex-ante expectations with the ex-post realizations. Define the counterfactual price $\hat{P}_0$ as the present value of the realized cash flows:

$$
\begin{aligned}
\hat{P}_0 &\equiv \sum_{t=1}^{T} \frac{D_t}{(1+y)^t} + \frac{P_T}{(1+y)^T} \\
&= \sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)+\varepsilon_t}{(1+y)^t} + \frac{\mathbb{E}_0(P_T)+e_T}{(1+y)^T} \\
&= P_0 + \sum_{t=1}^{T} \frac{\varepsilon_t}{(1+y)^t} + \frac{e_T}{(1+y)^T} \\
&\equiv P_0 + \xi, \qquad (2)
\end{aligned}
$$

where ε_t and e_T are the innovations in payouts and liquidating value, respectively. $\xi \equiv \sum_{t=1}^{T} \frac{\varepsilon_t}{(1+y)^t} + \frac{e_T}{(1+y)^T}$, is the aggregated innovation term, which has zero expectation: $\mathbb{E}_0(\xi) = 0$.

[5] We also show the key results with 10-year and 5-year horizons in Appendix C.

[6] The internal rate of return of an arbitrary cash flow stream might be non-unique in general. However, it is unique when all cash flows are positive.

[7] See, for example, Gebhardt, Lee, and Swaminathan (2001), Easton and Monahan (2005), Easton and Sommers (2007), Guay, Kothari, and Shu (2011), etc.

[8] See, for example, Ohlson and Juettner-Nauroth (2005), Hou, Van Dijk, and Zhang (2012), Levi and Welch (2017) etc.

According to Equations (1) and (2), the discount rate y can be represented as either a non-linear function of the true price or the counterfactual price:

$$
\begin{aligned}
y &= f\left(\{\mathbb{E}_0(D_t)\}_t, \mathbb{E}_0(P_T), P_0\right) \\
&= f\left(\{D_t\}_t, P_T, \hat{P}_0\right) = f\left(\{D_t\}_t, P_T, P_0 + \xi\right),
\end{aligned} \tag{3}
$$

where $f(\cdot)$ is the internal rate of return function that uncovers y from cash flows and prices.

Equation (3) shows that y can be uncovered from either the true price and the expected cash flows, or the counterfactual price and the realized cash flows. And in the second representation, only the counterfactual price is not directly observable.

To illustrate the intuition of our methodology, let's first take the first-order approximation of Equation (3) around P_0:

$$
\begin{aligned}
& y \approx f\left(\{D_t\}_t, P_T, P_0\right) + f_P\left(\{D_t\}_t, P_T, P_0\right)\xi \\
\Rightarrow\ & y \approx \mathbb{E}_0\left(f\left(\{D_t\}_t, P_T, P_0\right)\right) = \mathbb{E}_0(\tilde{y}).
\end{aligned} \tag{4}
$$

where the second line above takes the expectation on both sides of the first line and utilizes $\mathbb{E}_0(\xi) = 0$ to eliminate the unobservable ξ; and $\tilde{y} \equiv f\left(\{D_t\}_t, P_T, P_0\right)$ is the ex-post discount rate inferred from the realized cash flows for a given path.
Therefore, y and ξ can be approximated to the first order by:

$$
\hat{y}^{1st} = \hat{\mathbb{E}}\left(f\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\right) = \frac{1}{J}\sum_{j=1}^{J} \tilde{y}^j,
$$

$$
\tilde{\xi}^j = \frac{\hat{y}^{1st} - f\left(\{D_t\}_t^j, P_T^j, P_0^j\right)}{f_P\left(\{D_t\}_t^j, P_T^j, P_0^j\right)}. \tag{5}
$$

where $\hat{\mathbb{E}}(\cdot)$ denotes the sample mean; j is the index of a specific trajectory of payouts and prices; and J is the total number of trajectories.

Intuitively, $\hat{y}^{1st}$ is the sample average of $f\left(\{D_t\}_t, P_T, P_0\right)$, which is the internal rate of return of the realized cash flows. Therefore, as a first step, one can approximate y by simply computing the internal rate of return of the realized cash flows for portfolios formed at different points in time and then take an average.

However, the first-order approximation is biased because it approximates the discount rate $y \equiv f\left(\{\mathbb{E}_0(D_t)\}_t, \mathbb{E}_0(P_T), P_0\right)$ with $\mathbb{E}_0\left(f\left(\{D_t\}_t, P_T, P_0\right)\right)$. Therefore, the first-order approximation differs from the true discount rate by a Jensen's term because it changes the order of the expectation operator. And the Jensen's term corresponds to the high-order terms ignored by the first-order approximation of Equation (4). Therefore, one can better approximate y and try to make up for the Jensen's term by expanding Equation (3) to the higher orders:

$$y = f\left(\{D_t\}_t, P_T, P_0\right) + f_P\left(\{D_t\}_t, P_T, P_0\right)\xi + \frac{1}{2} f_{PP}\left(\{D_t\}_t, P_T, P_0\right)\xi^2 + \frac{1}{6} f_{PPP}\left(\{D_t\}_t, P_T, P_0\right)\xi^3 + \cdots, \tag{6}$$

or

$$\begin{aligned} y &= \mathbb{E}_0\left(f\left(\{D_t\}_t, P_T, P_0\right)\right) + \mathbb{E}_0\left(f_P\left(\{D_t\}_t, P_T, P_0\right)\xi\right) \\ &\quad + \frac{1}{2}\mathbb{E}_0\left(f_{PP}\left(\{D_t\}_t, P_T, P_0\right)\xi^2\right) + \frac{1}{6}\mathbb{E}_0\left(f_{PPP}\left(\{D_t\}_t, P_T, P_0\right)\xi^3\right) + \cdots \end{aligned} \tag{7}$$

where $f_P(\cdot)$, $f_{PP}(\cdot)$ and $f_{PPP}(\cdot)$ denote the first, second and third partial derivative of $f(\cdot)$ with respect to P_0.

Motivated by Equation (7), we adopt the third-order approximation[9] as the estimate of y:

$$\begin{aligned} \hat{y} \equiv \hat{y}^{3rd} &= \hat{\mathbb{E}}\left(f\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\right) + \hat{\mathbb{E}}\left(f_P\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\tilde{\xi}^j\right) \\ &\quad + \frac{1}{2}\hat{\mathbb{E}}\left(f_{PP}\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\left(\tilde{\xi}^j\right)^2\right) + \frac{1}{6}\hat{\mathbb{E}}\left(f_{PPP}\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\left(\tilde{\xi}^j\right)^3\right) \\ &= \hat{y}^{1st} + \frac{1}{2}\hat{\mathbb{E}}\left(f_{PP}\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\left(\tilde{\xi}^j\right)^2\right) + \frac{1}{6}\hat{\mathbb{E}}\left(f_{PPP}\left(\{D_t\}_t^j, P_T^j, P_0^j\right)\left(\tilde{\xi}^j\right)^3\right), \end{aligned}$$

where $\hat{\mathbb{E}}(\cdot)$ denotes the sample mean; and $\tilde{\xi}^j = \frac{\hat{y}^{1st} - f\left(\{D_t\}_t^j, P_T^j, P_0^j\right)}{f_P\left(\{D_t\}_t^j, P_T^j, P_0^j\right)}$ is from the first-order approximation of Equation (5).[10]

We discuss the accuracy of our discount rate estimates in Section 2.3.

Once the long-term discount rate, $\hat{y}$, is estimated, we define its difference with the 15-year zero-coupon risk-free rate, r_f^{15}, as the long-term risk premium.

[9] Simulations show that approximating y to the fourth order and beyond does not improve the accuracy of estimation any more.

[10] One could estimate $\left\{\tilde{\xi}^j\right\}$ recursively in each iteration of the Taylor approximations, but (untabulated) simulations show that the difference is negligible.

2.3 Accuracy of the Estimation Methodology

We provide two sets of analyses to evaluate the accuracy of our estimation methodology. The first approach measures the magnitude of the biases of our discount rate estimates with canonical structural cash flow processes. The second approach is model-free, whereby we non-parametrically evaluate the accuracy of our estimates with the realized cash flows in the data using block bootstrap techniques. We show that these two approaches arrive at the same conclusion that our estimation methodology is sufficiently accurate to study the cross-sectional patterns of long-term discount rates.

2.3.1 The Structural Approach

We study a number of parametric cash flow processes in this section, and show that the biases of our estimations are sufficiently small across these models with parameters iterated over large grids.

We run simulations with the following 6 cash flow specifications: 1) the original Fama and Babiak (1968) process; 2) the adjusted Fama and Babiak (1968) process; 3) the original Leary and Michaely (2011) process; 4) the adjusted Leary and Michaely (2011) process; 5) the long-run risk process of Bansal and Yaron (2004); and 6) the rare disaster process of Barro (2006).

For each model, we iterate parameters across large grids.[11] For each set of parameters, we measure the magnitude of estimation bias by taking the difference between the estimated discount rate and the true discount rate in the simulation. Table 2 reports the statistics of the biases across different models. The table shows that, in the most extreme case, the Barro (2006) process is associated with a bias of 46 bps, and the biases are much smaller than that in most cases. Across all models and all parameter values, the average bias in the simulation is 7 bps, the median is 3bps, and the 95th percentile is 29 bps. These biases are very small compared to the empirically-estimated spreads of discount rates across anomaly portfolios as will be presented in Section 3.

2.3.2 The Model-Free Approach

In addition to the structural approach, we also estimate the biases non-parametrically with a block bootstrap exercise using realized cash flows in the data for each anomaly portfolio.

Figure 1 illustrates the block bootstrap procedure. For a given anomaly decile, we construct a trajectory by forming a portfolio by the end of each year or month, and tracking its cash flows for 15 years since formation. We then divide the trajectories into 5-year blocks, and randomly draw the blocks across trajectories of the same anomaly decile to form new simulated trajectories.

[11] See Appendix A, for the details of the specifications of the models and parameter ranges.

To preserve the evolution of portfolio characteristics, blocks are indexed by levels. Levels 1, 2 and 3 represent the first, second and third 5-year episodes since portfolio formation. Each block records the time series of capital appreciation $\left\{Rx_t \equiv \frac{P_t}{P_{t-1}}\right\}_t$ and net-payout-to-price ratio $\left\{DP_t \equiv \frac{D_t}{P_t}\right\}_t$ of the original portfolio. When blocks are recombined into simulated trajectories, the time series of prices and cash flows are constructed from these two series within the blocks.

Once the cash flows of the simulated trajectories are constructed, their initial price P_0 is determined by discounting the average cash flows across the simulated trajectories with the discount rate estimate, $\hat{y}$, from the true trajectories. In other words, the estimated discount rate from the true trajectories acts as the true rate in the simulated environment and prices the simulated trajectories.

The block bootstrap procedure preserves the evolution of portfolio characteristics precisely within each 5-year block. The indexing of the blocks also attempts to capture portfolio evolution across 5-year episodes. Nonetheless, the breaking points at every 5-year-end when one block is stitched to another might introduce an inevitable wedge between the simulated trajectories and the true data generating process. To alleviate such a concern, Appendix B shows that such a wedge is small across the structural cash flow processes that we consider with parameters iterated over large grids.

To estimate the bias in $\hat{y}$, we take the difference between the discount rate estimate from the simulated trajectories and the true rate in the simulation. Table 3 reports the estimated biases for all the anomaly deciles. The table shows that the biases due to higher-order term truncation are in general very small. In the worst case scenario, decile 2 of the book-leverage-sorted portfolios has an estimated bias of -54 bps. The median absolute bias is much smaller and is about 21 bps.

Therefore, the model-free approach and the structural approach reach the same conclusion that the biases of our estimation methodology is sufficiently small in order to detect the patterns of long-term discount rates in the cross section of stocks.

2.4 Estimating the Relative Discount Factor

In addition to the long-term discount rate, we also define and estimate the relative discount factors of the anomaly portfolios to capture the effects of cash flow duration on firm's equity financing cost. The relative discount factor, or RDF, is defined as:

$$RDF \equiv \frac{P_0}{\sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{\left(1+r_{f,0,t}\right)^t} + \frac{\mathbb{E}_0(P_T)}{\left(1+r_{f,0,T}\right)^T}},$$

where $r_{f,0,t}$ is the t-year zero-coupon risk-free rate extracted from the term structure of interest rates observed at time 0.

The relative discount factor is a ratio between two prices: the actual price of the stock, and its counterfactual price if the cash flows were priced with the risk-free rates. In other words, the relative discount factor measures the wedge in funding cost between the firms in the anomaly portfolio and the US government when issuing default-free securities with the same expected cash flows.

Since the *RDF* is based on cash flow expectations, its empirical counterpart can be easily estimated with:

$$
\begin{aligned}
\frac{1}{RDF} &= \mathbb{E}_0\left(\frac{\sum_{t=1}^{T}\frac{D_t}{\left(1+r_{f,0,t}\right)^t}+\frac{P_T}{\left(1+r_{f,0,T}\right)^T}}{P_0}\right)\\
\widehat{\left(\frac{1}{RDF}\right)} &= \hat{\mathbb{E}}\left(\frac{\sum_{t=1}^{T}\frac{D_t}{\left(1+r_{f,0,t}\right)^t}+\frac{P_T}{\left(1+r_{f,0,T}\right)^T}}{P_0}\right)\\
\widehat{RDF} &= 1/\widehat{\left(\frac{1}{RDF}\right)}.
\end{aligned}
$$

When the sample size increases, the sample mean $\hat{\mathbb{E}}(\cdot)$ converges to the expectation $\mathbb{E}_0(\cdot)$. Therefore, the estimate of *RDF* is consistent. Its standard error is estimated along with the standard error of the long-term discount rate in the block bootstrap procedure described below.

2.5 Estimating the Standard Errors

As standard in the literature, we keep track of our portfolios for 15 years, and construct a new cohort of portfolios by the end of every year or month. As a result, our sample features non-trivial overlap. Conceptually, the estimation of the long-term discount rates is not necessarily less accurate than the short-term expected returns. On the one hand, since our focus is on the long term, we naturally have fewer independent observations in the data compared to the short-term returns. On the other hand, due to the amplification effect of cash flow duration, the long-term discount rates are also much less volatile than the short-term returns in the time-series.[12] Therefore, compared with the estimation of short-term expected returns, there are fewer observations to use in the estimation of long-term discount rates, but the long-term discount rates are also much less volatile than the short-term returns, which makes them easier to estimate.

[12] It is well documented that the long-horizon stock returns, which are closely related to our long-term discount rates, are less volatile than the short-term returns. See, for example, Fama and French (1988), Poterba and Summers (1988), Lo and MacKinlay (1988), Richardson and Stock (1989), Kim, Nelson, and Startz (1991), Richardson (1993), Seigel (2020), etc.

To correctly compute the standard errors of our long-term discount rate and discount factor estimates, we again perform the block bootstrap procedure presented in Section 2.3.2. To estimate the standard error of $\hat{y}$, we construct batches of the simulated trajectories. Each simulated batch contains the same number of trajectories as the true sample, and yields a discount rate estimate $\hat{y}^b$. And the standard error of $\hat{y}$ is estimated as the standard deviation of the discount rate estimates across batches, i.e. $se(\hat{y}) = std\left(\hat{y}^b\right)$. The standard error of RDF is estimated similarly.

3 Empirical Findings

Table 1 lists the set of prominent asset pricing anomalies that we study in this paper. We broadly classify these anomalies into three groups: 1) anomalies motivated by models of funding cost; 2) anomalies based on historical trading prices and volume; and 3) additional anomalies. Our classification is not a systematic approach, but simply an arrangement to facilitate the presentation of our empirical findings.

3.1 Anomalies Based on Funding Cost

We first consider a number of anomalies motivated by models of funding cost. They include the five factors summarized by Fama and French (2015) and a credit rating factor that sorts stocks by firm's credit rating. Table 4 presents our findings. Following the convention in the literature, we form 10 value-weighted portfolios for each anomaly, and calculate the difference between decile 10 and decile 1, i.e. HML. The indices of the portfolios increase with the sorting characteristic. In order to identify potential U-shape/hump-shape across portfolios, we also construct the extreme-minus-middle (EMM) portfolio, which is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5,6.

3.1.1 Fama-French Three Factors

Panels (a), (b) and (c) of Table 4 show that the shapes of the long-term discount rates of the Fama-French three factors are consistent with the shapes of the short-term factor premia.

As one of the most celebrated models in finance, the Capital Asset Pricing Model (CAPM) predicts that the cost of capital of a stock should be linear and increasing with its market beta. Graham and Harvey (2001) also show survey evidence that most CFOs adopt the CAPM for capital budgeting. However, empirical studies generally find that the average holding period returns of stocks in the cross section are too insensitive to their market betas according to the CAPM predictions.[13] Panel (a) of Table 4 shows that the point estimates of the long-term discount rates only slightly increase with market beta, and the high-minus-low spread is insignificant. Therefore, similar to the findings with the short-term expected returns, the long-term discount rate estimation does not generate a steep security market line.

Banz (1981) and Bhandari (1988) first show that the market capitalization and book-to-market ratio of a stock are positively correlated with its expected return. The size and book-to-market factors were then included in the celebrated Fama and French (1992) three-factor model. Moreover, Berk, Green, and Naik (1999) propose a production-based asset pricing model and illustrate how size and book-to-market ratio can be connected to firm's cost of capital. Panels (b) and (c) of Table 4 confirm the large differences in the short-term average returns for stocks with different sizes and book-to-market ratios. The panels also show that there are large spreads in the long-term discount rates and the relative discount factors along these two dimensions, consistent with the spread in the average holding period returns. Therefore, our exercise confirms that market capitalization and book-to-market ratio are significantly correlated with firm's equity financing cost. In addition, our estimates of the relative discount factor show that firms in the smallest decile are only able to raise 41 cents on a dollar, whereas firms in the largest decile are able raise 63 cents on a dollar. Similarly, extreme value firms are only able to raise 36 cents on a dollar, and extreme growth firms are able to raise 60 cents on a dollar. On average, the equity market in the US as a whole is able to raise 57 cents on a dollar (untabulated).

3.1.2 Gross Profitability and Investment

Novy-Marx (2013) and Titman, Wei, and Xie (2004) show, respectively, that gross profitability and investment rate are correlated with the expected short-term stock returns in the cross section. Fama and French (2015) incorporate these two findings into their five-factor model on the grounds that these two factors can be justified by a discount-cash-flow model and should be correlated with firm's financing cost. Panels (d) and (e) show that, indeed, the average short-term holding period returns show a strong gradient along gross profitability and investment rate with the same sign as the findings of Novy-Marx (2013) and Titman, Wei, and Xie (2004). However, the estimation of the long-term discount rates and relative discount factors reveals interesting and surprising findings. The panels show that, despite the large spread in the average short-term holding period returns, these is little or no HML spread in the long-term discount rates across portfolios sorted by these two characteristics. Instead, these two sets of portfolios show strong U-shape in the long-term discount rates. Both the most profitable and least profitable firms tend to face higher equity financing costs

[13] See, for example, Black et al. (1972), Baker, Bradley, and Wurgler (2011), Frazzini and Pedersen (2014), etc.

than others. And the findings are similar for firms with extreme investment rates.

We do not attempt to reconcile the wedge between the shapes of the long-term discount rates and the average short-term holding period returns in our study. We await future research for a complete reconciliation. That said, according to the following accounting identity, the wedge has to be caused by either the term structure of expected holding period returns or the co-movement between holding period returns and corporate payouts:

$$
\begin{aligned}
\sum_{t=1}^{\infty} \frac{\mathbb{E}_0(D_t)}{(1+y)^t} = P_0 &= \mathbb{E}_0\left[\frac{D_0+P_1}{(1+h_1)}\right] \\
&= \sum_{t=1}^{\infty} \mathbb{E}_0\left[\frac{D_t}{\Pi_{s=1}^{t}(1+h_s)}\right] \\
&= \underbrace{\sum_{t=1}^{\infty} \frac{\mathbb{E}_0(D_t)}{\Pi_{s=1}^{t}(1+\bar{h}_s)}}_{\text{Term Structure of Anomaly Premium}} + \underbrace{\sum_{t=1}^{\infty}\left\{\mathbb{E}_0\left[\frac{D_t}{\Pi_{s=1}^{t}(1+h_s)}\right] - \frac{\mathbb{E}_0(D_t)}{\Pi_{s=1}^{t}(1+\bar{h}_s)}\right\}}_{\text{Co-movement between Payouts and Returns}},
\end{aligned}
\tag{8}
$$

where $\bar{h}_s$ is the expected holding period return from period $s-1$ to s.

Since the long-term discount rates of several anomalies show a different shape compared to the average short-term holding period returns, our findings cannot be easily reconciled by the mean-reversion of factor premia alone as the mechanism assumed in Van Binsbergen and Opp (2019). Our discoveries strengthen the findings in a contemporaneous paper by Baba-Yara, Boons, and Tamoni (2020), where they show that the most recent observable characteristics of a firm are not sufficient statistics of its expected short-term return, and that there is a disconnect between the evolution of characteristic premia and the mean-reversion of the characteristics themselves. Our findings of the U-shape in the long-term discount rates for gross profitability and investment portfolios further suggest that the characteristic premia might not follow mean-reverting processes at all.

3.1.3 Credit Rating

Voluminous previous research shows that a firm's credit rating is strongly correlated with its borrowing cost. High credit rating firms pay lower yields on their corporate bonds than low credit rating firms. However, there is only very little evidence regarding the relation between a firm's credit rating and its equity financing cost. As one of the few exceptions, Avramov et al. (2009) argue that, surprisingly, firms with high credit ratings also generate higher average short-term holding period returns. Panel (f) of Table 4 replicates their work by sorting stocks based firms' S&P credit ratings. We confirm that firms with high credit ratings generate slightly higher average holding period returns than the low credit rating firms during the first year since portfolio formation, although the difference is small and insignificant because we have a different and longer sample compared to Avramov et al. (2009). However, the investigation of the long-term discount rates, again, reveals a striking finding that there is a large spread along firm's credit rating. According to our estimates, the high-credit-rating firms face much lower equity financing costs than the low-credit-rating firms. On average, the highest-credit-rating firms are able to raise 63 cents on a dollar, whereas the lowest-credit-rating firms are only able to raise 37 cents on a dollar.

This exercise demonstrates how our methodology reveals important patterns in firms' equity financing costs or stock price levels that would have been overlooked with tests centered around the short-term holding period returns of dynamic trading strategies.

3.2 Anomalies Based on Historical Trading Prices and Volume

Table 5 presents the findings regarding anomalies based on historical trading prices and volume. The panels in the table reveal an interesting pattern that low-frequency signals are more informative about long-term discount rates than high-frequency signals. The shape of the long-term discount rates is, again, not necessarily consistent with the shape of the average short-term holding period returns.

3.2.1 Short-Term Momentum and Long-Term Reversal

Jegadeesh and Titman (1993) show that stock return momentum is a salient anomaly that features large spread in the holding period returns of the stocks that have recently performed well compared to the stocks that have performed poorly. Panel (a) of Table 5 confirms the large momentum premium in our sample. The momentum anomaly has received significant amount of attention in the literature not only because of its striking magnitude, but also because it is difficult for structural models to generate a sizable momentum effect in firm's equity financing cost. Panel (a) shows that the long-term discount rates across momentum portfolios are almost flat. The shape is slightly non-monotonic with the discount rates of both the winning and losing stocks being higher than the stocks in the middle. The losing stocks also have a slightly higher a long-term discount rate than the winning stocks, by about 78 bps.

Again, our findings cannot be explained by the mean-reversion mechanism in the expected short-term holding period returns, but imply a reversal in the momentum premium. Figure 2 plots the term structure of the momentum premium. Consistent with the empirical findings regarding the reversal of momentum premium such as Lee and Swaminathan (2000) and Jegadeesh and Titman (2001), the figure shows that winning stocks generate significantly higher returns during the first year since portfolio formation than losing stocks, but they also generate significantly lower returns during the second year. Such a finding echoes the accounting identity of Equation (8) in the sense that it illustrates how the term structure in the expected holding period returns can drive a wedge between the shape of the long-term discount rates and the shape of the expected short-term returns. The almost flat long-term discount rates across momentum portfolios combined with the reversal in momentum premium is inconsistent with the mechanical channel proposed by Conrad and Kaul (1998) where the momentum effect arises mostly due to the cross-sectional heterogeneity in expected returns. Our findings are consistent with the overreaction channel proposed in behavioral models by Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999), etc, in that these models suggest that the momentum effect is caused by investors' temporary overreaction so that prices will eventually revert.

De Bondt and Thaler (1985) argue that the prices of stocks with poor performances over the most recent 5 years are too depressed compared to the long-term winning stocks and show that the long-term historical performance of a stock negatively predicts its short-term holding period return going forward. Panel (b) of Table 5 confirms their findings regarding the average short-term returns and also shows the presence of large spread in the long-term discount rates in the same direction as the short-term premium. Thus, the shape of the long-term discount rates is consistent with the behavioral channel proposed by De Bondt and Thaler (1985).

3.2.2 Idiosyncratic Volatility

Ang et al. (2006) present the striking finding that stocks with high idiosyncratic volatility tend to perform poorly compared to stocks with low idiosyncratic volatility. The premium of the idiosyncratic volatility anomaly is large and surprising as the more volatile stocks generate much lower returns the less volatile returns. Panel (c) of Table 5 replicates their exercise and confirms that, indeed, high idiosyncratic volatility stocks tend to underperform significantly compared low idiosyncratic volatility stocks short after portfolio formation. However, the pattern in the long-term discount rates is opposite to the average short-term holding period returns. Our estimates show that firms with more volatile stocks face about 2% higher equity cash flow discount rates than firms with less volatile stocks. Figure 3 suggests that the idiosyncratic volatility premium tends to revert after 10 years since portfolio formation. But the finding is inconclusive as the coefficients in the figure are insignificant. We leave it to future research to precisely pinpoint the cause of the wedge between the long-term discount rates and the average short-term returns across idiosyncratic volatility portfolios. While we do not resolve the Ang et al. (2006) anomaly by providing an explanation for the patterns in the short-term expected returns, we clarify that the idiosyncratic volatility anomaly is not a statement about firm's equity financing cost, but rather a effect in the short-term returns.

3.2.3 Abnormal Volume

Gervais, Kaniel, and Mingelgrin (2001) find that stocks experiencing unusually high (low) trading volume over a day or a week tend to appreciate (depreciate) over the course of the following month. Panel (d) of Table 5 confirms their finding of the significant high-volume premium. While Gervais, Kaniel, and Mingelgrin (2001) show that a substantive high-volume-premium spread persists for at least 6 months, the panel shows the effect dissipates over longer horizons. The long-term discount rates across abnormal volume portfolios are almost flat; although the 46 bps spread between deciles 10 and 1 is statistically significant, the economic magnitude is small. Consequently, the impact on long-term equity financing costs is modest.

3.3 Additional Anomalies

Campbell, Hilscher, and Szilagyi (2008) estimate firm-level probability of financial distress with a dynamic logit model and produce the puzzling finding that the average short-term holding period returns in cross section decrease with firm's probability of distress. Panel (a) of Table 6 confirms their finding regarding the surprising gradient in the average short-term holding period returns. However, the panel also shows that the long-term discount rates are flat across portfolios sorted by fail probability. Therefore, similar to the Ang et al. (2006) idiosyncratic volatility anomaly, the Campbell, Hilscher, and Szilagyi (2008)'s distress risk anomaly is mostly a short-term phenomenon and it does not have surprising implications for firm's long-term equity financing cost.

Panels (b) and (c) of Table 6 studies how leverage is correlated with firm's equity financing cost. Consistent with Fama and French (1992) and Gomes and Schmid (2010), book leverage does not seem to be correlated with expected stock return, whereas market leverage positively predicts expected stock returns in the cross section. The pattern of the long-term discount rates across leverage-sorted portfolios is consistent with that of the average short-term holding period returns.

Balakrishnan, Bartov, and Faurel (2010) report that the return on asset (ROA) metric positively predicts stock returns in the cross section shortly after quarterly announcements. Panel (d) of Table 6 confirms their finding regarding the pattern of the average short-term holding period returns, but the panel also shows that the pattern of the long-term discount rates is inverted. Therefore, high ROA firms tend to generate lower holding period returns shortly after announcements, but they also have higher long-term discount rates at the same time. Also, interestingly, the exercise shows that, as the two signals both related to firm's profitability, gross profitability and ROA generate similar patterns in the average short-term holding period returns, but different patterns in the long-term discount rates. We leave the reconciliation of such a difference to future research.

3.4 Out-of-Sample Performances

We decide to start our main sample from 1970 due to the availability of accounting variables, which are needed to construct most of the anomalies. That said, for a small set of anomalies that are based exclusively on prices or trading volume (momentum, long-term reversal, idiosyncratic volatility, abnormal volume, market beta and size), we are able to extend the sample back to 1926. We use the early data (1926-1970) to perform an out-of-sample test with these anomalies.

Table **??** presents the results, where for convenient comparisons we also reiterate the relevant results from the main sample (1970-2018). For momentum and long-term reversal, average short-term returns and long-term discount rates patterns are similar in both samples. For idiosyncratic volatility, the short-term premium disappears in the early sample, yet the long-term discount rates show the same pattern in both samples. For abnormal volume, the short-term premium is stronger in the early sample, but the spread in the long-term discount rates becomes weaker and insignificant. For market beta, the spread in the average short-term returns is insignificant for both samples; the long-term discount rates spread becomes significant in the early sample, while the spread in the relative discount factors is significant in both. Lastly, the size anomaly has a similar short-term premium for both samples, but the spread in the long-term discount rates is insignificant in the early sample, even though the relative discount factors show significant spread in both. For all of these anomalies, the point estimates of the spread in the long-term discount rates always have the same sign in both samples. Overall, these findings suggest that the long-term discount rates of this set of anomalies generally display similar patterns in the early and main samples.

4 Implications for Asset Pricing Theories, A Case Study with Kogan and Papanikolaou (2013)

To illustrate the implications of our empirical exercise for asset pricing theories, we conduct a case study with Kogan and Papanikolaou (2013), which is representative of a large class of structural models, by evaluating their production-based asset pricing model with our methodology.

Kogan and Papanikolaou (2013) propose a relatively simple channel that is able to generate *quantitatively* realistic short-term expected returns for multiple anomalies, including: idiosyncratic volatility, investment rate, gross profitability, etc.[14] The mechanism of their model can be summarized as the following. According to their theory, the value of a firm has two components – the

[14] Kogan and Papanikolaou (2013) is also able to match patterns along Tobin's Q, earnings-to-price ratio and market beta. We only focus on these three anomalies because we identified inconsistent patterns between short-term expected returns and the long-term discount rates in the data.

value of asset in place (VAP_f) and the present value of growth opportunities ($PVGO_f$):

$$V_f = VAP_f + PVGO_f.$$

The value of asset in place is the present value of the cash flows generated by all existing projects that have already been installed in the firm; and the present value of growth opportunities reflects the total NPV of the investment opportunities of the firm, which is the present value of all cash flows coming from potential future projects that have not yet been adopted by the firm.

Kogan and Papanikolaou (2013) argue that with the existence of investment-specific-technology shocks, the cash flows generated by these two components might have different risk profiles and would be discounted with different rates. Since the firm is a portfolio of asset in place and growth opportunities, the overall discount rate of the firm is thus determined by the ratio between $PVGO_f$ and VAP_f. They further show that, under certain assumptions, the ratio between $PVGO_f$ and VAP_f is monotonically related to various observable characteristics.

Therefore, in their model, firm characteristics reflect the ratio between $PVGO_f$ and VAP_f, which, in turn, determines the overall cash flow discount rate of the firm. The heterogeneity in firms' the overall discount rates in the cross section then generates the spreads in the short-term expected returns along various characteristics.[15]

Kogan and Papanikolaou (2013) show that a calibrated model is able to quantitatively match numerous anomalies regarding short-term expected returns. However, the mechanism of their model relies crucially on the nexus between long-term cash flow discount rates and short-term expected returns. In the model, firms have different short-term expected returns because they have different long-term discount rates. In other words, Kogan and Papanikolaou (2013) interpret the spreads in the short-term expected returns as a manifestation of the differences in the long-term discount rates, and then propose a mechanism to generate heterogeneous long-term discount rates in order to match the patterns in short-term expected returns. Therefore, an important implication of their model is that long-term discount rates must have the same pattern as short-term expected returns in the cross section.

Table 8 replicates Kogan and Papanikolaou (2013)'s simulation results and shows that their model is indeed able to closely match the spreads in the short-term expected returns along portfolios sorted by idiosyncratic volatility, investment rate and gross profitability. However, the table also shows that the long-term discount rates in the model always take the same shape as the short-term expected returns, which is evidently counterfactual. In the data, the shape of long-term discount rates is inverted for idiosyncratic volatility and exhibits strong U-shape for investment

[15]We refer the reader to their paper for the details of model specification and calibration.

and gross profitability.

Kogan and Papanikolaou (2013) is not just a single instance, but rather, it represents the view regarding asset pricing anomalies of a large class of structural models including: Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Li, Livdan, and Zhang (2009), Liu, Whited, and Zhang (2009), Li and Zhang (2010), Papanikolaou (2011), Belo, Lin, and Bazdresch (2014), Kogan and Papanikolaou (2014), Belo et al. (2017), Belo, Lin, and Yang (2019), Gofman, Segal, and Wu (2020), Dou, Ji, and Wu (2021), etc. This class of models interprets the observed spreads in the average returns of dynamically-rebalanced trading strategies as manifestations of the differences in firms' equity financing costs and valuation levels. However, such a connection between short-term expected returns and long-term discount rates, which serves as the crucial premise for all of these models, has not yet been carefully examined. Our simple non-parametric methodology fills this void. And our findings regarding the disconnect between the short term and the long term for a number of well-known anomalies present a challenge for this line of thinking.

5 Conclusion

In this paper, we propose a simple novel non-parametric methodology to estimate firms' long-term discount rates/discount factors with ex-post realized cash flows. Compared to the existing implied cost of capital (ICC) literature, our methodology has the advantage of being objective and model-free. Therefore, our methodology produces "clean" results that are not contaminated by analyst subjective biases or model mis-specifications.

By applying our methodology to the cross section of stocks, we discover that the patterns in long-term discount rates are substantially different from short-term expected returns for multiple well-known anomalies. The empirical findings of our paper provide a number of key implications for the cross-sectional asset pricing literature. First, our findings shed new light on the interpretation of several famous puzzles such as the idiosyncratic volatility anomaly by Ang et al. (2006) or the distress risk anomaly by Campbell, Hilscher, and Szilagyi (2008). For example, while high idiosyncratic volatility stocks do tend to generate much lower returns than low volatility stocks shortly after portfolio formation, we show that the long-term discount rates of the high volatility stocks are actually significantly higher than the low volatility stocks. Therefore, the idiosyncratic volatility anomaly is not a statement about firms' equity financing costs, but rather a short-term effect in the stock returns. Granted that we do not fully resolve the puzzle by explaining the cause of the pattern in short-term average returns, the new refined interpretation of this puzzle makes it arguably less puzzling. Second, our methodology enables us to identify new characteristics are that important for firms' equity financing costs but were previously overlooked by studies centered around short-term returns. For example, while an annually rebalanced trading strategy on firm's credit rating does not generate significant trading profits, we show that firms with low credit ratings do indeed face much higher long-term discount rates than high-credit-rating firms. Last but not least, our findings provide a new insight for a large class of structural models that aims to explain anomalies with rational forces. These models interpret the spreads in short-term average returns as manifestations of the differences in firms' equity financing costs, and propose various mechanisms relying on the nexus between short-term expected returns and long-term discount rates. However, we find that this assumed link between the short term and the long term is not always tenable, as we identify a group of prominent anomalies whose long-term discount rates exhibit substantially different patterns compared to the short-term expected returns. Therefore, reconciling the short term and the long term for these inconsistent anomalies would be a new challenge for this class of models.

Overall, our paper aims to stress that average returns generated by dynamically-rebalanced trading strategies do not necessarily reflect the equity financing costs faced by firms in the cross section. The short term and the long term are both important, but in different ways. Short-term expected returns inform the profitability of dynamic trading strategies, whereas long-term discount rates reflect firms' equity financing costs and valuation levels. And contrary to common beliefs, the patterns in the short term do not always coincide with the long term. As a methodological contribution, our simple non-parametric estimation can be regarded as the long-term counterpart of the Fama-French approach for investigating long-term discount rates in the cross section of stocks. With its help, we are able to uncover a host of new stylized facts regarding firms' equity financing costs. And these new findings can further inspire future research to better understand the mechanisms underlying various asset pricing anomalies as well as the determinants of stock price levels.

References

Ang, Andrew, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. “The cross-section of volatility and expected returns.” *The Journal of Finance* 61 (1):259–299.

Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov. 2009. “Credit ratings and the cross-section of stock returns.” *Journal of Financial Markets* 12 (3):469–499.

Baba-Yara, Fahiz, Martijn Boons, and Andrea Tamoni. 2020. “New and old sorts: Implications for asset pricing.” *Martijn and Tamoni, Andrea, New and Old Sorts: Implications for Asset Pricing (January 31, 2020)* .

Baker, Malcolm, Brendan Bradley, and Jeffrey Wurgler. 2011. “Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly.” *Financial Analysts Journal* 67 (1):40–54.

Balakrishnan, Karthik, Eli Bartov, and Lucile Faurel. 2010. “Post loss/profit announcement drift.” *Journal of Accounting and Economics* 50 (1):20–41.

Bansal, Ravi, Dana Kiku, and Amir Yaron. 2009. “An empirical evaluation of the long-run risks model for asset prices.” Tech. rep., National Bureau of Economic Research.

Bansal, Ravi and Amir Yaron. 2004. “Risks for the long run: A potential resolution of asset pricing puzzles.” *The journal of Finance* 59 (4):1481–1509.

Banz, Rolf W. 1981. “The relationship between return and market value of common stocks.” *Journal of financial economics* 9 (1):3–18.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny. 1998. “A model of investor sentiment.” *Journal of financial economics* 49 (3):307–343.

Barro, Robert J. 2006. “Rare disasters and asset markets in the twentieth century.” *The Quarterly Journal of Economics* 121 (3):823–866.

Belo, Frederico, Jun Li, Xiaoji Lin, and Xiaofei Zhao. 2017. “Labor-force heterogeneity and asset prices: The importance of skilled labor.” *The Review of Financial Studies* 30 (10):3669–3709.

Belo, Frederico, Xiaoji Lin, and Santiago Bazdresch. 2014. “Labor hiring, investment, and stock return predictability in the cross section.” *Journal of Political Economy* 122 (1):129–177.

Belo, Frederico, Xiaoji Lin, and Fan Yang. 2019. "External equity financing shocks, financial flows, and asset prices." *The Review of Financial Studies* 32 (9):3500–3543.

Berk, Jonathan B, Richard C Green, and Vasant Naik. 1999. "Optimal investment, growth options, and security returns." *The Journal of finance* 54 (5):1553–1607.

Bhandari, Laxmi Chand. 1988. "Debt/equity ratio and expected common stock returns: Empirical evidence." *The journal of finance* 43 (2):507–528.

Black, Fischer, Michael C Jensen, Myron Scholes et al. 1972. "The capital asset pricing model: Some empirical tests." *Studies in the theory of capital markets* 81 (3):79–121.

Botosan, Christine A. 1997. "Disclosure level and the cost of equity capital." *Accounting review* :323–349.

Boudoukh, Jacob, Roni Michaely, Matthew Richardson, and Michael R Roberts. 2007. "On the importance of measuring payout yield: Implications for empirical asset pricing." *The Journal of Finance* 62 (2):877–915.

Bradshaw, Mark T, Scott A Richardson, and Richard G Sloan. 2001. "Do analysts and auditors use information in accruals?" *Journal of Accounting research* 39 (1):45–74.

Brealey, Richard A, Stewart C Myers, Franklin Allen, and Pitabas Mohanty. 2012. *Principles of corporate finance*. Tata McGraw-Hill Education.

Campbell, John Y. 2017. *Financial decisions and markets: a course in asset pricing*. Princeton University Press.

Campbell, John Y, Jens Hilscher, and Jan Szilagyi. 2008. "In search of distress risk." *The Journal of Finance* 63 (6):2899–2939.

Campbell, John Y and Robert J Shiller. 1988. "The dividend-price ratio and expectations of future dividends and discount factors." *The Review of Financial Studies* 1 (3):195–228.

Campbell, John Y and Tuomo Vuolteenaho. 2004. "Bad beta, good beta." *American Economic Review* 94 (5):1249–1275.

Carhart, Mark M. 1997. "On persistence in mutual fund performance." *The Journal of finance* 52 (1):57–82.

Carlson, Murray, Adlai Fisher, and Ron Giammarino. 2004. "Corporate investment and asset price dynamics: Implications for the cross-section of returns." *The Journal of Finance* 59 (6):2577–2603.

Chan, KC and Nai-Fu Chen. 1991. "Structural and return characteristics of small and large firms." *The journal of finance* 46 (4):1467–1484.

Chen, Long, Ralitsa Petkova, and Lu Zhang. 2008. "The expected value premium." *Journal of Financial Economics* 87 (2):269–280.

Cho, Thummim and Christopher Polk. 2019. "Asset Pricing with Price Levels." *Available at SSRN 3499681* .

Clement, Michael B. 1999. "Analyst forecast accuracy: Do ability, resources, and portfolio complexity matter?" *Journal of Accounting and Economics* 27 (3):285–303.

Cochrane, John H. 2008. "The dog that did not bark: A defense of return predictability." *The Review of Financial Studies* 21 (4):1533–1575.

———. 2011. "Presidential address: Discount rates." *The Journal of finance* 66 (4):1047–1108.

Cohen, Randolph B, Christopher Polk, and Tuomo Vuolteenaho. 2003. "The value spread." *The Journal of Finance* 58 (2):609–641.

———. 2009. "The price is (almost) right." *The Journal of Finance* 64 (6):2739–2782.

Conrad, Jennifer and Gautam Kaul. 1998. "An anatomy of trading strategies." *The Review of Financial Studies* 11 (3):489–519.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. "Investor psychology and security market under-and overreactions." *the Journal of Finance* 53 (6):1839–1885.

Das, Somnath, Carolyn B Levine, and Konduru Sivaramakrishnan. 1998. "Earnings predictability and bias in analysts' earnings forecasts." *Accounting Review* :277–294.

Davis, James L, Eugene F Fama, and Kenneth R French. 2000. "Characteristics, covariances, and average returns: 1929 to 1997." *The Journal of Finance* 55 (1):389–406.

De Bondt, Werner FM and Richard Thaler. 1985. "Does the stock market overreact?" *The Journal of finance* 40 (3):793–805.

Dou, Winston Wei, Yan Ji, and Wei Wu. 2021. "Competition, profitability, and discount rates." *Journal of Financial Economics* 140 (2):582–620.

Dugar, Amitabh and Siva Nathan. 1995. "The effect of investment banking relationships on financial analysts' earnings forecasts and investment recommendations." *Contemporary Accounting Research* 12 (1):131–160.

Easton, Peter D. 2004. "PE ratios, PEG ratios, and estimating the implied expected rate of return on equity capital." *The accounting review* 79 (1):73–95.

Easton, Peter D and Steven J Monahan. 2005. "An evaluation of accounting-based measures of expected returns." *The Accounting Review* 80 (2):501–538.

Easton, Peter D and Gregory A Sommers. 2007. "Effect of analysts' optimism on estimates of the expected rate of return implied by earnings forecasts." *Journal of Accounting Research* 45 (5):983–1015.

Fama, Eugene F and Harvey Babiak. 1968. "Dividend Policy: An Empirical Analysis." *Journal of the American Statistical Association* :1132–1161.

Fama, Eugene F and Kenneth R French. 1988. "Permanent and temporary components of stock prices." *Journal of political Economy* 96 (2):246–273.

———. 1992. "The cross-section of expected stock returns." *the Journal of Finance* 47 (2):427–465.

———. 1993. "Common risk factors in the returns on stocks and bonds." *Journal of* .

———. 1996. "Multifactor explanations of asset pricing anomalies." *The journal of finance* 51 (1):55–84.

———. 1997. "Industry costs of equity." *Journal of financial economics* 43 (2):153–193.

———. 1999. "The corporate cost of capital and the return on corporate investment." *The Journal of Finance* 54 (6):1939–1967.

———. 2002. "The equity premium." *The Journal of Finance* 57 (2):637–659.

———. 2015. "A five-factor asset pricing model." *Journal of financial economics* 116 (1):1–22.

Francis, Jennifer, Per Olsson, and Dennis R Oswald. 2000. “Comparing the accuracy and explainability of dividend, free cash flow, and abnormal earnings equity value estimates.” *Journal of accounting research* 38 (1):45–70.

Francis, Jennifer and Donna Philbrick. 1993. “Analysts’ decisions as products of a multi-task environment.” *Journal of Accounting Research* 31 (2):216–230.

Frazzini, Andrea and Lasse Heje Pedersen. 2014. “Betting against beta.” *Journal of Financial Economics* 111 (1):1–25.

Gabaix, Xavier. 2012. “Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance.” *The Quarterly journal of economics* 127 (2):645–700.

Gebhardt, William R, Charles MC Lee, and Bhaskaran Swaminathan. 2001. “Toward an implied cost of capital.” *Journal of accounting research* 39 (1):135–176.

Gervais, Simon, Ron Kaniel, and Dan H Mingelgrin. 2001. “The high-volume return premium.” *The Journal of Finance* 56 (3):877–919.

Gofman, Michael, Gill Segal, and Youchang Wu. 2020. “Production networks and stock returns: The role of vertical creative destruction.” *The Review of Financial Studies* 33 (12):5856–5905.

Gomes, Joao, Leonid Kogan, and Lu Zhang. 2003. “Equilibrium cross section of returns.” *Journal of Political Economy* 111 (4):693–732.

Gomes, Joao F and Lukas Schmid. 2010. “Levered returns.” *The Journal of Finance* 65 (2):467–494.

Gordon, Myron J. 1959. “Dividends, earnings, and stock prices.” *The review of economics and statistics* 41 (2):99–105.

Graham, John R and Campbell R Harvey. 2001. “The theory and practice of corporate finance: Evidence from the field.” *Journal of financial economics* 60 (2-3):187–243.

Guay, Wayne, SP Kothari, and Susan Shu. 2011. “Properties of implied cost of capital using analysts’ forecasts.” *Australian Journal of Management* 36 (2):125–149.

Hansen, Lars Peter, John C Heaton, and Nan Li. 2008. “Consumption strikes back? Measuring long-run risk.” *Journal of Political economy* 116 (2):260–302.

Harvey, Campbell R, Yan Liu, and Heqing Zhu. 2016. "... and the cross-section of expected returns." *The Review of Financial Studies* 29 (1):5–68.

Hong, Harrison and Jeffrey D Kubik. 2003. "Analyzing the analysts: Career concerns and biased earnings forecasts." *The Journal of Finance* 58 (1):313–351.

Hong, Harrison, Jeffrey D Kubik, and Amit Solomon. 2000. "Security analysts' career concerns and herding of earnings forecasts." *The Rand journal of economics* :121–144.

Hong, Harrison and Jeremy C Stein. 1999. "A unified theory of underreaction, momentum trading, and overreaction in asset markets." *The Journal of finance* 54 (6):2143–2184.

Hou, Kewei, Mathijs A Van Dijk, and Yinglei Zhang. 2012. "The implied cost of capital: A new approach." *Journal of Accounting and Economics* 53 (3):504–526.

Hou, Kewei, Chen Xue, and Lu Zhang. 2017. "Replicating anomalies." *The Review of Financial Studies* .

Jegadeesh, Narasimhan and Sheridan Titman. 1993. "Returns to buying winners and selling losers: Implications for stock market efficiency." *The Journal of finance* 48 (1):65–91.

———. 2001. "Profitability of momentum strategies: An evaluation of alternative explanations." *The Journal of finance* 56 (2):699–720.

Keloharju, Matti, Juhani T Linnainmaa, and Peter Nyberg. 2019. "Long-term discount rates do not vary across firms." Tech. rep., National Bureau of Economic Research.

Kim, Myung Jig, Charles R Nelson, and Richard Startz. 1991. "Mean reversion in stock prices? A reappraisal of the empirical evidence." *The Review of Economic Studies* 58 (3):515–528.

Kogan, Leonid and Dimitris Papanikolaou. 2013. "Firm characteristics and stock returns: The role of investment-specific shocks." *The Review of Financial Studies* 26 (11):2718–2759.

———. 2014. "Growth opportunities, technology shocks, and asset prices." *The journal of finance* 69 (2):675–718.

Leary, Mark T and Roni Michaely. 2011. "Determinants of dividend smoothing: Empirical evidence." *The Review of Financial Studies* 24 (10):3197–3249.

Lee, Charles MC and Bhaskaran Swaminathan. 2000. "Price momentum and trading volume." *the Journal of Finance* 55 (5):2017–2069.

Levi, Yaron and Ivo Welch. 2017. "Best practice for cost-of-capital estimates." *Journal of Financial and Quantitative Analysis* 52 (2):427–463.

Li, Dongmei and Lu Zhang. 2010. "Does q-theory with investment frictions explain anomalies in the cross section of returns?" *Journal of Financial Economics* 98 (2):297–314.

Li, Erica XN, Dmitry Livdan, and Lu Zhang. 2009. "Anomalies." *The Review of Financial Studies* 22 (11):4301–4334.

Lim, Terence. 2001. "Rationality and analysts' forecast bias." *The journal of Finance* 56 (1):369–385.

Lintner, John. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *The Review of Economics and Statistics* :13–37.

Liu, Laura Xiaolei, Toni M Whited, and Lu Zhang. 2009. "Investment-based expected stock returns." *Journal of Political Economy* 117 (6):1105–1139.

Lo, Andrew W and A Craig MacKinlay. 1988. "Stock market prices do not follow random walks: Evidence from a simple specification test." *The review of financial studies* 1 (1):41–66.

———. 1989. "The size and power of the variance ratio test in finite samples: A Monte Carlo investigation." *Journal of econometrics* 40 (2):203–238.

Lu, Zhongjin. 2016. "The Cross Section of Long-Term Expected Returns." *Available at SSRN 2804130* .

Malkiel, Burton G and Eugene F Fama. 1970. "Efficient capital markets: A review of theory and empirical work." *The journal of Finance* 25 (2):383–417.

McLean, R David and Jeffrey Pontiff. 2016. "Does academic research destroy stock return predictability?" *The Journal of Finance* 71 (1):5–32.

Newey, Whitney K, Kenneth D West et al. 1987. "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica* 55 (3):703–708.

Novy-Marx, Robert. 2013. "The other side of value: The gross profitability premium." *Journal of Financial Economics* 108 (1):1–28.

Ohlson, James A and Beate E Juettner-Nauroth. 2005. "Expected EPS and EPS growth as determinantsof value." *Review of accounting studies* 10 (2-3):349–365.

Papanikolaou, Dimitris. 2011. "Investment shocks and asset prices." *Journal of Political Economy* 119 (4):639–685.

Pástor, L'uboš, Meenakshi Sinha, and Bhaskaran Swaminathan. 2008. "Estimating the intertemporal risk–return tradeoff using the implied cost of capital." *The Journal of Finance* 63 (6):2859–2897.

Poterba, James M and Lawrence H Summers. 1988. "Mean reversion in stock prices: Evidence and implications." *Journal of financial economics* 22 (1):27–59.

Richardson, Matthew. 1993. "Temporary components of stock prices: A skeptic's view." *Journal of Business & Economic Statistics* 11 (2):199–207.

Richardson, Matthew and James H Stock. 1989. "Drawing inferences from statistics based on multiyear asset returns." *Journal of financial economics* 25 (2):323–348.

Ross, Stephen. 1976. "The arbitrage theory of capital asset pricing." *Journal of Economic Theory* 13 (3):341–360.

Seigel, Jeremy J. 2020. *Stocks for the long run*. MGH.

Sharpe, William F. 1964. "Capital asset prices: A theory of market equilibrium under conditions of risk." *The journal of finance* 19 (3):425–442.

Shiller, Robert J. 1981. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *The American Economic Review* 71 (3):421–436.

Shumway, Tyler. 1997. "The delisting bias in CRSP data." *The Journal of Finance* 52 (1):327–340.

Shumway, Tyler and Vincent A Warther. 1999. "The delisting bias in CRSP's Nasdaq data and its implications for the size effect." *The Journal of Finance* 54 (6):2361–2379.

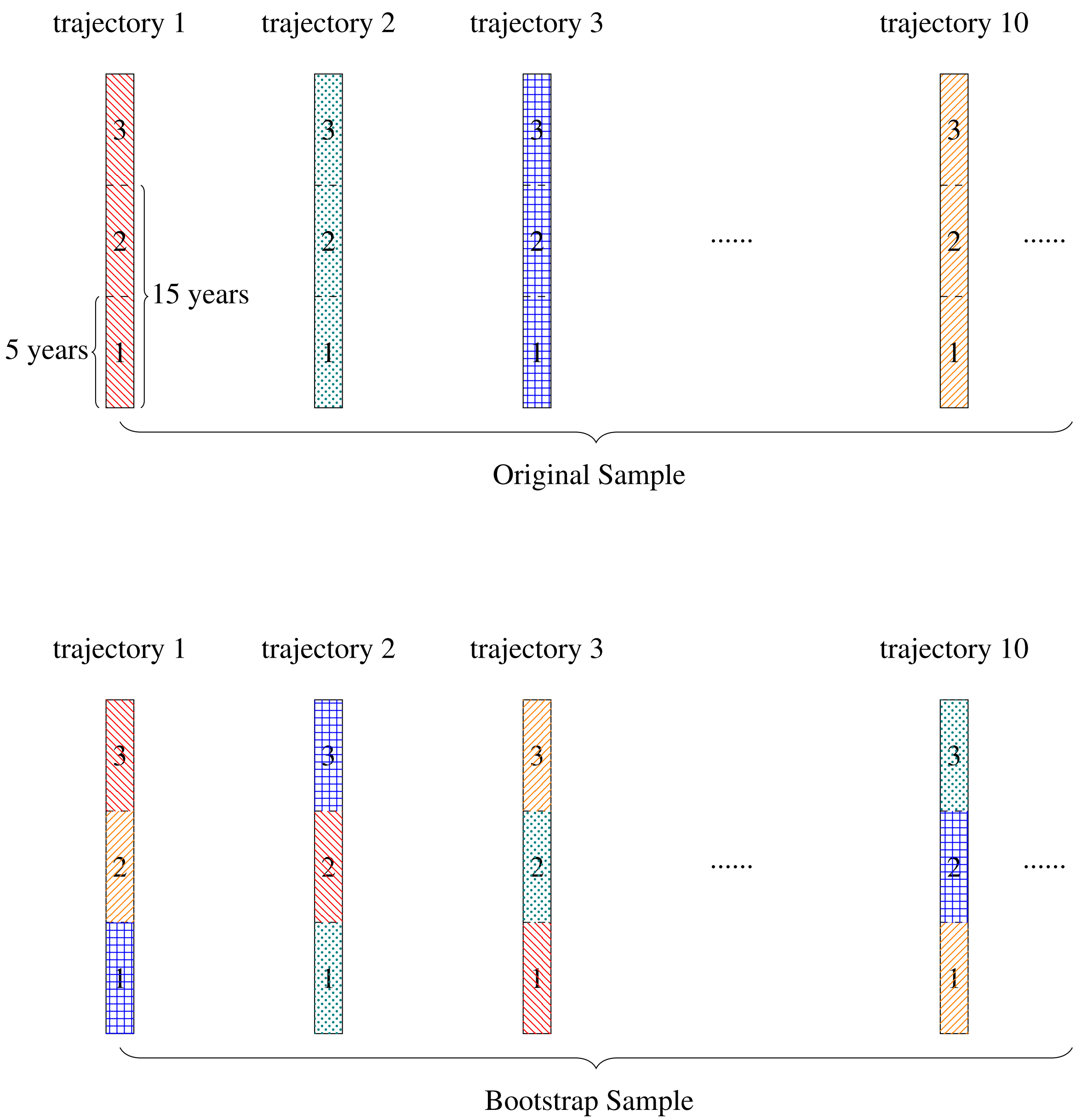
Titman, Sheridan, KC John Wei, and Feixue Xie. 2004. "Capital investments and stock returns." *Journal of financial and Quantitative Analysis* 39 (4):677–700.

Van Binsbergen, Jules H and Christian C Opp. 2019. "Real anomalies." *The Journal of Finance* 74 (4):1659–1706.

Zhang, Lu. 2005. "The value premium." *The Journal of Finance* 60 (1):67–103.

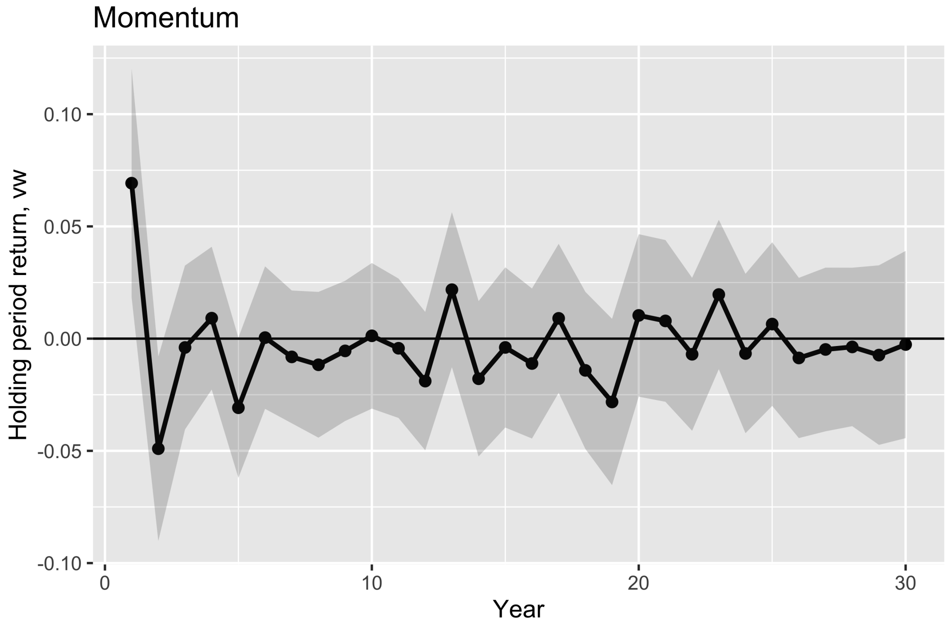
Figures

Figure 1: Block Bootstrap Illustration



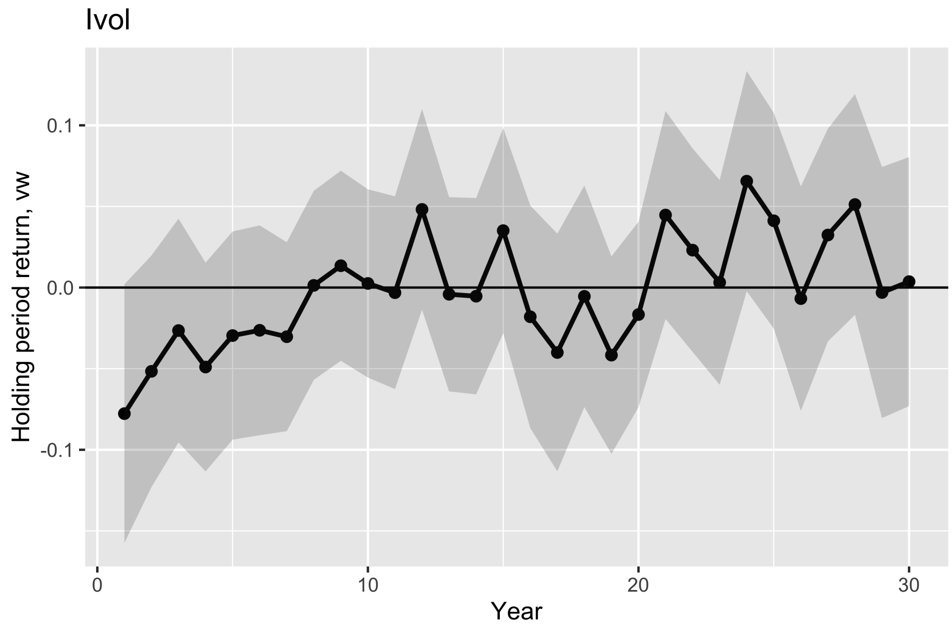
This figure demonstrates how the block bootstrap sample is generated from the original sample. A trajectory tracks the cash flows of a portfolio formed at a particular point in time for 15 years since formation. Each trajectory is then split into three 5-year blocks. The blocks are indexed by levels 1, 2 and 3, representing the first, second and third 5-year episodes since portfolio formation. The bootstrap sample has the same number of trajectories as the original sample. Each trajectory within the bootstrap sample is simulated by randomly drawing blocks with the same levels from the original sample.

Figure 2: Term Structure of the Momentum Premium



This figure plots the evolution of the average returns of the momentum strategy since portfolio formation. The solid line is the difference in the value-weighted average returns between the top decile and bottom decile $\left\{\bar{h}_t^{10} - \bar{h}_t^{1}\right\}_t$. The shaded area is the 95% confidence internal.

Figure 3: Term Structure of the Idiosyncratic Volatility Premium



This figure plots the evolution of the average returns of the trading strategy that longs high idiosyncratic volatility stocks and shorts low idiosyncratic volatility stocks. The solid line is the difference in the value-weighted average returns between the top decile and bottom decile $\left\{\bar{h}_t^{10} - \bar{h}_t^1\right\}_t$. The shaded area is the 95% confidence internal.

Tables

Table 1: List of Anomalies

Name	Literature	Sample Period	Frequency
Market Beta	Lintner (1965), Sharpe (1964), Fama and French (1992)	1970-2018	Y
Size	Banz (1981), Fama and French (1992)	1970-2018	Y
Book-to-Market	Chan and Chen (1991), Fama and French (1992)	1970-2018	Y
Gross Profitability	Novy-Marx (2013), Fama and French (2015)	1970-2018	Y
Investment	Titman, Wei, and Xie (2004), Fama and French (2015)	1970-2018	Y
Credit Rating	Avramov et al. (2009)	1985-2018	Y
Momentum	Jegadeesh and Titman (1993)	1970-2018	M
Long-Term Reversal	De Bondt and Thaler (1985)	1970-2018	M
Idiosyncratic Volatility	Ang et al. (2006)	1970-2018	M
Abnormal Volume	Gervais, Kaniel, and Mingelgrin (2001)	1970-2018	M
Fail Probability	Campbell, Hilscher, and Szilagyi (2008)	1975-2018	M
Return on Asset	Balakrishnan, Bartov, and Faurel (2010)	1970-2018	M
Book Leverage	Fama and French (1992), Gomes and Schmid (2010)	1970-2018	Y
Market Leverage	Fama and French (1992), Gomes and Schmid (2010)	1970-2018	Y

This table reports the anomalies studied in this paper. "Frequency" denotes the rebalancing frequency of the anomaly portfolios.

Table 2: Estimated Biases in Discount Rate Estimates (Structural Approach)

	FB	FB ADJ	LM	LM ADJ	BY	RD
mean	0.003	0.03	0.03	0.04	0.13	0.26
min	0.0	0.001	0.0002	0.001	0.01	0.11
25%	0.0002	0.01	0.02	0.02	0.06	0.19
50%	0.001	0.02	0.02	0.03	0.12	0.25
75%	0.003	0.04	0.04	0.06	0.16	0.31
max	0.03	0.14	0.06	0.15	0.34	0.46

This table reports the estimated biases in discount rate estimates with the structural approach. The biases for each model are estimated as the differences between the estimated discount rates and the specified discount rates across large parameter grids as detailed in Appendix A. We simulate across a wide range of discount rates with $y \in [8\%, 20\%]$. "FB" denotes the Fama and Babiak (1968) process. "FB ADJ" denotes the adjusted Fama and Babiak (1968) process where earnings growth follows geometric random walk. "LM" denotes the Leary and Michaely (2011) process. "LM ADJ" denotes the adjusted Leary and Michaely (2011) process where earnings growth follows geometric random walk. "BY" denotes the long-run risk process in Bansal and Yaron (2004). "RD" denotes the rare disaster process in Barro (2006). The reported statistics are from large grids of parameter specifications. The biases are recorded in absolute values. All units are in percentage points.

Table 3: Estimated Biases in Discount Rate Estimates (Bootstrap Approach)

Anomaly	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	EMM
Market Beta	0.13	0.23	0.33	0.23	0.29	0.21	0.18	0.29	0.12	0.44	0.30	-0.02
Book-to-Market	-0.03	0.18	0.12	0.18	0.20	0.22	0.25	0.18	0.28	-0.01	0.02	-0.11
Size	0.22	0.20	0.22	0.29	0.29	0.24	0.17	0.29	0.18	0.21	-0.01	-0.07
Gross Profitability	0.46	0.21	0.10	0.38	0.21	0.17	0.28	0.39	0.47	0.29	-0.17	0.17
Investment	0.06	0.29	0.33	0.26	0.22	0.32	0.27	0.05	0.06	0.00	-0.06	-0.17
Credit Rating	0.38	0.12	-0.54	-0.04	0.11	-0.03	0.29	0.25	0.12	0.01	-0.36	0.12
Momentum	-0.02	-0.13	0.05	0.09	0.15	0.17	0.20	0.32	0.48	0.39	0.41	0.02
Idiosyncratic Volatility	0.16	0.27	0.24	0.06	0.01	0.12	0.19	0.20	0.23	-0.15	-0.31	0.06
Long-Term Reversal	0.05	0.10	0.15	0.10	0.12	0.19	0.17	0.19	0.23	-0.18	-0.23	-0.11
Abnormal Volume	0.31	0.24	0.23	0.28	0.27	0.32	0.27	0.29	0.28	0.35	0.04	-0.01
Fail Probability	-0.22	0.11	0.36	0.19	0.15	0.16	0.10	0.10	-0.11	-0.26	-0.04	-0.28
Return on Asset	0.25	0.34	0.39	0.26	0.12	0.31	0.17	0.16	0.08	-0.11	-0.37	-0.07
Book Leverage	0.21	-0.54	0.12	0.19	0.29	0.37	0.31	0.35	0.30	0.16	-0.05	-0.30
Market Leverage	-0.16	0.09	0.15	0.14	0.28	0.15	0.10	0.26	0.22	-0.25	-0.09	-0.24

This table reports the estimated biases in discount rate estimates with the block bootstrap approach. The bias for each portfolio is estimated in block bootstrap simulations as the difference between the estimated discount rate in the simulations and the specified discount rate. The specified discount rates in the simulations adopt the estimates in the true data. All units are in percentage points.

Table 4: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^{15}$	5.76	6.19	6.44	6.67	6.66	6.51	5.98	6.37	7.06	8.01	2.25	1.59	0.16	0.33
RDF	59.95	58.72	57.56	55.28	55.79	55.85	58.35	56.98	51.36	47.44	-12.52**	-2.15	-1.45	-0.67
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
Panel (b): Size														
$y-r_f^{15}$	9.33	9.67	9.73	9.71	9.0	8.2	7.62	7.27	7.27	5.33	-4.0***	-3.22	-0.7**	-2.32
RDF	41.41	42.08	42.35	42.74	45.43	47.4	49.98	51.57	51.3	63.35	21.94***	4.41	3.12*	1.85
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
Panel (c): Book-to-Market														
$y-r_f^{15}$	5.37	5.3	6.32	6.51	7.22	8.36	7.87	8.28	9.95	12.5	7.13***	4.6	0.49	1.2
RDF	59.78	60.97	55.46	55.29	52.31	46.88	50.51	48.2	43.88	35.75	-24.02***	-3.04	0.5	0.27
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
Panel (d): Gross Profitability														
$y-r_f^{15}$	10.4	8.88	4.81	5.45	6.01	6.06	6.14	6.63	6.94	8.81	-1.59**	-1.96	2.73***	4.92
RDF	40.9	47.38	64.6	62.09	57.02	57.33	57.5	56.56	53.71	45.71	4.81	1.48	-10.25***	-4.35
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
Panel (e): Investment														
$y-r_f^{15}$	8.57	7.88	7.47	6.7	6.54	6.45	6.16	6.35	7.28	7.39	-1.18	-0.98	1.28**	2.51
RDF	45.71	50.7	53.06	55.52	55.89	56.7	58.03	55.51	51.46	49.42	3.71	0.62	-6.97***	-2.85
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
Panel (f): Credit Rating														
$y-r_f^{15}$	11.71	8.21	7.74	7.99	6.5	5.84	6.07	5.73	7.19	5.32	-6.39***	-4.23	1.94***	3.27
RDF	36.66	45.41	48.78	49.9	59.76	59.57	61.54	62.5	54.18	62.92	26.25***	3.71	-9.87***	-4.0
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53

This table documents the anomalies motivated by models of funding cost. "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 5: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^{15}$	7.6	7.45	7.2	6.69	6.34	6.41	6.48	6.36	6.67	6.82	-0.78**	-2.12	0.76***	5.13
RDF	47.78	48.83	51.46	55.13	56.99	56.55	56.7	57.93	56.43	55.68	7.9***	4.11	-4.59***	-6.38
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^{15}$	9.1	8.36	8.06	7.52	7.17	6.63	6.34	5.85	5.37	4.87	-4.22***	-11.55	0.03	0.21
RDF	40.71	45.18	47.59	50.48	52.69	55.52	56.6	59.85	62.56	65.02	24.32***	10.85	-0.74	-1.02
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
RDF	62.91	56.68	53.19	52.27	49.77	47.82	47.98	50.24	51.84	48.04	-14.87***	-6.47	6.07***	7.52
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^{15}$	6.42	6.15	6.09	6.3	6.07	6.23	6.21	6.31	6.41	6.88	0.46***	2.72	0.31***	3.74
RDF	58.17	59.08	59.73	58.77	59.56	58.79	59.04	58.51	58.33	56.34	-1.83**	-2.22	-1.2***	-2.81
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 6: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
						Panel (a): Fail Probability								
$y-r_f^{15}$	6.29	6.53	6.16	5.59	5.63	5.79	5.7	5.81	6.17	6.2	-0.09	-0.21	0.58***	3.55
RDF	56.15	57.25	59.99	61.46	61.25	60.1	60.48	60.28	57.03	54.75	-1.4	-0.58	-4.37***	-4.54
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71
						Panel (b): Book Leverage								
$y-r_f^{15}$	5.91	5.71	5.63	5.74	5.84	6.27	6.11	6.84	6.93	6.5	0.59	0.81	0.21	0.5
RDF	56.3	58.92	60.71	57.22	58.13	57.99	57.74	54.04	52.62	54.4	-1.9	-0.39	-2.5	-1.15
$\bar{h}_1-r_f$	4.87	6.44	5.39	6.54	5.55	6.46	6.7	7.3	8.25	7.35	2.49	1.21	3.73**	2.3
						Panel (c): Market Leverage								
$y-r_f^{15}$	5.03	5.64	5.88	6.58	6.62	6.97	7.2	7.46	8.11	8.2	3.17**	2.25	-0.05	-0.12
RDF	61.66	58.78	58.04	55.02	55.11	53.26	51.54	48.89	48.27	44.9	-16.76*	-1.9	-0.78	-0.28
$\bar{h}_1-r_f$	5.26	5.5	5.83	6.11	8.01	7.93	10.11	10.14	8.33	10.86	5.6*	1.96	3.5**	2.2
						Panel (d): Return on Asset								
$y-r_f^{15}$	7.4	8.72	8.15	9.48	6.48	5.68	6.07	6.01	5.78	5.19	-2.21***	-6.15	0.69***	4.41
RDF	50.99	47.87	50.02	45.69	55.89	61.03	57.57	57.28	57.48	61.95	10.95***	5.52	-3.89***	-4.83
$\bar{h}_1-r_f$	-4.13	1.74	2.16	4.7	5.96	6.86	6.81	7.36	6.74	8.36	12.49***	4.07	-0.03	-0.01

This table documents several additional anomalies. "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 7: Out-of-Sample Performances

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
Early Sample: 1926-1970														
$y-r_f^{15}$	11.72	10.69	10.28	10.54	10.06	10.35	10.42	10.53	10.69	10.9	-0.82**	-2.17	0.79***	4.62
RDF	26.72	30.41	31.39	31.4	32.41	32.47	32.12	31.95	31.48	31.95	5.23***	5.36	-2.3***	-5.14
$\bar{h}_1-r_f$	2.71	5.63	4.21	7.54	8.41	8.84	10.46	11.79	12.79	16.33	13.62***	2.97	5.05**	2.07
Main Sample: 1970-2018														
$y-r_f^{15}$	7.6	7.45	7.2	6.69	6.34	6.41	6.48	6.36	6.67	6.82	-0.78**	-2.12	0.76***	5.13
RDF	47.78	48.83	51.46	55.13	56.99	56.55	56.7	57.93	56.43	55.68	7.9***	4.11	-4.59***	-6.38
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
Early Sample: 1926-1970														
$y-r_f^{15}$	12.36	11.13	11.16	11.25	11.0	10.68	10.61	9.98	9.64	9.84	-2.52***	-6.12	-0.1	-0.67
RDF	23.65	27.84	29.02	29.41	30.37	31.98	32.51	34.34	35.51	34.13	10.48***	9.63	-0.89**	-2.26
$\bar{h}_1-r_f$	14.01	12.86	11.32	10.27	10.47	9.44	9.73	9.12	7.81	5.6	-8.41**	-2.14	5.1**	2.1
Main Sample: 1970-2018														
$y-r_f^{15}$	9.1	8.36	8.06	7.52	7.17	6.63	6.34	5.85	5.37	4.87	-4.22***	-11.55	0.03	0.21
RDF	40.71	45.18	47.59	50.48	52.69	55.52	56.6	59.85	62.56	65.02	24.32***	10.85	-0.74	-1.02
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
Early Sample: 1926-1970														
$y-r_f^{15}$	9.81	9.69	10.56	11.44	11.67	12.26	13.15	10.6	11.49	11.84	2.03***	4.97	-1.26***	-6.72
RDF	32.57	34.35	32.79	30.29	29.83	28.59	26.73	28.07	28.02	22.06	-10.51***	-9.15	0.04	0.09
$\bar{h}_1-r_f$	8.43	10.17	10.32	10.62	12.35	11.18	10.51	12.69	13.39	14.2	5.78	1.09	5.66**	2.54
Main Sample: 1970-2018														
$y-r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
RDF	62.91	56.68	53.19	52.27	49.77	47.82	47.98	50.24	51.84	48.04	-14.87***	-6.47	6.07***	7.52
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (d): Abnormal Volume														
Early Sample: 1926-1970														
$y-r_f^{15}$	10.19	10.27	10.6	10.29	10.35	10.41	10.45	10.24	10.5	10.49	0.3	1.42	-0.02	-0.12
RDF	33.38	33.2	32.15	32.9	33.37	33.24	32.72	33.55	32.99	33.06	-0.32	-0.52	-0.15	-0.49
$\bar{h}_1-r_f$	3.78	7.78	7.06	7.85	9.59	8.75	12.67	10.64	12.36	14.3	10.52***	6.24	4.97***	2.76
Main Sample: 1970-2018														
$y-r_f^{15}$	6.42	6.15	6.09	6.3	6.07	6.23	6.21	6.31	6.41	6.88	0.46***	2.72	0.31***	3.74
RDF	58.17	59.08	59.73	58.77	59.56	58.79	59.04	58.51	58.33	56.34	-1.83**	-2.22	-1.2***	-2.81
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (e): Market Beta														
Early Sample: 1926-1970														
$y-r_f^{15}$	8.06	9.61	10.54	10.81	11.34	11.08	10.79	10.66	10.24	11.1	3.05**	2.58	-1.46***	-3.41
RDF	34.52	31.86	29.73	29.32	27.49	28.33	30.55	28.55	29.92	26.41	-8.11**	-2.17	2.77**	2.13
$\bar{h}_1-r_f$	6.81	7.71	6.88	9.98	10.58	10.82	10.68	12.72	10.92	14.18	7.36	1.54	4.55**	2.07
Main Sample: 1970-2018														
$y-r_f^{15}$	5.76	6.19	6.44	6.67	6.66	6.51	5.98	6.37	7.06	8.01	2.25	1.59	0.16	0.33
RDF	59.95	58.72	57.56	55.28	55.79	55.85	58.35	56.98	51.36	47.44	-12.52**	-2.15	-1.45	-0.67
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (f): Size														
Early Sample: 1926-1970														
$y-r_f^{15}$	11.44	12.02	13.43	13.63	12.29	11.87	11.02	11.11	10.39	9.66	-1.78	-1.11	-1.2**	-2.31
RDF	19.59	21.15	24.56	25.3	27.18	28.02	30.31	30.9	32.71	33.22	13.63***	3.68	-0.93	-1.24
$\bar{h}_1-r_f$	18.88	15.82	12.32	12.64	11.9	12.34	11.07	9.71	9.94	7.91	-10.96**	-2.2	7.08***	2.84
Main Sample: 1970-2018														
$y-r_f^{15}$	9.33	9.67	9.73	9.71	9.0	8.2	7.62	7.27	7.27	5.33	-4.0***	-3.22	-0.7**	-2.32
RDF	41.41	42.08	42.35	42.74	45.43	47.4	49.98	51.57	51.3	63.35	21.94***	4.41	3.12*	1.85
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45

This table compares the long-term discount rates of several anomalies for the main sample (1970-2018) versus an early sample (1926-1970). "$y-r_f^{15}$" is the the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 8: Case Study with Kogan and Papanikolaou (2013)

Panel (a): Idiosyncratic Volatility														
Data														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
$y - r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
Model														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	14.01	12.93	12.08	11.25	10.45	9.67	8.96	8.33	7.89	7.88	-6.13		0.61	
$y - r_f$	13.56	12.56	11.75	11.01	10.3	9.66	9.12	8.75	8.67	9.48	-4.08		1.09	

Panel (b): Investment														
Data														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
$y - r_f^{15}$	8.57	7.88	7.47	6.7	6.54	6.45	6.16	6.35	7.28	7.39	-1.18	-0.98	1.28**	2.51
Model														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	13.47	13.09	12.83	12.24	11.45	10.61	9.79	9.33	8.72	7.57	-5.9		-0.32	
$y - r_f$	12.72	12.95	13.08	12.67	12.08	11.49	11.05	10.87	10.73	10.57	-2.15		-0.04	

Panel (c): Gross Profitability														
Data														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
$y - r_f^{15}$	10.4	8.88	4.81	5.45	6.01	6.06	6.14	6.63	6.94	8.81	-1.59**	-1.96	2.73***	4.92
Model														
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	7.67	10.15	11.2	11.7	11.83	11.82	11.53	11.21	10.88	10.54	2.87		-2.02	
$y - r_f$	8.7	10.58	11.47	11.81	11.85	11.68	11.36	11.0	10.59	10.27	1.56		-1.73	

This table compares the estimates of the short-term expected returns and the long-term discount rates between the data and the Kogan and Papanikolaou (2013) model for idiosyncratic volatility, investment and gross profitability anomalies.

Appendix A. Specifications of the Structural Cash Flow Processes

Section 2.3.1 evaluates the biases of discount rate estimates using 6 structural cash flow processes across large parameter grids. The details of the cash flow specifications and parameter ranges are described below.

A1. The Fama and Babiak (1968) Process

Fama and Babiak (1968) estimated a payout model where a firm's dividend process is co-integrated with its earnings process. We adopt this model in our simulations to understand how the auto-correlation in dividends affects the bootstrap exercise.

According to Fama and Babiak (1968), the firm's earnings follow:

$$E_{t+1} = (1+\lambda) E_t + e_{t+1},$$

and its dividends are co-integrated with the earnings:

$$\Delta D_{t+1} = \beta_1 D_t + \beta_2 E_t + u_{t+1}$$

where

$$e_t \sim N\left(0, \sigma_e^2\right),\ u_t \sim N\left(0, \sigma_u^2\right),\ Corr\left(e_t, u_t\right) = 0$$

We adopt the following parameter ranges in our simulations: $\beta_1 \in [-0.45 - 2\times 0.15, -0.45 + 2\times 0.15]$, $\beta_2 = [0.15 - 2\times 0.05, 0.15 + 2\times 0.05]$, $\lambda = 5\%$, $\sigma_u = \frac{1}{0.6745}$, $\sigma_e = \frac{0.21}{0.6745}$, $E_0 = 100$, $D_0 = 33$, $y \in [8\%, 20\%]$. The ranges for β_1 and β_2 are 95% confidence interval reported by Table 2 of the paper. The values of σ_e, σ_u and the ratio between D_0 and E_0 are from page 1143 of the paper.

A2. The Adjusted Fama and Babiak (1968) Process

Fama and Babiak (1968) aimed to study the properties of corporate payouts. The model assumes random shocks to the level of the earnings process, or an arithmetic random walk. To better suit our purposes, we adjusted the earnings process to a geometric random walk so that the shocks affect the growth rate of the earnings:

$$E_{t+1} = \exp\left(g_{t+1}\right) E_t$$

$$g_{t+1} = \lambda + \sigma \cdot e_{t+1}$$

The co-integration between the dividend process and the earnings process remains the same. For the volatility of earnings growth, we adopt the range: $\sigma \in [7.5\%, 15.5\%]$, which is the 95% confidence interval of dividend growth volatility documented in Table II of Bansal, Kiku, and Yaron (2009). λ is assumed to be 5%.

A3. The Leary and Michaely (2011) Process

Similar to Fama and Babiak (1968), Leary and Michaely (2011) also proposed a model of corporate payout where earnings and dividends are co-integrated. The processes are specified as:

$$\begin{aligned} \Delta E_{t+1} &= \delta + \gamma \times \Delta E_t + \omega_{t+1} \\ \Delta D_{t+1} &= \beta \times (TPR \times E_{t+1} - D_t) + \varepsilon_{t+1}, \end{aligned}$$

where

$$\omega_t \sim N\left(0, \sigma_\omega^2\right),\ \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right),\ Corr\left(\omega_t, \varepsilon_t\right) = 0$$

We adopt the following parameter ranges in our simulations: $\beta \in [0.1, 0.5]$, $TPR = 0.3$, $\delta = 0.1$, $\gamma = -0.2$, $\sigma_\omega = 0.7$, $\sigma_\varepsilon = 0.1$, $E_0 = 10$, $D_0 = 3$, $y \in [8\%, 20\%]$. The range of β and the values for TPR, δ, γ, σ_ω are from page 3245 of Leary and Michaely (2011); the value of σ_ε is from Figure 2 of the working paper version of the paper; and the ratio between D_0 and E_0 are determined by the steady state of the dividend equation, i.e. $D_0/E_0 = TPR$.

A4. The Adjusted Leary and Michaely (2011) Process

Similar to the adjusted Fama and Babiak (1968) process, we consider a variant of the Leary and Michaely (2011) process where the earnings growth follows a geometric random walk:

$$E_{t+1} = \exp\left(g_{t+1}\right) E_t$$

$$g_{t+1} = \lambda + \sigma \cdot e_{t+1}$$

with $\lambda = 5\%$ and $\sigma \in [7.5\%, 15.5\%]$.

A5. The Bansal and Yaron (2004) Process

We consider the long-run risk process as specified in Bansal and Yaron (2004):

$$
\begin{aligned}
D_{t+1} &= D_t \cdot \exp\left(g_{d,t+1}\right) \\
x_{t+1} &= \rho x_t + \phi_e \sigma e_{t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \phi_d \sigma u_{t+1} \\
e_{t+1}, u_{t+1} &\sim N \; i.i.d \; (0,1).
\end{aligned}
$$

We adopt the following parameter ranges in our simulations: $\rho = 0.979$, $\phi_e = 0.044$, $\sigma = 0.0078$, $\mu_d = 0.004$, $\phi = \{0,3\}$, $\phi_d = [3.5, 5.5]$, $y \in [8\%, 20\%]$. The values of ρ, ϕ_e and σ adopt the calibration in Bansal and Yaron (2004). The specification of the model is of monthly frequency. We adopt $\mu_d = 0.004$ so that the annualized average dividend growth rate is close to 5%. We adopt $\phi_d = [3.5, 5.5]$ so that the annualized dividend growth volatility is in the range of $[7.5\%, 15.5\%]$, which is the 95% confidence interval estimated by Bansal, Kiku, and Yaron (2009). We consider two values of ϕ. When $\phi = 0$, the long-run risk component is switched off, and the dividend process follows a geometric random walk. When $\phi = 3$, the long-run risk component is present in the dividend growth rate and the parameter value is provided by Bansal and Yaron (2004).

A6. The Barro (2006) Process

We consider the rare disaster risk process as specified in Barro (2006):

$$
\begin{aligned}
D_{t+1} &= D_t \cdot \exp\left(g_{t+1}\right) \\
g_{t+1} &= \mu + \sigma u_{t+1} + v_{t+1} \\
u_{t+1} &\sim N \; i.i.d \; (0,1) \\
v_{t+1} &\sim \begin{cases} e^{-p} & 0 \\ 1 - e^{-p} & \log(1-b) \end{cases},
\end{aligned}
$$

where b is the random disaster loss following the empirical distribution provided by Barro (2006) page 832 Panel B.

We adopt the following parameter ranges in our simulations: $\mu = 5\%$, $\sigma \in [7.5\%, 15.5\%]$, $p \in [0.01, 0.05]$, $y \in [8\%, 20\%]$. The range of dividend growth volatility, $\sigma \in [7.5\%, 15.5\%]$, adopts the 95% confidence interval estimated by Bansal, Kiku, and Yaron (2009). Our range of disaster probability, $p \in [0.01, 0.05]$, covers the parameter choices in Barro (2006) ($p = 0.017$) and Gabaix

(2012) ($p = 0.0363$).

Appendix B. Robustness of the Block Bootstrap Procedure

Section 2.3.2 explains that one concern regarding the block bootstrap procedure might arise due to the possible wedge between the simulated trajectories and the true data generating process. To alleviate such a concern, we reuse the structural cash flow processes presented in Appendix A to show that our block bootstrap procedure is reliable across these models with parameters iterated over large grids.

Specifically, for each model, we iterate parameters across large grids. For each set of parameters, we generate two samples of cash flows: one from the true data generating process and the other from the block bootstrap simulations. We then compare the biases and standard errors estimated from these two samples, respectively, and record their differences. Finally, we adopt a different set of parameters from the grid and repeat the process.

Table 9 reports the differences between the estimated biases and standard errors from the model generated samples and the bootstrap simulated samples. In the most extreme case, bootstrap bias differs from the true value by 38 bps, and the bootstrap standard error differs from the true value by 26 bps. The median differences are even much smaller. The table shows that the block bootstrap procedure does a great job simulating the true data generating processes and produces bias and standard error estimates that are close to the true values.

Table 9: Accuracy of the Block Bootstrap Simulations

	FB	FB ADJ	LM	LM ADJ	BY	RD
mean	0.001	0.04	0.01	0.06	0.08	0.11
min	0.0	0.0001	0.001	0.001	0.002	0.02
25%	0.0002	0.01	0.01	0.01	0.02	0.06
50%	0.0005	0.02	0.01	0.03	0.08	0.11
75%	0.001	0.05	0.01	0.07	0.1	0.16
max	0.01	0.26	0.03	0.24	0.22	0.38

(a) Differences in Biases

	FB	FB ADJ	LM	LM ADJ	BY	RD
mean	0.004	0.04	0.05	0.06	0.09	0.09
min	0.0	0.0003	0.002	0.001	0.02	0.02
25%	0.001	0.02	0.01	0.02	0.05	0.04
50%	0.002	0.03	0.03	0.03	0.07	0.08
75%	0.004	0.04	0.08	0.08	0.12	0.12
max	0.03	0.22	0.12	0.18	0.26	0.26

(b) Differences in Standard Errors

This table reports the absolute differences in discount rate estimates biases and standard errors between the sample generated by the block bootstrap procedure and the true data generating process across several dividend processes. "FB" denotes the Fama and Babiak (1968) process. "FB ADJ" denotes the adjusted Fama and Babiak (1968) process where earnings growth follows geometric random walk. "LM" denotes the Leary and Michaely (2011) process. "LM ADJ" denotes the adjusted Leary and Michaely (2011) process where earnings growth follows geometric random walk. "BY" denotes the long-run risk process in Bansal and Yaron (2004). "RD" denotes the rare disaster process in Barro (2006). The reported statistics are from large grids of parameter specifications. All units are in percentage points.

Appendix C. Key Results for 10-Year and 5-Year Horizons

C.1 Key Results for the 10-Year Horizon

Table 10: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
							Panel (a): Market Beta							
$y-r_f^{15}$	6.07	6.96	6.77	7.13	7.04	7.37	6.96	7.5	8.57	10.15	4.07**	2.22	0.74	1.15
RDF	67.2	64.47	65.03	62.45	62.98	61.75	63.04	60.94	54.8	49.77	-17.43***	-2.95	-3.31	-1.53
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
							Panel (b): Size							
$y-r_f^{15}$	10.8	11.05	10.84	10.84	10.1	8.82	8.42	8.09	8.18	6.02	-4.78***	-3.02	-0.45	-1.16
RDF	45.75	46.73	48.07	48.31	51.17	54.75	56.69	57.96	57.42	68.55	22.8***	4.27	1.65	0.98
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
							Panel (c): Book-to-Market							
$y-r_f^{15}$	6.39	5.89	7.0	7.33	8.16	9.38	8.64	8.98	11.35	14.71	8.32***	4.42	0.81*	1.81
RDF	64.19	66.73	61.93	60.9	57.92	52.87	56.72	54.93	48.28	38.74	-25.45***	-3.4	-0.91	-0.53
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
							Panel (d): Gross Profitability							
$y-r_f^{15}$	10.82	9.99	5.52	5.95	6.85	6.81	6.84	8.11	7.89	10.16	-0.66	-0.72	2.88***	4.21
RDF	51.42	51.57	68.66	68.06	62.82	63.45	63.42	60.11	61.0	52.57	1.15	0.39	-9.0***	-3.88
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
							Panel (e): Investment							
$y-r_f^{15}$	8.53	9.07	8.42	7.38	6.87	7.39	6.99	6.99	8.52	9.06	0.53	0.36	1.66***	2.64
RDF	55.31	55.11	58.28	61.74	63.29	61.97	63.87	62.37	56.12	53.37	-1.94	-0.32	-7.65***	-3.05
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
							Panel (f): Credit Rating							
$y-r_f^{15}$	14.08	8.3	10.49	10.24	7.25	6.93	7.36	6.6	9.45	7.59	-6.48***	-3.58	2.76***	3.8
RDF	43.0	57.62	46.57	51.61	63.78	62.87	63.95	66.14	55.17	61.51	18.51***	2.78	-9.0***	-4.39
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53

This table documents the anomalies motivated by models of funding cost. "$y-r_f^{10}$" is the the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{10}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 11: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^{15}$	8.63	8.15	7.68	7.36	6.94	6.99	7.22	7.26	7.96	8.25	-0.38	-0.84	1.28***	7.22
RDF	52.67	54.22	58.78	61.26	63.29	63.04	62.72	63.39	61.13	58.86	6.19***	3.31	-6.44***	-9.05
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^{15}$	10.79	9.59	9.12	8.34	7.92	7.28	6.96	6.53	6.32	5.8	-4.99***	-11.02	0.53***	3.25
RDF	45.75	50.44	53.13	56.59	59.13	61.91	62.96	65.25	66.51	69.02	23.26***	10.6	-2.59***	-3.66
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^{15}$	5.81	7.31	8.17	8.54	8.98	9.53	9.33	8.66	7.73	8.19	2.38***	3.88	-2.0***	-10.55
RDF	68.73	62.87	58.61	56.16	55.22	51.79	50.75	54.05	56.67	49.03	-19.7***	-7.05	5.82***	6.73
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^{15}$	7.39	7.22	7.01	7.34	7.04	7.15	7.24	7.45	7.41	7.93	0.54***	2.7	0.39***	4.0
RDF	62.88	63.15	64.05	63.03	64.09	63.67	63.28	62.24	62.53	60.73	-2.15***	-2.68	-1.56***	-3.96
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. "$y-r_f^{10}$" is the the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{10}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 12: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Fail Probability														
$y-r_f^{15}$	8.01	8.72	7.67	6.79	6.98	6.96	6.69	6.62	7.18	6.96	-1.05**	-2.29	0.75***	3.92
RDF	54.13	58.5	62.53	64.75	63.23	63.19	64.38	64.58	61.15	59.39	5.27**	2.35	-4.92***	-5.38
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71
Panel (b): Book Leverage														
$y-r_f^{15}$	6.42	7.56	6.51	6.31	6.68	6.92	6.81	7.19	7.93	7.81	1.39	1.63	0.63	1.2
RDF	62.22	59.17	66.21	64.08	63.7	64.08	64.13	62.71	58.42	58.04	-4.18	-0.91	-4.43**	-2.11
$\bar{h}_1-r_f$	4.87	6.44	5.39	6.54	5.55	6.46	6.7	7.3	8.25	7.35	2.49	1.21	3.73**	2.3
Panel (c): Market Leverage														
$y-r_f^{15}$	6.1	6.4	6.52	7.1	7.3	7.69	8.39	9.01	9.48	9.69	3.59**	2.03	0.42	0.84
RDF	65.44	64.17	64.1	62.01	61.67	59.64	55.59	54.01	53.14	49.18	-16.26*	-1.82	-2.68	-1.08
$\bar{h}_1-r_f$	5.26	5.5	5.83	6.11	8.01	7.93	10.11	10.14	8.33	10.86	5.6*	1.96	3.5**	2.2
Panel (d): Return on Asset														
$y-r_f^{15}$	6.74	8.4	8.94	10.56	7.35	6.47	6.46	6.51	6.46	6.54	-0.2	-0.46	0.13	0.72
RDF	65.7	58.35	57.51	51.21	61.21	66.36	65.33	64.82	64.39	63.82	-1.88	-1.01	-0.72	-0.92
$\bar{h}_1-r_f$	-4.13	1.74	2.16	4.7	5.96	6.86	6.81	7.36	6.74	8.36	12.49***	4.07	-0.03	-0.01

This table documents several additional anomalies. "$y-r_f^{10}$" is the the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^{10}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

C.2 Key Results for the 5-Year Horizon

Table 13: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^5$	6.16	7.42	7.36	7.67	7.68	7.86	7.39	7.85	9.76	11.62	5.46**	2.12	0.97	1.19
RDF	79.66	76.51	76.6	75.14	74.98	74.01	75.17	74.81	68.36	65.65	-14.0**	-2.12	-1.95	-0.95
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
Panel (b): Size														
$y-r_f^5$	14.18	13.12	11.94	12.26	10.69	9.84	9.57	9.23	8.64	6.34	-7.84***	-4.43	0.31	0.66
RDF	58.75	61.41	64.15	63.47	67.44	69.0	69.44	70.54	71.72	78.69	19.94***	5.19	-0.58	-0.52
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
Panel (c): Book-to-Market														
$y-r_f^5$	6.16	6.48	8.3	8.46	8.85	10.54	9.56	10.2	13.34	17.91	11.75***	5.06	1.27*	1.95
RDF	78.77	77.53	72.57	71.89	71.1	67.04	69.65	67.43	61.02	50.65	-28.12***	-4.72	-2.08	-1.26
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
Panel (d): Gross Profitability														
$y-r_f^5$	10.54	10.74	6.51	6.63	7.2	7.23	6.71	8.83	9.09	10.85	0.31	0.22	3.09***	3.76
RDF	69.31	66.91	77.36	77.8	75.69	75.71	78.13	72.85	72.43	67.41	-1.91	-0.58	-6.69***	-3.11
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
Panel (e): Investment														
$y-r_f^5$	10.36	9.67	9.48	8.32	7.16	7.59	7.89	7.9	8.76	9.34	-1.02	-0.59	2.16***	2.6
RDF	67.68	70.72	70.46	73.01	76.33	74.98	73.9	72.91	71.19	68.84	1.16	0.25	-6.05***	-2.9
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
Panel (f): Credit Rating														
$y-r_f^5$	16.25	8.73	14.51	11.32	8.94	7.73	8.19	8.03	10.73	9.14	-7.12*	-1.85	2.88**	1.99
RDF	56.59	72.94	55.1	64.49	71.29	73.88	73.24	73.76	66.51	70.26	13.68	1.46	-6.01*	-1.67
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53

This table documents the anomalies motivated by models of funding cost. "$y-r_f^5$" is the the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^5$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 14: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^5$	9.36	8.5	8.11	7.71	7.33	7.54	8.08	8.4	9.55	9.81	0.45	0.67	1.87***	6.25
RDF	69.09	71.39	72.94	74.34	75.62	75.08	73.75	73.46	70.74	68.4	-0.7	-0.39	-5.45***	-7.12
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^5$	12.65	11.95	11.08	9.69	8.69	8.26	7.86	7.35	7.22	7.1	-5.56***	-8.99	1.26***	4.86
RDF	60.73	62.3	65.06	68.66	71.89	72.94	74.09	75.61	75.87	73.13	12.41***	7.62	-4.41***	-6.61
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^5$	6.21	8.08	9.08	9.39	10.21	11.27	11.55	9.56	8.71	8.04	1.83***	2.75	-2.98***	-9.85
RDF	78.93	73.88	70.88	70.04	67.04	64.78	65.58	71.18	71.93	73.37	-5.57**	-2.44	8.62***	11.67
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^5$	8.01	7.87	7.67	7.96	7.92	8.04	7.79	8.12	8.22	8.74	0.73**	2.57	0.23*	1.68
RDF	74.27	74.38	75.03	74.35	74.25	74.14	74.72	73.8	73.59	72.45	-1.82**	-2.37	-0.52	-1.41
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. "$y-r_f^5$" is the the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^5$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.

Table 15: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Fail Probability														
$y-r_f^5$	9.7	11.17	9.13	8.13	8.22	8.21	7.52	7.95	8.04	8.15	-1.55**	-2.24	1.05***	3.13
RDF	66.69	63.56	72.04	73.74	72.85	72.91	74.8	73.89	72.56	71.26	4.57**	2.41	-4.36***	-4.77
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71
Panel (b): Book Leverage														
$y-r_f^5$	6.68	7.18	8.0	6.02	7.36	7.5	7.81	8.42	9.56	8.98	2.3	1.34	0.67	0.89
RDF	77.27	70.98	72.9	79.65	75.38	76.04	74.68	73.01	70.15	71.22	-6.05	-1.13	-3.3	-1.53
$\bar{h}_1-r_f$	4.87	6.44	5.39	6.54	5.55	6.46	6.7	7.3	8.25	7.35	2.49	1.21	3.73**	2.3
Panel (c): Market Leverage														
$y-r_f^5$	6.11	7.22	7.09	7.73	8.22	9.24	8.94	11.65	11.77	12.36	6.25***	2.74	0.64	0.93
RDF	76.33	75.33	76.1	73.74	73.05	70.12	70.39	64.84	63.64	61.96	-14.36**	-2.22	-2.27	-1.2
$\bar{h}_1-r_f$	5.26	5.5	5.83	6.11	8.01	7.93	10.11	10.14	8.33	10.86	5.6*	1.96	3.5**	2.2
Panel (d): Return on Asset														
$y-r_f^5$	5.78	8.12	9.5	11.0	8.7	6.9	6.86	7.15	7.01	7.54	1.76***	3.05	-0.69**	-2.07
RDF	80.92	73.67	71.02	66.49	71.23	77.15	76.89	75.94	76.53	73.78	-7.14***	-3.22	2.04*	1.83
$\bar{h}_1-r_f$	-4.13	1.74	2.16	4.7	5.96	6.86	6.81	7.36	6.74	8.36	12.49***	4.07	-0.03	-0.01

This table documents several additional anomalies. "$y-r_f^5$" is the the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "RDF" is the relative discount factor. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-stats for "$y-r_f^5$" and "RDF" are based on the standard errors estimated from block bootstrap simulations. The t-stats for "$\bar{h}_1-r_f$" are the standard OLS t-stats.