The Stick or the Carrot? The Role of Regulation and Liquidity in Activist Short-Termism

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Abstract

I study a model of activist short-termism, where the activist can sell his stake in the target before the impact of his intervention is realized. Changes in liquidity or policies that make activists' exit harder can *increase* firm value if there is only moral hazard (where activist's intervention creates more value if he exerts effort) or only adverse selection (where some interventions destroy value while others create value). However, these changes *destroy* total firm value when both moral hazard and adverse selection are present. Policies that reward long-termism can also destroy total firm value, but with a lower likelihood.

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1 Introduction

Activist shareholders are often criticized for sacrificing long-term value for short-term gains. Laurence Fink, the CEO of the world's largest asset manager BlackRock, has argued that "the proliferation of activist shareholders seeking immediate returns" pressures "companies to meet short-term financial goals at the expense of building long-term value", along with State Street which has expressed similar concerns.[1] In the empirical literature, while several papers report that the stock price response to activist campaigns is positive *on average*, others report evidence consistent with activist shareholders destroying long-term value in *some* of their interventions due to short-termist motives.[2]

An important reason for this short-termism criticism is that the market often cannot immediately assess the long-term consequences of activist interventions, and therefore activists can sell their stake in the target before the effect of their actions on firm value is revealed. In particular, there are two important channels through which this problem can arise.

The first one is adverse selection: When there are activists whose interventions improve firm value, other activists who do not have a value-increasing strategy might be tempted to make a profit by intervening in firms, creating the perception that they are improving firm value with their intervention (by pooling with activists who do increase firm value), and then selling their stake, even if such interventions destroy firm value in reality. For example, in 2006, hedge fund activist William Ackman pressured Wendy's to spin-off Tim Hortons and sold his stake in Wendy's shortly after the spin-off, making a profit due to the short-term increase in Wendy's share price from about $15 to $20. However, over the following years, the share price declined all the way down to $4-5 and remained in this range for years. Many have argued that the loss of synergies with Tim Hortons played an important role in Wendy's poor share performance and criticized Ackman for destroying firm value with the spin-off.[3]

Motivated by such activist interventions, many proposals have been made to curb activists

[1] See Business Insider, "BlackRock CEO Larry Fink tells the world's biggest business leaders to stop worrying about short-term results," 04/14/2015, and State Street Global Advisors, "Protecting Long-Term Shareholder Interests In Activist Engagements," 10/10/2016. For a similar remark made by BlackRock about activists in a separate statement, see HLS Forum on Corporate Governance and Financial Regulation, "Getting Along with BlackRock," 11/06/2017.

[2] For extensive surveys, see, e.g., Brav, Jiang, and Kim (2009), Brav, Jiang, and Kim (2015), and Brav, Jiang, and Li (2022). For evidence regarding negative effects, see Cremers, Masconale, and Sepe (2017), deHaan, Larcker, and McClure (2019), and Baker (2021).

[3] Forbes has commented that "Ackman successfully extracted short-term value from the company while hurting long-term shareholders" and "Wendy's never recovered from the loss of Tim Horton's." See Forbes, "Some Activist Investors Come Up Short In Creating Shareholder Value," 05/17/2016.

that destroy value. Some of these proposals punish short-termism by making activists' exit more difficult: In her campaign for the 2016 presidential election, Hillary Clinton called out "hit-and-run activists whose goal is to force an immediate payout" and proposed to substantially increase the short-term capital gains tax rate and increase the horizon it applies to, changes which were also put forward by Laurence Fink and endorsed by many other prominent figures such as John Bogle and Warren Buffett.[4] In 2022, the SEC formally proposed reducing activists' time window to make a disclosure when they start selling their stake.[5] In contrast, another set of policies suggested to combat short-termism take a different route and instead reward activists for keeping their stake longer, such as reducing long-term capital gains taxes (which was also part of Clinton's and Fink's proposals mentioned above), or giving activists additional perks for holding their stake long-term (e.g., additional voting rights or a higher dividend rate).[6]

A second channel through which activist short-termism can manifest itself is moral hazard. In particular, the activist can increase firm value by exerting hidden effort, where this effort can represent the activist's due diligence, dedication of his resources, and behind-the-scenes monitoring of the management to implement the intervention more effectively (e.g., if the intervention is a spin-off, then structuring, pricing, and executing the spin-off optimally; if it is firing the CEO, then finding a better replacement for the CEO; if it is reducing costs, then cutting costs efficiently). However, as already established by the theoretical blockholder literature, activists' ability to sell their stake undermines their incentives to exert effort.[7] Therefore, this literature posits that making it more difficult for activists to exit should alleviate the moral hazard problem, by incentivizing the activist to keep his stake and thus, to exert effort. While a particular focus of this literature has been on liquidity, the same reasoning implies that policies that punish short-termism, as well as those that reward long-termism (such as policies mentioned above), should mitigate the moral hazard problem as well.

[4]See NYT DealBook, "Hillary Clinton Aim Is to Thwart Quick Buck on Wall Street," 7/27/2015; NYT DealBook, "BlackRock's Chief, Laurence Fink, Urges Other C.E.O.s to Stop Being So Nice to Investors," 4/13/2015; and Aspen Institute, "Overcoming Short-termism," 9/9/2009.

[5]Specifically, the SEC proposed reducing the filing window for 13D amendments to *one* business day (see HLS Forum on Corp. Governance and Financial Regulation, "SEC Proposes Updates to Schedule 13D/G Reporting," 2/15/2022).

[6]For instance, the Florange law in France doubles a shareholder's voting rights after holding the shares for two years. To encourage long-term investing, Toyota issued a special type of loyalty shares in 2015 with an annual dividend yield increasing from 0.5% in the first year to 2.5% in the fifth year (see WSJ, "Q&A: Toyota's New Type of Share, and Why It's Controversial", 6/17/2015).

[7]See Section 5.1.2 in the survey by Edmans and Holderness (2017). The history of this argument goes back to Coffee (1991) and Bhide (1993).

In this paper, I build a theoretical model of activist short-termism to analyze the impact of liquidity and the policies aimed at combatting short-termism. The main questions that the paper addresses are as follows: If stricter regulation or lower liquidity eliminates the value-destroying activists *and* at the same time provides incentives for other activists to create more value, does it always increase the value created in interventions? Is there any tension between policies that aim to enhance governance *forcefully* and activists' *endogenous* incentives to improve firm value? And if there is such a tension, then what kind of policies are more likely to create value than others?

I find that making the activist's exit more difficult can *increase* firm value if there is only moral hazard or only adverse selection. However, it *destroys* total firm value (that is, the total value of all firms) when both moral hazard and adverse selection are present at the same time. Moreover, even policies that reward long-termism can destroy total firm value. Nevertheless, when I compare the effectiveness of different policies, I find that policies rewarding long-termism are least likely to destroy total firm value. The key result driving these implications is that a higher number of value-destroying activists can result in a higher probability of effort by the value-creating activists, and as a result of this higher effort, average firm value can increase.

I start with the following baseline model in Section 2. There is a firm and an activist with a stake in this firm. The activist's intervention in the firm is either "good" or "bad", which is the private information of the activist: In the former case, the activist's intervention increases the firm's value and the activist can further increase this value by exerting (unobservable) effort when intervening. In the latter, the activist's intervention destroys firm value.[8] The main friction in the model is that, as alluded to earlier, it takes time for the impact of the activist's intervention on firm value to be revealed (i.e., become public information), and the activist can sell his stake in the company (i.e., "exit") before the change in firm value is revealed. However, with some probability, the activist has to sell his stake due to a liquidity shock, and therefore the market cannot perfectly distinguish whether the activist has to sell for this exogenous (liquidity) reason or because he chooses to sell even if he does not have to.

The exit opportunity of the activist creates short-term incentives for him not to exert effort. To see this, consider the case where the activist's intervention is always good. Then, for any cost of effort, the activist shirks with some probability. This is because if he exerted effort with certainty, then the stock price would incorporate this expectation, and therefore the activist would have incentives to shirk and sell his stake instead. In other words, in equilibrium, the

[8] To simplify the exposition, I assume that the activist with a bad intervention cannot improve the effect of his intervention on firm value by exerting effort. That said, the main results are robust to relaxing this assumption, as I discuss in Section 4.1.

first-best is not achieved due to the possibility of making short-term trading profits.

The primary result of the baseline model is that if the moral hazard problem is binding (that is, the activist with a good intervention sometimes shirks and sometimes exerts effort), then average firm value is higher when the probability that the activist is implementing a *bad* intervention is higher. This is because the value-destroying activist (i.e., the activist that implements a bad intervention) always sells his stake, and therefore as the probability that the activist is destroying value increases, the stock price reacts more negatively upon the activist's sale decision. As a result, the incentives of the value-creating activist (i.e., the activist that implements a good intervention) to keep his stake until the change in firm value is revealed increases. In turn, this activist internalizes the effect of his actions on the actual firm value more, and therefore his incentives to exert effort to further increase value also get stronger.

Importantly, I show that this positive effect dominates the greater value destruction induced by the higher probability that the activist is implementing a value-destroying intervention. The intuition is as follows. As the probability that the activist is destroying value increases, the activist with a good intervention increases the probability that he exerts effort to a certain point. In particular, his higher effort increases the average stock price upon his exit to the point where he becomes once again indifferent between exerting effort (and keeping his stake unless hit by a liquidity shock) vs. shirking (and selling his stake). The critical observation is that the impact of a higher probability of effort has more positive impact on the *unconditional* average value of the firm compared to the stock price *conditional* on the exit of the activist. This is because upon the exit of the activist, the market assigns a lower probability that the activist has exerted effort, since such an activist does not exit unless he has to. Overall, to put it differently, one kind of short-termism (i.e., not exerting sufficient effort while still creating value) stemming from moral hazard can be alleviated with what can be seen as even more severe short-termism (i.e., outright value destruction) stemming from adverse selection.

Next, I introduce and analyze the main model of the paper in Section 3 in order to study policy and liquidity implications. In particular, I generalize the baseline model to a multiple firm and multiple activist setup, where each activist is matched to a different firm and now decides whether to acquire a stake in his matched firm or not, as well as whether to intervene or not.

I find that the implications of greater liquidity (i.e., higher noise trading, which makes it easier for the activist to camouflage his trade), as well as policy implications, are vastly different when there is only moral hazard or adverse selection, compared to the case where both are present. To begin with, suppose that there is only moral hazard but no adverse selection. In other words, there is no uncertainty about the value of the activist's intervention if he does not exert effort, but the activist can increase this value by exerting hidden effort. Then, in parallel with the aforementioned literature on blockholder intervention and trading, I find that lower liquidity at the exit stage (i.e., when the activist decides whether to sell his stake or keep it) as well as other policies that make the activist's exit more difficult (e.g., higher short-term capital gains taxes or tighter disclosure thresholds for selling) can increase total firm value, because they incentivize the activist to hold his stake, and in turn, to exert effort in the first place.

Next, suppose that there is only adverse selection but no moral hazard (i.e., the activist has private information about the value of his intervention, but he cannot increase it by exerting effort). Then, lower liquidity at the exit stage, as well as other policies that make activists' exit more difficult, can again increase total firm value. This is because these changes reduce the profit that activists can make in equilibrium, thereby reducing the number of activists that implement value-destroying interventions. Note that an activist with a value-destroying intervention can still make a profit because when the activist sells his stake, the market cannot perfectly distinguish him from activists with value-creating interventions. For the same reason, a value-destroying activist can prefer intervening over not intervening, because the latter gives him a payoff of zero.

Then, I combine moral hazard and adverse selection, and I find very different results from above. In particular, I find that if the moral hazard problem is binding and there is adverse selection in equilibrium, then lower liquidity at the exit stage as well as other policies that make activists' exit more difficult *destroy* total firm value. Strikingly, this is true even if these policies only eliminate the activists with value-destroying interventions and the number of activists who implement good interventions does not change.

Importantly, this result arises because as the number of activists who implement bad interventions decreases, activists with good interventions exert effort with much lower probability, as it was shown in the baseline model. Therefore, another way to rephrase this result is that as *formal liquidity* (i.e., noise trading) decreases or other policies that make activists' exit *more difficult* are implemented, due to the resulting reduction in bad interventions, *implicit liquidity* imposed on activists with good interventions *increases* so much that they shirk and exit with much higher probability.

Next, I analyze the implications of policies that reward long-termism, i.e., lowering long-term capital gains taxes or giving the activist additional perks for holding the stake long-term (e.g., a higher dividend rate or more voting power).[9] I find that when moral hazard and adverse

[9] Another example is proxy access, which has been adopted by 76% of S&P 500 firms as of 2019 (compared to

selection are both present, then policies that reward long-termism can hurt total firm value as well. The core intuition for this result is similar to the one explained above: To enjoy the benefits from holding the stake long-term, some activists with value-destroying interventions switch from intervening and exiting to not intervening and keeping their stake. However, since the market rationally anticipates this effect and exit price adjusts, this has a negative impact on the *endogenous* incentives of the activists with good interventions to exert effort. Nevertheless, as I explain in more detail in the paper, compared to policies that make the activist's exit more difficult as well as lower liquidity at the exit stage, I find that policies that reward long-termism are less likely to destroy total firm value.

Finally, to demonstrate the robustness of the results, I discuss several extensions in Section 4, including (*i*) allowing the activist with a bad intervention to improve the impact of his intervention on firm value by exerting effort, (*ii*) micro-founding the activist's effort as a search among different projects, (*iii*) relaxing the assumption that the activist can intervene unilaterally, (*iv*) allowing for partial sale as well as dynamic disclosure, (*v*) incorporating the activist's career concerns and skill, and (*vi*) choosing among multiple activists.

It is important to note that the model has implications beyond shareholder activism as well, as I also discuss in Section 4. In particular, the activist in the baseline model can also represent other kinds of blockholders who are subject to short-termism: For example, venture capitalists and entrepreneurs are often criticized for pushing so many "unicorns" to IPOs despite not demonstrating profitability and fooling investors with infeasible business plans. Therefore, the baseline model implies the optimality of value destruction by such types of blockholders as well.

1.1 Related literature

This paper contributes to multiple strands of the literature. The first strand is the large literature on short-termism that primarily goes back to Narayanan (1985) and Stein (1988, 1989). In this literature, the closest papers to mine are the ones that argue short-termism might have positive effects due to some frictions (e.g., Bolton, Scheinkman, and Xiong (2006), Laux (2012), Hackbarth, Rivera, and Yong (2018), Xiong and Jiang (2021), Donaldson, Malenko, and Piacentino (2020), Thakor (2021)). Marinovic and Varas (2021) study short-termism by

1% in 2014) and allows shareholders to nominate directors to the board on the company's proxy ballot if they have held a minimum stake for a certain period of time (typically three years), making such nominations much less costly for the qualifying shareholders. See HLS Forum on Corp. Governance and Financial Regulation, "Proxy Access: A Five-Year Review," 2/4/2020.

combining moral hazard and adverse selection and focusing on IPOs for their implications. In contrast to these papers, the focus in my paper is to analyze the impact of liquidity and different policies on activist short-termism. I also show that the presence of value-destroying activists can be optimal for firm value, because it strengthens the incentives of other activists to exert more effort and create more value as a result.

The second strand my paper is closely related to is the blockholder literature, which is broadly divided into two branches: "voice" and "exit".[10] My paper falls under the voice category, where the blockholder intervenes in a firm to influence the actions or projects undertaken (e.g., Shleifer and Vishny (1986), Admati et al. (1994), Burkart et al. (1997), Cohn and Rajan (2013), Corum and Levit (2019), Corum (2023)). Under the voice category, a further subset of papers study the relation between liquidity and the blockholder's incentives to engage in an intervention (e.g., Kyle and Vila (1991), Bolton and von Thadden (1998), Kahn and Winton (1998), Maug (1998), Noe (2002), Aghion, Bolton, and Tirole (2004), Faure-Grimaud and Gromb (2004), Back et al. (2018)). Among these, the closest paper to mine is Maug (1998), who shows that higher liquidity might increase firm value. However, in his model, this occurs because the blockholder utilizes higher liquidity to *increase* his stake. In contrast, my paper combines moral hazard with adverse selection, and it shows that higher liquidity can strictly increase total firm value even if the blockholder *never* increases his stake as a result but utilizes this higher liquidity only to *sell* his shares. I also show that other policies that make the blockholders' exit more difficult (e.g., tightening disclosure rules for selling or raising short-term taxes) can result in a lower total firm value.

Finally, the literature on governance through the threat of exit studies how a blockholder can influence the decisions of the manager of the firm, not through voice, but because the exit of the blockholder can reveal negative information about the firm and hence punish the manager (e.g., Admati and Pfleiderer (2009), Edmans (2009), Cvijanovic, Dasgupta, and Zachariadis (2022)).[11] In contrast to these papers, in my paper, there is no other player that affects the firm value other than the blockholder.

[10] For surveys on voice and exit, see, e.g., Edmans (2014) and Edmans and Holderness (2017).

[11] Some papers combine voice and exit (e.g., Edmans and Manso (2011), Dasgupta and Piacentino (2015), Song (2015), Fos and Kahn (2019), Edmans, Levit, and Reilly (2019), Levit (2019)). For other models where a blockholder's trading impacts the manager's decisions, see, e.g., Khanna and Mathews (2012) and Goldman and Strobl (2013).

2 The Baseline Model

2.1 Setup

There is a firm and an activist shareholder (in the baseline model, the activist can alternatively represent an entrepreneur or a venture capitalist; see Section 4.8 for details). The only other player in the game is the market maker. The activist's intervention affects the value of the firm (I use "intervention" and "action" interchangeably). I denote the value of the firm in status quo (i.e., if the activist does not intervene) by Q_0. The activist has α stake in the firm, and he can have either of the two types of interventions: good and bad. Whether the activist's intervention is good or bad is his private information. The activist with a bad intervention always destroys firm value, and I denote the impact of the bad intervention on the firm value by $\Delta_L < 0$. In contrast, the activist with a good intervention always increases the value of the firm. In particular, if he exerts an (unobservable) effort, then the firm value increases by $\Delta_H > 0$ but the activist incurs a cost of $c > 0$. If he does not exert effort, then the firm value increases by $\Delta_M > 0$, where $\Delta_M < \Delta_H$. I also refer to the activist with a good (bad) intervention as the value-creating (value-destroying) activist. I denote the time at which the activist intervenes by $t = 1$ (i.e., the "intervention" stage). At this stage, the activist simultaneously decides whether to intervene and whether to exert effort (for a discussion of this assumption, see Section 4.1, where I also relax the assumption that the activist with a bad intervention cannot improve firm value by exerting effort). Note that at $t = 1$, NPV of the intervention (i.e., the effect of the intervention on the firm value) is the private information of the activist, as well as whether he has exerted effort. That said, if the activist intervenes, then this is immediately observable.

The main friction of the model is that it takes time for the NPV of the activist's intervention to be revealed (i.e., become public information), and the activist can sell his stake before then. In particular, the activist can sell his stake by $t = 2$ (i.e., the "exit" stage), and the NPV becomes public information at $t = 3$ (i.e., the "realization" stage). At the exit stage, with probability ϕ the activist has to sell his stake (e.g., due to a liquidity shock or an attractive outside opportunity), and with probability $1-\phi$ the activist has the flexibility to choose whether to sell his stake or not. If the activist decides to sell, he has to disclose this information before selling his stake, which is an assumption I relax in the full model (Section 3). The disclosure is observed by the market maker, who determines the stock price based on this information. For simplicity, I assume that the activist can sell either all of his stake or none of it. I relax this assumption in Section 4.3.

There is no discounting in the game. I also ignore the short-term and long-term capital gains taxes in the baseline model. I incorporate taxes as well as many other aspects in the full model in Section 3.

2.2 Analysis under No Voluntary Exit

In this brief section, I analyze a simplified version of the baseline model, which constitutes the first-best benchmark of the model. Specifically, in this benchmark, the only difference with respect to the baseline model is that the activist cannot voluntarily exit (i.e., sell his stake) before the NPV is revealed, unless he is hit with a liquidity shock. Therefore, the activist has to exit with probability ϕ (which is the probability of a liquidity shock to the activist), and has to keep his stake with probability $1 - \phi$. While the assumption that the activist cannot voluntarily exit is artificial, it will help convey the crucial role that voluntary exit plays in the next section. In particular, under this assumption, the result below follows.

Lemma 1 *Suppose that the activist cannot sell his stake before the NPV is revealed unless he is hit with a liquidity shock. Then, when the activist implements a good intervention, he never exerts effort if* $c > \bar{c}$ *and always exerts effort if*

$$c < \bar{c} \equiv (1 - \phi)\,\alpha\,(\Delta_H - \Delta_M)\,. \tag{1}$$

Note that I solve for the Perfect Bayesian Equilibria of the game. The intuition of Lemma 1 is as follows. With probability ϕ, the activist has to exit, and at this stage the market cannot distinguish if he has exerted effort or not. Therefore, conditional on intervening, the stock price at the exit stage is the same regardless of whether he has exerted effort or not. For this reason, in this case, he receives no benefit for exerting effort. However, with probability $1 - \phi$, the activist has to keep his stake α until the NPV is revealed, and hence his payoff is $\alpha(\Delta_H - \Delta_M)$ higher if his intervention is good and he has exerted effort. Therefore, when the activist implements a good intervention, he never exerts effort if $c > \bar{c}$, and he always exerts effort if $c < \bar{c}$, where c is the activist's cost of effort. As a result, the first-best achieved when $c < \bar{c}$, and increasing the probability that the intervention is bad always decreases firm value. However, as I show in the next section, this result changes once the activist is allowed to exit voluntarily.

2.3 Analysis under Voluntary Exit

In this section, I solve the baseline model without imposing any additional restriction. Recall that at the exit stage (after the intervention is implemented but before the NPV of intervention is revealed), with probability ϕ the activist has to sell his stake (e.g., due to a liquidity shock), and with probability $1-\phi$ he chooses whether to sell his stake or keep it until the NPV is revealed.

I again solve for the Perfect Bayesian Equilibria of the game. All proofs are in the appendix. I denote by ρ the probability that the activist has exerted effort when implementing a good intervention, and I denote by q the probability that the implemented intervention is bad. Also, conditional on the activist intervening and then exiting (i.e., selling his stake before the NPV is realized), I denote the expected NPV of the intervention by P. Recall that before the activist sells his stake, he has to disclose that he will sell (which is an assumption I relax in the full model in Section 3). Since the value of the firm when no intervention is implemented is Q_0, once the activist discloses that he will exit the stock price becomes $Q_0 + P$, which I refer to as the "exit price". Since Q_0 and α (where the latter is the activist's stake) are exogenous constants, Q_0 has no effect on any of the results, and therefore I normalize $Q_0 = 0$ throughout the text in this section to simplify the notation (for a discussion of endogenous α, see Section 4.4). All of the proofs hold for any $Q_0 \geq 0$. I start the analysis with the following lemma.

Lemma 2 *Conditional on implementing the intervention:*

(*i*) *For any $q \in [0,1)$, the activist with a good intervention never exerts effort (i.e., $\rho^* = 0$) if $c > \bar{c}$ and exerts effort with positive probability (i.e., $\rho^* > 0$) if $c < \bar{c}$, where $\bar{c}$ is given by (1).*

(*ii*) *Suppose that $c < \bar{c}$. If the activist's intervention is good with certainty (i.e., $q = 0$), then he shirks with positive probability (i.e., $\rho^* < 1$).*

Let us first understand the intuition behind part (i) of Lemma 2. Conditional on intervening, the payoff of the activist with a good intervention from shirking is given by $\alpha[\phi P + (1-\phi)\max\{P, \Delta_M\}]$, since the payoff of the stock is Δ_M if the activist waits until the NPV is revealed, and it is P if he does not wait and chooses to exit instead. In contrast, the payoff of the activist from exerting effort is given by $\alpha[\phi P + (1-\phi)\Delta_H] - c$, since the exit price P always satisfies $P \leq \Delta_H$ (in other words, the activist weakly prefers to keep his stake if he exerts effort, since the market's expectation is weakly worse at the exit stage). Therefore, the activist exerts effort only if

$$\begin{aligned}\alpha[\phi P+(1-\phi)\Delta_H]-c &\geq \alpha[\phi P+(1-\phi)\max\{P,\Delta_M\}] \\ &\Leftrightarrow\ c\leq(1-\phi)\,\alpha\,(\Delta_H-\max\{P,\Delta_M\})\,. \end{aligned}\tag{2}$$

However, if $c>\bar{c}$, then (2) is never satisfied, and therefore the activist never exerts effort. This is because then, even if the exit price P is so low that the activist prefers to keep his stake, he still does not exert effort because for him, it is not worth to incur the cost c to increase the stock payoff by $\Delta_H-\Delta_M$. For this reason, throughout the analysis in the baseline model, I will focus on the case where $c<\bar{c}$.

In turn, part (ii) of Lemma 2 introduces the core inefficiency in the paper: It states that even given $c<\bar{c}$, if an activist always implements a good intervention, then he sometimes shirks. Note that this in contrast with Lemma 1 in the previous section, where the channel of voluntary exit was shut down, and it was shown that as a result, the first-best was achieved if $c<\bar{c}$.

To see the reason behind this result, suppose for a moment that if an activist intervenes, it is always a good intervention and he always exerts effort ($q=0$ and $\rho^*=1$). Then the exit price is $P^*=\Delta_H$, which is also equal to the eventual payoff of the stock if the activist keeps his stake after exerting effort. However, then the activist gains nothing by exerting effort and keeping his stake until the NPV is revealed. For this reason, the activist would instead strictly prefer to shirk and exit at the price of $P^*=\Delta_H$, and save the cost of effort by doing so. In other words, such an exit price is too high to incentivize the activist to exert effort, since it instead motivates the activist to shirk and exit.

For this reason, when the implemented intervention is good with certainty, the activist does not always exert effort, even though the first-best would require him to exert effort. As mentioned above, this is the efficiency at the heart of the paper. Interestingly, as we will see further below, increasing the likelihood that the activist's intervention is destroying value actually alleviates this problem. Before showing this result, however, we need to characterize the equilibrium. To that end, I solve the model backwards: In particular, I first derive the activist's equilibrium behavior at the exit stage (in the lemma below), and then use it to solve for the probability of effort.

Lemma 3 *In any equilibrium for any $q\in[0,1)$, conditional on implementing the intervention:*

(*i*) *The activist with a bad intervention always exits.*

(ii) If the activist with a good intervention exerts effort, he does not exit unless hit by a liquidity shock.

(iii) If the activist with a good intervention shirks, he always exits.

It is relatively straightforward to understand the exit strategy of the activist if he has a good intervention and has exerted effort, or he has a bad intervention, given by parts (i) and (ii) of Lemma 3, respectively. Let us start with the former. As long as the implemented intervention is not always bad (that is, for any $q \in [0, 1)$), the exit price P always satisfies $P^* > \Delta_L$, because there is a positive probability that the activist has implemented a good intervention and is selling his stake involuntarily. However, the activist that has implemented a bad intervention knows that the market value of the firm will be Δ_L if he keeps his stake, and therefore strictly prefers to sell his stake at the exit stage before the NPV is revealed.

On the other end of the spectrum, the activist that has implemented a good intervention never exits if he has exerted effort, because the NPV is not revealed at the exit stage yet. In other words, if he is going to exit before he consequences of his effort is revealed, then he has no reason to exert effort in the first place, yielding part (ii) of Lemma 3.

Finally, part (iii) of Lemma 3 states that the activist always exits if he has implemented a good intervention and shirked. Intuitively, this activist keeps his stake only if the exit price is weakly lower than the actual value of the firm (that is, $P \leq \Delta_M$). However, whenever this is the case, the activist is strictly better off by exerting effort, and keeping his stake unless hit by a liquidity shock. This is because by doing so, he increases the firm value from Δ_M to Δ_H, and hence his payoff increases by $(1 - \phi)\alpha(\Delta_H - \Delta_M) - c$, which is positive due to $c < \bar{c}$ and (1). Therefore, if the exit price is indeed low such that $P \leq \Delta_M$, the activist with a good intervention will always exert effort. In other words, this activist does not exert effort only if the exit price is high ($P > \Delta_M$), and then, it is always profitable for him to exit.

Note that an implication of Lemma 3 is that as a function of ρ and for any given q, the exit price P^* in any equilibrium is given by

$$P^* (\rho; q) = \frac{(1 - q)\, \rho\phi\Delta_H + (1 - q)\,(1 - \rho)\, \Delta_M + q\Delta_L}{1 - (1 - q)\, \rho\, (1 - \phi)}, \tag{3}$$

because (3) is the expected NPV of the implemented intervention conditional on the information that the activist is exiting. In turn, the exit price P determines the incentives of the activist with a good intervention to exert effort. In particular, when implementing a good intervention, Lemma 3 implies that the activist chooses between shirking (and exiting) vs.

exerting effort (and keeping his stake unless hit by a liquidity shock). The former gives the activist a payoff of αP, while the latter gives him a payoff of $\alpha[\phi P + (1-\phi)\Delta_H] - c$. Therefore, the activist with a good intervention is indifferent between these two options if and only if

$$\alpha(1-\phi)P = \alpha(1-\phi)\Delta_H - c, \tag{4}$$

Plugging (3) in (4) yields the solution (6) given in Proposition 1 below. Importantly, if (6) does not exceed 1, then it is the equilibrium level of effort ρ^*. To see why, note that if ρ^* were any larger than (6), then (3) implies that the resulting P^* would be strictly larger.[12] Intuitively, the market incorporates the higher probability of effort by the activist in the exit price, since the activist that exerts effort still has to exit with probability ϕ. As a result, the activist would instead strictly prefer to shirk and exit, since the exit price would be too tempting for the activist. In contrast, if ρ^* were any smaller than (6), then P^* would be too low, and hence the activist would strictly prefer to keep his stake (unless hit with a liquidity shock). Then, since the activist would rationally anticipate that he will keep his stake, he would exert effort (as also implied by part (ii) of Lemma 3).

The situation where the activist with a good intervention does not always exert effort (that is, $\rho^* < 1$) arises if and only if q (i.e., the probability that the implemented intervention is bad) is not too large, that is $q < \underline{q}$. The formal expression for the threshold $\underline{q}$ is given by (5) in Proposition 1 below. This is because intuitively, as mentioned above, if the activist with a good intervention always exerted effort then the exit price P would become too large to incentivize the activist to exert effort.

Importantly, in this region (where $q < \underline{q}$), as q increases ρ^* increases as well. The reason is that for a given ρ, if the likelihood that the activist is destroying value is higher (i.e., if q is higher), this has a more negative impact on the exit price P. In turn, the resulting lower exit price incentivizes the value-creating activist (i.e., the activist that implements a good intervention) to keep his stake until the NPV is revealed. Therefore, the activist internalizes the impact of his actions on the firm more, and exerts effort with a higher probability (i.e., ρ^* increases). In particular, to counteract the negative impact of the value-destroying activist (i.e., the activist that implements a bad intervention), the value-creating activist increases the likelihood that he exerts effort until the exit price reaches its original level, that is, until (4) is established once again.

In contrast, if q is larger (i.e., $q \geq \underline{q}$), then for any probability of effort, the resulting

[12] I show this result formally in the proof of Proposition 1.

P^* is always low (in particular, it is lower than the level implied by (4)), and therefore the value-creating activist is always incentivized to exert effort (i.e., $\rho^* = 1$). This threshold of q is formally given in Proposition 1 below, which summarizes what is described above and characterizes the equilibrium effort level ρ^* and exit price P^*.

Proposition 1 *Let*

$$\underline{q} \equiv \frac{\frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \Delta_L - \frac{c}{\alpha}}, \tag{5}$$

where $\underline{q} \in (0, 1)$. For any q, the equilibrium is unique, and in equilibrium:

(i) If $q < \underline{q}$, then

$$\rho^* = \frac{1}{1-q}\left(1 + \frac{q(\Delta_M - \Delta_L) - \frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \frac{c}{\alpha} - \Delta_M}\right) \in (0,1), \tag{6}$$

$$P^* = \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}. \tag{7}$$

Moreover, ρ^* is strictly increasing in q.

(ii) If $q \geq \underline{q}$, then $\rho^* = 1$, and

$$P^* = \frac{(1-q)\phi\Delta_H + q\Delta_L}{1-(1-q)(1-\phi)} \tag{8}$$

As mentioned earlier, if q is not too large, then ρ^* increases as q increases. In other words, as the probability that the implemented intervention is bad increases, then the activist with a good intervention exerts effort with a higher probability. However, which of these effects dominate for the value of the firm? Denoting the expected NPV of the intervention by V, and noting that it is given by

$$V(\rho; q) = (1-q)\rho\Delta_H + (1-q)(1-\rho)\Delta_M + q\Delta_L \tag{9}$$

and that V also represents the unconditional firm value, the next proposition formalizes that the higher probability of effort dominates the negative impact of value destruction and the firm value is maximized at $q = \underline{q}$.

Proposition 2 *V^* is strictly increasing in q if $q < \underline{q}$, attains its unique maximum at $q = \underline{q}$, and it is strictly decreasing in q if $q > \underline{q}$.*

As q increases, why does the positive effect of a higher likelihood of effort dominate the negative effect of a higher likelihood of value destruction? As the probability that activist is implementing a value-destroying intervention increases, the activist that is implementing a good intervention increases the probability that he exerts effort to a certain point. In particular, as explained previously, the probability of effort increases until the stock price P^* upon his exit reaches the point given by (4), where he becomes once again indifferent between exerting effort (and keeping his stake) vs. not exerting effort (and selling his stake).

The critical observation is that the impact of higher probability of effort has more positive impact on the *unconditional* average value of the firm V compared to the stock price *conditional* on the exit of the activist, P. This is because upon the exit of the activist, the market correctly assigns a lower probability that the activist has exerted effort, since such an activist does not exit unless he has to. Therefore, as the value-creating activist exerts more effort and the activist's exit price P reaches its original level, this higher effort results in even a larger increase in the unconditional firm value V and pushes it up beyond its original level. This is how the average firm value increases with the likelihood that the activist is implementing a value-destroying intervention, and it holds all the way up until this likelihood is sufficiently high (i.e., $q = \underline{q}$) so that the value-creating activist always exerts effort.

However, beyond $\underline{q}$, increasing q does not provide any additional benefit but only harm to the firm value. Intuitively, the value-creating activist always exerts effort at this point, and therefore it cannot increase any further with the likelihood that the activist is implementing a value-destroying intervention. As a result, the average firm value V^* decreases as q increases further, and hence the maximum firm value is attained at $q = \underline{q}$.

An implication of Propositions 1 and 2 is that the region where $V^*(q)$ increases with q is not a narrow parameter space: Indeed, this result holds as long as the moral hazard problem is binding, that is, $\rho^* < 1$. Moreover, $\underline{q}$ strictly increases in c. In other words, V^* increases in q for a larger range of q if the moral hazard problem is more severe, because then the activist with a good intervention needs more incentives to exert effort. These results are laid out in part (i) of the corollary below.

Corollary 1 *(i)* $V^*(q)$ *strictly increases with* q *if the moral hazard problem is binding, that is, if* $\rho^* < 1$*. Moreover,* $\underline{q}$ *strictly increases with the cost of exerting effort,* c*.*

(ii) $\underline{q}$ *strictly decreases and* $V^*(\underline{q})$ *strictly increases as* Δ_L *decreases.*

Another interesting result, which is described by part (ii), is that the maximum firm value that can be achieved across all q (i.e., $V^*(\underline{q})$) actually increases as Δ_L decreases. Therefore, it might be desirable to have the activist to destroy a lot of value when he has a bad intervention, and this might provide an advantage that cannot be achieved by simply increasing the probability that the activist is implementing a bad intervention that destroys less value.

While this result may seem counter-intuitive, the reason is as follows. At $q = \underline{q}$, the value-creating activist always exerts effort, and hence exits with only probability ϕ. However, the value-destroying activist always exits. Therefore, the unconditional probability that the activist exits is given by $\underline{q} + \phi(1 - \underline{q})$, and the average value of the firm conditional on the activist's exit is equal to exit price, P^*. On the other hand, the activist does not exit with an unconditional probability of $(1 - \underline{q})(1 - \phi)$, which is given by the probability that the activist has a good intervention and is not hit with a liquidity shock. Conditional on this event, the firm value is given by Δ_H. Combining, the unconditional firm value $V^*(\underline{q})$ can be summarized as

$$V^*(\underline{q}) = [\underline{q} + \phi(1 - \underline{q})]P^* + (1 - \underline{q})(1 - \phi)\Delta_H, \tag{10}$$

where P^* is given by (7).[13] Importantly, (7) does not depend on Δ_L or $\underline{q}$, and therefore the expression given by (10) strictly increases as $\underline{q}$ decreases.

In other words, the unconditional firm value $V^*(\underline{q})$ is given by a weighted average of the firm value conditional on the activist's exit and conditional on activist keeping his stake. Importantly, the value of the firm conditional on either of these events do not change with $\underline{q}$ or Δ_L. This is because if the activist is not exiting, then it must be he has implemented a good intervention and exerted effort, and therefore the firm value must be Δ_H. Similarly, if the activist is exiting, then the conditional expected firm value is given by the price P^* that keeps the activist with a good intervention indifferent between exerting effort and not, that is, the price P^* that satisfies (4).[14] In other words, the conditional firm value in either of these two cases does not have anything to do with the likelihood of a bad intervention per se, or how much value it destroys.

However, the probabilities of these two events (the activist exiting vs. not exiting) does depend on $\underline{q}$. Importantly, as $\underline{q}$ decreases, the probability of no exit increases, since the value-destroying activist exits more frequently than the value-creating activist that has exerted effort. Therefore, as $\underline{q}$ decreases, the unconditional firm value (10) increases.

Going back to the impact of Δ_L, the reason why $V^*(\underline{q})$ increases as Δ_L decreases is that

[13] Plugging (5) in (8) reveals that (7) applies at $q = \underline{q}$ as well. Note that this is also implied by the continuity of $P^*(q)$ in q.

[14] In particular, $\underline{q}$ is the highest value of q such that this indifference holds.

$\underline{q}$ decreases as a result. Intuitively, $\underline{q}$ is the smallest value of q for which the value-creating activist always exerts effort. The activist is motivated to exert effort only if the exit price is low, and the value-destroying activist has a more negative impact on the exit price if he destroys even more value. Therefore, if Δ_L is smaller, full effort is reached at a smaller q, implying that $\underline{q}$ is smaller. To put it differently, if the activist with a bad intervention destroys even more value, a lower likelihood of a bad intervention is sufficient to motivate the activist with a good intervention to separate himself and keep his stake, in turn inducing him to exert effort.

3 Main Model

In this section, I generalize the baseline model to study policy implications as well as implications of liquidity. All proofs are in the online appendix.

3.1 The Setup

While the setup in this section shares many characteristics with the baseline model, there are also many differences. In particular, I generalize the baseline model to a multiple firm and multiple activist setup, where each activist is matched to a different firm and now decides whether to acquire a stake in his matched firm or not, as well as whether to intervene or not. At the same time, I also incorporate into the model: (*i*) trading disclosure thresholds for the activist when he purchases as well as sells his stake, (*ii*) short-term vs. long-term capital gains tax rates that apply if the activist exits vs. keeps his stake, respectively, (*iii*) additional perks for holding the stake long-term (e.g., receiving a higher dividend rate or more voting power if he holds the shares longer), and (*iv*) the activist's cost for intervention. The details are as follows.

To begin with, instead of one firm and one activist, I now assume that there are measure μ_F of identical firms and a measure $\bar{\mu}_{BL}$ of ex-ante identical activists. The only players in the game are these activists and a market maker. I add an initial stage (denoted by $t = 0$) to the timeline in the baseline model where each activist is matched with a different firm. Upon matching, each activist has a good intervention with probability σ, and a bad intervention with probability $1 - \sigma$. I denote the measure of activists with good (bad) interventions by $\bar{\mu}_G$ ($\bar{\mu}_B$). Note that $\bar{\mu}_G = \sigma\bar{\mu}_{BL}$ and $\bar{\mu}_B = (1 - \sigma)\,\bar{\mu}_{BL}$. I assume that the measure of good interventions is smaller than the measure of firms, that is, $\bar{\mu}_G < \mu_F$. Note that whether his intervention is good or bad is the private information of each activist. As in the baseline model, I also refer to the activist with a good (bad) intervention as a value-creating (value-destroying) activist.

Upon observing the quality of his intervention, each activist decides whether to acquire a stake α in his matched firm or not (the stake size α is exogenous, an assumption which I discuss in Section 4.4). I assume that the market is perfectly liquid and hence the market maker cannot infer any information about the activist's stock accumulation if the activist does not make any disclosure (I relax this assumption in Section 3.4). If the activist decides to buy a stake, he does this purchase continuously, with a disclosure in between: First, he continuously buys shares until accumulating a stake of α_0, and once he has accumulated this stake, he has to disclose it. In this disclosure, he also needs to state whether he intends to intervene.[15] Importantly, following the activist's disclosure, the market maker determines the stock price based on this information. After the disclosure, the activist accumulates the remaining stake of $\alpha - \alpha_0$. I denote $\eta \equiv \alpha_0/\alpha$, where a lower (higher) η represents a tighter (looser) disclosure threshold that applies when the activist buys shares.

Note that in reality, the disclosure threshold might depend on whether the activist intends to influence firm value (e.g., due to filing 13G rather than 13D). Therefore, the disclosure threshold might be different if the activist does not intend to intervene upon buying shares. To capture this possibility, I denote the threshold for this case via α_0', where $\alpha_0' > 0$. That said, as I show in the analysis, this particular disclosure threshold does not make any difference for the results.

After the activist buys shares, at $t = 1$ he decides whether to intervene, and if he has a good intervention, whether to exert effort (these decisions are simultaneous for simplicity; see Section 4.1 for a discussion). As in the baseline model, if the activist with a good intervention exerts an (unobservable) effort, he incurs a cost of $c > 0$, and increases the NPV of the intervention from Δ_M to Δ_H, where $\Delta_H > \Delta_M > 0$. Also recall that the NPV of the bad intervention is Δ_L and cannot be improved by exerting effort (for a discussion of this assumption, also see Section 4.1). In addition, I also include a cost for intervention: Specifically, the activist needs to incur a cost of κ ($\kappa + \Delta\kappa$) to implement a good (bad) intervention, where $\kappa > 0$ and $\Delta\kappa > 0$. I assume that the activist can always intervene if he would like to do so (in the scenario that the activist needs to convince the management of the firm or gather the support of other shareholders to implement the intervention, κ includes these costs – see Section 4.2 for a discussion). Here, $\Delta\kappa > 0$ represents the additional cost to the activist, for example, due to potential legal

[15] Indeed, when filing 13D, investors have to also complete state their "purpose of transaction". For further details, e.g., see https://www.investopedia.com/terms/s/schedule13d.asp.

liability because of the value that the bad intervention destroys, or due to the extra measures that the activist needs to take (such as hiding or falsifying some details about the value that the intervention will generate) so that the market cannot immediately recognize that this is a bad intervention. Therefore, a higher $\Delta\kappa$ is a proxy for increased oversight, accountability, or liability. Note that the cost κ was not relevant in the baseline model, because all the results were conditional on implementation of the intervention.

As in the baseline model, if the activist intervenes, then this information (but not the NPV of the intervention) is immediately observed at the end of $t = 1$. At $t = 2$, the activist decides whether to sell his stake (i.e., exit), which is before the NPV is revealed. In contrast to the baseline model, however, I include disclosure thresholds at the exit stage as well. The activist's sale of his stake shares the same properties with the previous stage where he bought the shares: The market is again perfectly liquid, so the market maker cannot make any inference about the activist's trading other than observing disclosures (which is an assumption I also relax in Section 3.4). The activist sells his shares until he sells a stake of α_1 (where $\alpha_1 < \alpha$), then he has to disclose that he has sold this stake, and finally he sells his remaining stake $\alpha - \alpha_1$. To keep the analysis simple, I restrict the activist's decision to a binary space: He either keeps all of his stake or sells all of it (in Section 4.3, I relax this assumption and allow partial sales). I denote $\lambda \equiv \alpha_1/\alpha$, where a lower (higher) λ represents a tighter (looser) disclosure threshold that applies when the activist sells shares.[16]

Again, like in the baseline model, at the exit stage ($t = 2$) the activist has to sell his stake with probability ϕ (e.g., due to a liquidity shock or an outside option). However, with probability $1 - \phi$, he has the flexibility to decide whether to exit at $t = 2$ or keep his stake until $t = 3$. While making this decision, there are several factors that he considers. First of all, the NPV is revealed at $t = 3$. Second, I incorporate capital gain taxes. In particular, the activist pays a tax rate of τ_s on his short-term gains (that is, if he sells at the exit stage), and a tax rate of τ_l on his long-term gains (that is, if he keeps his stake until the NPV is revealed), where $\tau_s \geq \tau_l$. Third, I also include additional perks for holding the shares longer (e.g., a higher dividend rate or more voting power). In particular, if the activist keeps his stake until $t = 3$ rather than selling it at $t = 2$, then he gains an additional $b \geq 0$ per share.

Parameter Specifications. Throughout the analysis, I make the parameter specifications below. Importantly, all of the specifications in this model are generalizations of the parameter

[16] Note that this disclosure threshold might be different as well if the activist has not intervened (again, for example, due to filing 13G rather than 13D). Therefore, I denote the threshold for this case via α_1', where $\alpha_1' > 0$. However, I also show in the analysis that this particular threshold does not matter for the results.

specifications made in the baseline model, where it was assumed that $\tau_s = \tau_l = 0$, $\lambda = 0$, $b = 0$, and c satisfies (1).

To begin with, I assume that the cost of effort is not too high, that is, $c < \min\{\bar{c}, \hat{c}\}$, where

$$\bar{c} \equiv \alpha(1-\tau_l)(1-\phi)(\Delta_H - \Delta_M), \quad (11)$$

$$\hat{c} \equiv \alpha(1-\phi)\{(\tau_s - \tau_l)\eta\Delta_H + (1-\tau_s)(1-\lambda)(\Delta_H - \Delta_M) + b\}. \quad (12)$$

Here, $c < \min\{\bar{c}, \hat{c}\}$ holds under the assumptions of the baseline model. The reason for these specifications is because if $c > \bar{c}$, then the activist with a good intervention never exerts effort (as I show in Lemma 5 in Online Appendix B) and hence making the moral hazard problem irrelevant, and if $c > \hat{c}$, then multiple equilibria exist that makes the analysis more complicated.[17]

I also assume that the additional benefit b from holding the stake longer is not so high that it completely offsets the cost of intervention or the cost of effort, that is,

$$\alpha b < \min\{\kappa, c\}.$$

This assumption is automatically satisfied in the baseline model (where $b = 0$). Finally, I also assume that the short-term tax rate is not too high compared to long-term tax rate, that is,

$$\tau_s < \bar{\tau}_s \equiv \tau_l + \frac{c - (1-\phi)\alpha b}{\eta(1-\phi)\alpha\Delta_H}. \quad (13)$$

This is because if this assumption is violated, as I show in Lemma 5 in Online Appendix B, then the activist with a good intervention always exerts effort (because he always keeps his stake unless hit by a liquidity shock), making the moral hazard problem once again irrelevant. Note that under the assumptions of the baseline model ($\tau_s = \tau_l = 0$ and $b = 0$), (13) always holds.

3.2 Robustness of the Results from the Baseline Model

Before moving on to the analysis of the policy implications, I first show that the results derived in the baseline model continue to hold in this model as well. Moreover, the derivations made in this section will also be useful to understand the policy implications better. For the proofs

[17] Nevertheless, I find in an unreported analysis that the main results continue to hold even if the assumption $c > \hat{c}$ is relaxed.

as well as the formalization of the results mentioned but not formalized in this section, see Online Appendix B.

To begin with, in the online appendix I show that Lemmas 2 and 3 from the baseline model continue to hold, with the exception that $\bar{c}$ in (1) in Lemma 2 is replaced with (11). Note that as a result, as in the baseline model, the activist keeps his stake if and only if he exerts effort and is not hit with a liquidity shock.

Importantly, as I formalize in the proposition below, I show that all of the other results from the baseline model (Propositions 1 and 2 and Corollary 1) continue to hold as well, with the exception that it no longer specifies the closed form solutions for P^*, and for ρ^* when $q < \underline{q}$.

Proposition 3 *For any $q \in [0, 1)$, an equilibrium ρ^* always exists and it is unique. Moreover, ρ^* is continuous in q, and there exists $\underline{q} \in (0, 1)$ such that in equilibrium:*

(i) $\rho^* \in (0, 1)$ if $q = 0$, ρ^* strictly increases with q if if $q < \underline{q}$, and $\rho^* = 1$ if $q \geq \underline{q}$.

(ii) $V^*(q)$ is strictly increasing in q if $q < \underline{q}$, attains its unique maximum at $q = \underline{q}$, and is strictly decreasing in q if $q > \underline{q}$.

Moreover, $\underline{q}$ strictly increases with c and Δ_L, and $V^*(\underline{q})$ strictly increases as Δ_L decreases.

Notably, among other things, this proposition states that the key result from the baseline model continues to hold: As the probability that the implemented intervention is bad increases (i.e., as q increases), the activist that is implementing a good intervention exerts effort with higher likelihood (i.e., ρ^* increases), and moreover, the average firm value V^* increases since the latter effect dominates. The intuition behind this result is similar to the baseline model, with more subtlety involved. In particular, to understand this result, let us first derive the payoffs of the activist from shirking and from exerting effort.

To that end, note that the activist pays $\alpha [Q_0 + (1 - \eta)V^*]$ to buy his stake if he intends to intervene. To see why, note that when accumulating shares, he first buys a stake of α_0 without the need to do any disclosure. Since the market is perfectly liquid, the market maker cannot make any inference about this trade in the absence of disclosure. Therefore, the activist pays Q_0 per share, where Q_0 is the value of the firm in status quo (i.e., if no intervention is implemented).[18] On the other hand, once the activist buys a share of α_0, he has to make

[18] Note that here, to avoid complicating the analysis further, I ignore the market maker's calculation that an activist might be buying a stake in this firm, which will be negligible for example if there are too few activists compared to the total number of firms. Nevertheless, I incorporate this aspect to the model in an unreported analysis and show that all results throughout Section 3 (including the policy implications) continue to hold even if the measure of activists is large.

a disclosure. In particular, he has to disclose this stake and that he intends to intervene.[19] Therefore, the market maker correctly updates the expected value of the firm to $Q_0 + V^*$, where V^* is the expected NPV conditional on the implementation of the intervention. For this reason, the activist pays $(\alpha - \alpha_0)(Q_0 + V^*)$ to buy the remaining shares. Combining, the activist pays $\alpha_0 Q_0 + (\alpha - \alpha_0)(Q_0 + V^*)$ to buy his total stake of α. Since $\eta = \alpha_0/\alpha$ by definition of η, an easier way to express this payment is $\alpha\left[Q_0 + (1-\eta)V^*\right]$. Note that a lower η represents a stricter disclosure threshold when buying the shares (note that a related result that will be useful in the later sections is that using the same arguments, an activist that does not intend to intervene pays αQ_0 to buy his stake, because his disclosure will include his intent and hence prompt the market maker to keep the share price at Q_0).

Next, after intervening, if the activist exits (i.e., sell his stake before the NPV is revealed), then he sells his stake for $\alpha\left[Q_0 + \lambda V^* + (1-\lambda)P^*\right]$. The reason is that when selling, the activist can sell a stake of α_1 without disclosing it. Since the market maker cannot infer this trade, the share price remains at $Q_0 + V^*$ during this time. However, then the activist has to disclose that he has sold a stake of α_1, and hence the market maker updates the stock price to $Q_0 + P^*$, where P^* is the expected NPV conditional on the activist's exit. Overall, the activist collects a total of $\alpha_1(Q_0 + V^*) + (\alpha - \alpha_1)(Q_0 + P^*)$ for the sale. Since $\lambda = \alpha_1/\alpha$ by definition of λ, these proceeds can also be expressed as $\alpha\left[Q_0 + \lambda V^* + (1-\lambda)P^*\right]$. Again, note that a lower λ represents a tighter disclosure when selling.

Combining the calculations above, the net profit of the activist from buying a stake α, implementing a good intervention, *shirking*, and then exiting is given by

$$\pi^*_{exit} - \kappa = \alpha(1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right] - \kappa, \tag{14}$$

where κ is the cost of implementing a good intervention, and π^*_{exit} is the activist's profit on his stake when he exits. Therefore, (14) is the net profit of the activist with a good intervention from shirking, since he always exits as noted earlier.

On the other hand, the net profit of the activist from buying a stake α, implementing a good intervention, *exerting effort*, and then keeping his stake until the NPV is revealed (unless hit by a liquidity shock) is given by

$$\phi\pi^*_{exit} + (1-\phi)\alpha\left\{(1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] + b\right\} - \kappa - c, \tag{15}$$

because with probability ϕ, he gets hit with a liquidity shock and has to exit, and with proba-

[19] As mentioned in Section 3.1, this assumption is in parallel with the 13D filing requirements.

bility $1-\phi$, he is able to keep his stake (in which case he is subject to the long-term tax rate τ_l). Therefore, the whole expression given by (15) is the net profit of the activist with a good intervention from exerting effort, because also as noted earlier, he keeps his stake unless hit by a liquidity shock.

Note that the value-creating activist (i.e., the activist that is implementing a good intervention) exerts with some probability due to the assumption $c < \bar{c}$. Therefore, if he is not always exerting effort (i.e. $\rho^* < 1$), he must be indifferent between exerting effort and shirking. In turn, this implies that (14) and (15) must be equal, which implies that

$$(1-\tau_s)\left[\lambda V^* + (1-\lambda) P^* - (1-\eta)V^*\right] = (1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] + b - c/\left[\alpha(1-\phi)\right] \quad (16)$$

or equivalently,

$$(1-\tau_s)\left[\lambda V^* + (1-\lambda) P^*\right] + (\tau_s - \tau_l)(1-\eta)V^* = (1-\tau_l)\Delta_H + b - c/\left[\alpha(1-\phi)\right]. \quad (17)$$

Importantly, in equilibrium, the LHS in (17) is strictly decreasing in q given ρ, and is strictly increasing in ρ given q.[20] Therefore, this equation yields two critical results, as also formalized in Proposition 3: First, it yields that ρ^* strictly increases in q if $\rho^* < 1$, that is, the value-creating activist (i.e., the activist that implements a good intervention) exerts effort with a higher probability as the likelihood of a bad intervention increases. Intuitively, as in the baseline model, as q increases for a given ρ, exiting becomes less attractive for value-creating activist. This induces him to keep his stake, and in turn, to exert effort.

Second, equation (17) also yields the main result, that is, the average firm value $V^*(q)$ strictly increases with q if $\rho^* < 1$ (i.e., whenever the moral hazard problem is binding). Importantly, noting that the LHS of (17) is just a weighted sum of V^* and P^*, this result follows due to the exact same *insight* from the baseline model: The impact of higher effort ρ^* has more positive impact on the *unconditional* average value of the firm V compared to the stock price *conditional* on the exit of the activist, P. Therefore, as q increases and ρ^* increases as a result, for the LHS of (17) to stay constant, it must be that $V^*(q)$ is strictly increasing and $P^*(q)$ is strictly decreasing.

A more intuitive way to understand the importance of the insight mentioned above and its connection to the main result is by setting $\tau_s = \tau_l$. Then, as also reflected in (16), as q increases, $\lambda V^* + (1-\lambda) P^*$ has to stay constant for the value-creating activist to remain indifferent between exerting effort and shirking. This is because $\lambda V^* + (1-\lambda) P^*$ represents

[20] I formally show this result in the proof of Proposition 3.

the average price at which the activist can sell his stake at (before the NPV is revealed), and if this price is any higher (lower), then it incentivizes the value-creating activist to always shirk and exit (exert effort and keep his stake unless hit by a liquidity shock), pushing the price back down (up). In turn, the insight mentioned above again implies that $V^*(q)$ has to strictly increase with q for $\lambda V^*(q) + (1-\lambda) P^*(q)$ to stay constant. Note that an increase in $V^*(q)$ also increases the cost basis of the activist, that is, the amount he needs to pay to acquire the stake. Therefore, when $\tau_s > \tau_l$, the increase in $V^*(q)$ w.r.t. q is dampened, since the increase in the activist's cost basis decreases his incentives to keep his stake (because it reduces the tax savings he can have from keep his stake and achieving the long-term tax rate τ_l). However, this latter effect cannot completely reverse the increase of $V^*(q)$ w.r.t. q, because the whole reason this effect exits is the increase in $V^*(q)$. As a result, $V^*(q)$ strictly increases with q whenever $\rho^* < 1$.

As also stated in Proposition 3, since ρ^* also increases in q (as established earlier), it follows that there exists a $\underline{q} \in (0, 1)$ such that $\rho^* = 1$ if and only if $q \geq \underline{q}$. Since increasing q any further cannot have any benefit on the firm value, V^* attains its maximum at $q = \underline{q}$, just like in the baseline model. Moreover, again as in the baseline model, the range of q for which $V^*(q)$ increases in q expands with the severity of the moral hazard problem (that is, the set $[0, \underline{q})$ strictly expands in c).

3.3 Policy Implications

In this section, I study the policy implications of the model (implications of liquidity are analyzed later, in Section 3.4, since they utilize the formal results in Section 3.3). In particular, the model allows me to analyze several policies, which can be broadly categorized under four distinct groups as outlined below. Some of these policies are voluntarily adopted by firms, some of them are determined by regulations, and some overlap. For example, the SEC's formal proposal in 2022 would reduce λ and η and increase κ.[21] Indeed, these policies have been in

[21] Specifically, in addition to proposing (*i*) reducing the filing window for 13D amendments to one business day (which would reduce λ), the SEC also proposed (*ii*) reducing the time window for activists' initial 13D filing (which is the public filing activists have to make once they cross the 5% ownership threshold) from ten to five business days (which would reduce η), as well as (*iii*) broadening the definition of who collaborate with activists. The latter would not only make such collaborations more expensive (increasing κ), but also make a 13D disclosure more likely even if the activist himself does not exceed the 5% threshold. See Bloomberg, "The SEC Wants to Stop Activism," 3/24/2022; WSJ, "SEC Proposes Giving Activist Investors Less Time to Report Positions," 2/10/2022; and HLS Forum on Corp. Governance and Financial Regulation, "SEC Proposes Updates to Schedule 13D/G Reporting," 2/15/2022.

the SEC's agenda for a long time, with the intention to fight activist short-termism.[22] Before the SEC formally proposed them in 2022, a number of senators including Bernie Sanders and Elizabeth Warren introduced them in 2016 in a bipartisan legislation called Brokaw Act, which would "fight against increasing short-termism in our economy by promoting transparency and strengthening oversight of activist hedge funds."[23]

I categorize the policies as follows.

(A) *Policies that punish short-termism by making activists' exit more difficult*, which hurt the activist if he exits before the NPV is revealed: increasing short-term taxes τ_s, or tightening the disclosure rule λ for selling.

(B) *Policies that reward long-termism*, which benefit the activist if he keeps his stake until the NPV is revealed: reducing long-term taxes τ_l, or giving additional perks b for holding the stake long-term (such as a higher dividend rate, more voting power, or proxy access).

(C) *Policies that act as blanket punishments*, which hurt the activist regardless of whether he exits or not: The policies include tightening the disclosure rule η for buying, or raising the firm's defenses against the activist, which would make it more difficult for the activist to intervene and hence increase κ. For the latter, examples that impact κ include staggered boards, super majority voting requirements, restrictions on shareholders' ability to call special meetings, and dual class share structures. Alternatively, κ can also be increased by regulation (e.g., by the SEC's proposal mentioned above).

(D) *Policies that make it more difficult to implement value-destroying interventions*: increasing $\Delta\kappa$ (for example, by increasing oversight, liability, or accountability).

3.3.1 Exogenous q or ρ

The proposition below formalizes the policy implications, holding q constant (i.e., for a given probability that the intervention implemented by the activist is bad). Note that this includes the special case $q = 0$, i.e., the case where the adverse selection channel is shut down. Also note that while the condition $\pi^*_{exit} > 0$ is imposed in this proposition, as I show in the next section,

[22] In 2015, the SEC Commissioner Daniel Gallagher conveyed that the "most obvious issue" the SEC faced in terms of its role to combat activist short-termism was "Section 13 reporting obligations." (see sec.gov, "Activism, Short-Termism, and the SEC," 06/23/2015)

[23] See baldwin.state.gov, "U.S. Senator Tammy Baldwin Introduces Bipartisan Legislation to Strengthen Oversight of Predatory Hedge Funds," 8/31/2017, and davispolk.com, "Legislation Introduced to Increase Activist Hedge Fund Disclosure," 3/24/2016.

this condition is always satisfied in equilibrium where entry of the activists is endogenized (because otherwise some activists make a loss in equilibrium).

Proposition 4 *Consider any $q < 1$, and suppose that the equilibrium satisfies $\pi^*_{exit} > 0$, where π^*_{exit} is given by (14). Then, for any given $q < 1$, ρ^* and V^* weakly (strictly) increase in τ_s and b and weakly (strictly) decrease in τ_l and λ (if $q < \underline{q}$).*

The proposition above shows that for a given q, policies that punish short-termism (i.e., policies that make the activist's exit more difficult) as well as policies that reward long-termism increase firm value, because both of these changes incentivize the activist with a good intervention to exert effort. This result is intuitive, because both of them incentivize the activist with a good intervention to keep his stake, and in turn, to exert effort.

Next, I hold ρ (i.e., the probability that the activist with good intervention is exerting effort) constant and endogenize q. Note that this includes the special case $\rho = 0$, i.e., the case where the moral hazard channel is shut down. As I show in the proposition below, I find that policies that punish short-termism increase firm value, because they decrease the profit that an activist with bad intervention can make from intervention, and thus discourage him from intervening. Similarly, policies that reward long-termism increase firm value as well, because they incentivize activists with bad interventions to switch from intervening and exiting to not intervening and keeping their stake, so that they can enjoy the benefits of keeping their stake.

Proposition 5 *Consider any $\rho \in [0, 1]$, and fix it. Suppose that the equilibrium satisfies $\pi^*_{exit} > 0$ and $q^* > 0$. Then, q^* strictly decreases in τ_s and b, strictly increases in λ, and weakly increases in τ_l. Therefore, V^* strictly increases in τ_s and b, strictly decreases in λ, and weakly decreases in τ_l.*

However, as we will see in the next section, these policies can have completely opposite implications for the value created in the economy when both moral hazard and adverse selection are taken into account simultaneously, despite the *additional* incentive provided by these policies for the value-creating activists to exert effort.

3.3.2 Endogenous q and ρ

In this section, to better study policy implications, I simultaneously endogenize the activists' effort and entry, and hence, ρ and q. Here, I use the term "entry" to refer to an activist's purchase of a stake in a firm and implementation of the intervention. Endogenizing the adverse selection aspect in addition to moral hazard is important because as also explained in the introduction, many regulations and policies aimed at mitigating short-termism intend to do so by curtailing activists' value destruction.

Indeed, I show in this section that under endogenous entry, many kinds of policies can destroy total firm value, including policies that punish short-termism as well as policies that reward long-termism, even if the number of activists implementing good interventions does not change. Nevertheless, I also find that compared to all other policies I study, policies that reward long-termism is the least likely to destroy total firm value. However, the effect on firm value significantly varies even across the policies that reward long-termism.

Recall that there are multiple firms and multiple activists, and each activist is matched to a different firm at the beginning of the game. After observing the quality of their interventions, each activist decides whether to acquire a stake α in his matched firm and whether to intervene (while α is exogenous here, I relax this assumption in Section 4.4). I denote the total measure of activists with good (bad) interventions by $\bar{\mu}_G$ ($\bar{\mu}_B$), and the measure of activists with good (bad) interventions that implement their interventions by μ_G (μ_B). Note that $q = \frac{\mu_B}{\mu_G+\mu_B}$ by definition of q.

I start the analysis by showing a preliminary result. In particular, as described in the following lemma, there exits a threshold $\bar{\kappa}$ such that if the activist's cost κ of implementing a good intervention is higher than this threshold then no activist intervenes, and if $\kappa < \bar{\kappa}$ then all activists with good interventions implement their interventions. This is because the profit of the value-destroying activist from entering is lower than that of the value-creating activist.[24]

Lemma 4 *There exists a unique threshold $\bar{\kappa}$ such that*

(i) If $\kappa > \bar{\kappa}$, then no activist intervenes (i.e., $\mu_B^* = \mu_G^* = 0$).

(ii) If $\kappa < \bar{\kappa}$, then all of the activists with good interventions implement their interventions (i.e., $\mu_G^* = \bar{\mu}_G$).[25]

Moreover, there exists $\bar{b} > 0$ such that $0 < \bar{\kappa}$ if $b < \bar{b}$ and $\eta > 1 - \max\{\lambda, \Delta_M/\Delta_H\}$.

[24] In turn, this is because the cost $\kappa + \Delta\kappa$ of implementing a bad intervention is higher than the cost κ of implementing a good intervention, as explained in Section 3.1. However, $\Delta\kappa > 0$ is not a necessary condition for the results, but rather a sufficient condition to ensure unique equilibrium. Indeed, if $\Delta\kappa = 0$, then whenever there is an equilibrium where a positive measure of activists intervene, any $\mu_G^* \in (0, \bar{\mu}_G]$ is an equilibrium, because all of the value-creating activists break even by entering.

[25] Note that if $\kappa < \bar{\kappa}$, depending on the off-equilibrium-path beliefs, the equilibrium where no activist intervenes may exist as well. However, whenever an equilibrium where a positive measure of activists intervene, I select that equilibrium.

Due to Lemma 4, I assume $\kappa < \bar{\kappa}$ in the remainder of the analysis.[26] Therefore, all of the value-creating activists intervene.

In order to find the equilibrium measure of value-destroying activists that intervene, we need to compare their payoff from doing so to their outside option. If $b = 0$, the activist's outside option is zero, since there is no way he can get a positive payoff without intervening. However, if $b > 0$, then he gets an expected payoff of $\alpha(1-\phi)b$ by buying a stake in his matched firm, not intervening, and then keeping his stake unless hit by a liquidity shock. This is because as explained after Proposition 3 in Section 3.2, if the activist does not intend to intervene, he pays αQ_0 for his stake. Since the market observes that the activist does not intervene, the value of his stake remains at αQ_0. However, if the activist keeps his stake (which he can do with probability $1 - \phi$), he gets an additional payoff of b per share, giving him an expected payoff of $\alpha(1 - \phi)b$.

Next, note that the payoff of the value-destroying activist that buys a stake and intervenes is

$$\pi^*_{exit} - (\kappa + \Delta\kappa) = \alpha(1 - \tau_s)\left[\lambda V^* + (1 - \lambda)\, P^* - (1 - \eta)V^*\right] - (\kappa + \Delta\kappa)\,, \tag{18}$$

because he always exits, where π^*_{exit} is the activist's profit on his stake when he exits, and $\kappa + \Delta\kappa$ is the cost of implementing a bad intervention.[27] Note that a value-destroying activist can make profit by intervening because he can sell his stake before the NPV is revealed. In particular, he strictly prefers to buy a stake and intervene if (18) is strictly larger than $\alpha(1-\phi)b$, he is indifferent if they are equal, and he strictly prefers not to intervene otherwise. However, as more activists implement bad interventions and q increases as a result, the market incorporates this expectation into the stock price P^* conditional on the activist's exit, thereby driving down the profit π^*_{exit} of these activists (in other words, $\lambda V^*(q) + (1 - \lambda)\, P^*(q) - (1 - \eta)V^*(q)$ decreases as q increases). Therefore, the measure μ^*_B of value-destroying activists that intervene is determined such that the net profit of the activist from doing so is $\alpha(1 - \phi)b$. The next proposition formalizes this result.

Proposition 6 *$\pi^*_{exit}(q)$ is strictly decreasing with q if $\pi^*_{exit}(q) \geq 0$. Therefore, the equilibrium is unique, and if $\bar{\mu}_B > (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$, then $\mu^*_B < \bar{\mu}_B$ and the equilibrium is given by*

$$q^* = \begin{cases} 0, & \text{if } \kappa + \Delta\kappa \geq \bar{\kappa}, \\ q \in (0,1) \text{ such that } \pi^*_{exit}(q) - (\kappa + \Delta\kappa) = \alpha(1 - \phi)b, & \text{otherwise.} \end{cases} \tag{19}$$

[26] That said, I do not impose the condition described at the end of Proposition 4, which is a sufficient condition for $\kappa < \bar{\kappa}$ but not necessary.

[27] For the details of the derivation of the payoff (18), see the derivation of (14) in Section 3.2.

Note that in equilibrium q^* pins down μ_B^* and vice versa, because $q^* = \frac{\mu_B^*}{\mu_G^*+\mu_B^*}$ and $\mu_G^* = \bar{\mu}_G$. The condition $\bar{\mu}_B > (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$ means that the measure of activists with bad interventions is sufficiently high compared to those with good interventions, reflecting the idea that good ideas are scarce and bad ideas are abundant. I will keep this assumption throughout the rest of the section to simplify the analysis.[28]

Policy implications

Now that the equilibrium q^* is characterized, we can turn our attention to study the policy implications. To that end, the proposition below characterizes the comparative statics of q^* and V^*, and the resulting impact on the total firm value $(\mu_B^* + \mu_G^*)V^*$ (that is, the combined value of all firms, where Q_0 is ignored because it is a constant).[29]

Proposition 7 *There exits $\underline{\kappa}$ such that $\underline{\kappa} < \bar{\kappa}$ and:*

(i) If $\kappa + \Delta\kappa > \bar{\kappa}$, then $q^* = 0$. Moreover, V^* and $(\mu_B^* + \mu_G^*)V^*$ do not change in κ and $\Delta\kappa$, strictly increase in τ_s and b, weakly increase in η, and strictly decrease in τ_l and λ.

(ii) If $\kappa + \Delta\kappa \in (\underline{\kappa}, \bar{\kappa})$, then $q^* \in (0, \underline{q})$, and:

- *(a) q^* strictly decreases in κ, $\Delta\kappa$, τ_s, and strictly increases in η and λ.*
- *(b) V^* strictly decreases in κ, $\Delta\kappa$, and τ_l, strictly increases in η and b, and does not change in τ_s and λ.*
- *(c) $(\mu_B^* + \mu_G^*)V^*$ strictly decreases in κ, $\Delta\kappa$, τ_s, and τ_l, and strictly increases in η and λ.*

(iii) If $\kappa + \Delta\kappa < \underline{\kappa}$, then $q^* > \underline{q}$, and q^* strictly decreases in κ, $\Delta\kappa$, τ_s, and b, strictly increases in λ and η, and does not change in τ_l. Moreover, V^* and $(\mu_B^* + \mu_G^*)V^*$ strictly increase in κ, $\Delta\kappa$, τ_s, and b, strictly decrease in λ and η, and do not change in τ_l.

The region where $q^* = 0$ (there is no adverse selection, but moral hazard is binding)**:** This region corresponds to part (i), where $\kappa + \Delta\kappa$ is so high that none of the activists that have bad

[28] If $\bar{\mu}_B \leq (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$, then in equilibrium, all of the activists with bad interventions might implement their interventions (i.e., $\mu_B^* = \bar{\mu}_B$). However, this is not an interesting case, because then μ_B^* (and hence q^*) does not respond to policies, and a main goal of this section is to study the effects of policies on endogenous q^*.

[29] In particular, the full expression for total firm value is $\mu_F Q_0 + (\mu_B^* + \mu_G^*)V^*$, where μ_F is the measure of all firms, Q_0 is the value of a firm if no intervention is implemented in that firm, and V^* is the expected NPV of the intervention conditional on implementation of the intervention.

interventions implements it (i.e., $\mu_B^* = 0$ and hence $q^* = 0$). Then, policies that punish short-termism (i.e., raise τ_s or decrease λ) as well as policies that reward long-termism (i.e., reduce τ_l or increase b) strictly increase the expected NPV V^* of an implemented intervention, because it incentivizes the value-creating activist to keep his stake, and in turn, internalize his impact on the firm and exert more effort (i.e., increase ρ^*) in the first place. This is not a surprising result, since we have already seen in the previous section that keeping q constant, these policies increase V^*. Since the number of value-creating activists remain the same ($\mu_G^* = \bar{\mu}_G$), this also implies that the total firm value $(\mu_B^* + \mu_G^*)V^*$ increases as well.

The region where $q^* \in (0, \underline{q})$ (there is adverse selection, and moral hazard is binding): The impact of the policies above are very different in the region that corresponds to part (ii), where $\kappa + \Delta\kappa$ is in the intermediate region such that some value-destroying activists intervene but not too many, so that not all of the value-creating activist exert effort (i.e., μ_B^* is such that $q^* \in (0, \underline{q})$ and hence $\rho^* < 1$). To begin with policies that punish short-termism (i.e., raise τ_s or decrease λ), they *always* hurt total firm value $(\mu_B^* + \mu_G^*)V^*$ in this region, even though the measure of value creating activists that intervene remains at $\mu_G^* = \bar{\mu}_G$. This is because these policies strictly reduce μ_B^* while not increasing V^*. Here, it is straightforward to understand the former effect: μ_B^* strictly decreases because the profit of the value-destroying activist from his stake decreases. However, the surprising result is that V^* does not change. To see why, note that combining the value-destroying activist's entry indifference equation $\pi_{exit}^* - (\kappa + \Delta\kappa) = \alpha(1 - \phi)b$ from (19) with (18) and the value-creating activist's effort indifference equation (16) immediately yields

$$\kappa + \Delta\kappa = \alpha(1 - \tau_l)\left[\Delta_H - (1 - \eta)V^*\right] + \alpha\phi b - c/(1 - \phi), \tag{20}$$

which shows the comparative statics of V^* to any parameter in this region very clearly, including that V^* does not change in τ_s or λ. Specifically, (20) shows that in the determination of V^*, the profit π_{exit}^* that the activist makes on his stake by exiting does *not* play a role (due to the combination of the two indifference equations as mentioned above). As a result, neither τ_s nor λ shows up in (20), and hence does not impact V^*.

Intuitively, punishing short-termism discourages value-destroying activists from intervening, driving μ_B^* (and hence q^*) down. This has such a negative impact on the incentives of value-creating activists to exert effort that it dominates even though (*i*) now fewer value-destroying activists intervene (which was also present in the baseline model) and (*ii*) these policies provide direct incentives to exert effort (which was *not* present in the baseline model). Therefore, the total firm value $(\mu_B^* + \mu_G^*)V^*$ decreases, even though μ_G^* remaining constant (i.e., $\mu_G^* = \bar{\mu}_G$).

Moreover, the reason why more value-creating activists choose to shirk is because more of them choose to exit (instead of keeping their stake and internalizing their impact on the firm value). In other words, while trying to mitigate short-termism, these policies might turn so many value-creating activists into short-termist activists (despite the additional incentive provided by these policies to keep their stake longer) that it can result in value destruction, even though value-destroying short-termist activists are driven out.

Due to the same reasons, blanket punishment policies (i.e., raising κ or reducing η) as well as policies that make it more difficult to implement value-destroying interventions (i.e., increasing $\Delta\kappa$) also destroy total firm value. Importantly, note that these policies (including punishing short-termism) destroy value in the entire region where $q^* > 0$ and $\rho^* < 1$, that is, whenever there is adverse selection and the moral hazard problem is binding. More generally, these results show that under short-termism, there is a tension between improving governance *forcefully through policies* versus *endogenously through market forces*. As it was just shown, the former can suppress the latter too much, making overall governance worse.

Moreover, even policies that reward long-termism can result in value destruction in this region. Specifically, increasing b (i.e., giving the activists more perks for holding their stakes longer) can have such an effect. The reason is that even though (20) implies that V^* increases with b, at the same time μ_B^* (and hence q^*) can strictly decrease with b. This is because increasing b makes the outside option more attractive for the value-destroying activist, which is not intervening and instead keeping his stake unless hit by a liquidity shock (so that he can enjoy an expected payoff of $\alpha(1-\phi)b$ as explained after Lemma 4 above). The resulting decrease in q^* can disincentivize the value-creating activist so much that the total firm value can decrease.[30] Nevertheless, increasing b can also increase total firm value, so its effect can go either way. That said, I show in Section 4.5 that under an alternative modeling of b that is more accurate for proxy access, increasing b destroys total firm value in the entire region of $q^* > 0$ and $\rho^* < 1$, just like the other policies above.

On the other hand, I find that reducing long-term capital gains tax rate τ_l always strictly increases total firm value in this region, in contrast to all of the other policies I study. This is because τ_l does not have a direct impact on the value-destroying activists' payoff from intervening (because then they always exit before the NPV is revealed), and it also does not impact the activist's payoff of $\alpha(1-\phi)b$ from his outside option. In contrast, reducing τ_l

[30] A numerical example that yields this result is $\Delta_H = 100$, $\Delta_M = 1$, $\Delta_L = -1$, $\bar{\mu}_G = 1$, $\phi = 0.01$, $\alpha = 0.1$, $c = 6.5$, $\tau_s = 0.3$, $\tau_l = 0.2$, $\lambda = 0.1$, $\eta = 0.83$, $\kappa = 0.18$, $\Delta\kappa = 0.01$. Indeed, in this numerical example, the total firm value strictly decreases w.r.t. b if $b \leq 0.2$.

directly motivates the value-creating activist to keep his stake, and in turn, to exert effort. As a result, total value increases.[31] Moreover, since total firm value increases under lower long-term tax rate, this can even result in a higher amount of total taxes collected, so it may not create an issue for keeping a balanced budget. That said, we will see that in the next region below, reducing τ_l does not improve total firm value but increasing b does.

The region where $q^* > \underline{q}$ (there is adverse selection, but moral hazard is not binding): This region corresponds to part (iii) of Proposition 7. To begin with, in this region total firm value does not change with τ_l, because all of the value-creating activists are already exerting effort (i.e., $\rho^* = 1$), and τ_l does not directly impact the incentives of the value-destroying activists as explained above. In contrast, increasing b strictly increases total firm value, because it motivates the value-destroying activists switch from intervening and exiting to not intervening and keeping their stake to enjoy these perks b. In contrast to the previous region, reducing μ_B^* is beneficial in this region, because it is too high (because the marginal value-destroying activist has no impact on ρ^* since $\rho^* = 1$, that is, the moral hazard problem is not binding). Similarly, in this region all other policies increase total firm value as well, because they decrease the profit of the value-creating activists from intervening, and hence drive their number down.

Combining all three regions above, the corollary below summarizes some of the important policy implications.

Corollary 2 *(i) Policies that punish short-termism, act as blanket punishments, or make it more difficult to implement a value-destroying intervention destroy total firm value whenever there is adverse selection (i.e.,* $\mu_B^* > 0$*) and moral hazard is binding (i.e.,* $\rho^* < 1$*), even though they only decrease* μ_B^* *and not* μ_G^**.*

(ii) Compared to any policy in part (i), any policy that rewards long-termism is less likely to destroy total firm value, and moreover, increasing b *is a dominating policy (i.e., whenever any policy in part (i) increases total firm value, increasing* b *increases total firm value as well).*

(iii) However, increasing b *does not dominate reducing* τ_l *or vice versa. That said, reducing* τ_l *is even less likely to destroy total firm value. Specifically:*

[31] If higher b represents a higher dividend rate, then b may increase as τ_l decreases. Then, it may be possible that total firm value decreases as τ_l decreases. However, as already mentioned, increasing b sometimes increases total firm value in this region as well. Therefore, this still would not change the overall result that among all the policies I study, reducing τ_l is the least likely to destroy total firm value.

(a) *If* $\mu_B^* = 0$*, then increasing* b *and reducing* τ_l *both increase total firm value.*

(b) *If* μ_B^* *is positive but not too high (i.e.,* $\mu_B^*/(\mu_B^* + \bar{\mu}_G) \in (0, \underline{q})$*), then reducing* τ_l *increases total firm value, while the effect of increasing* b *is ambiguous.*

(c) *If* μ_B^* *is too high (i.e.,* $\mu_B^*/(\mu_B^* + \bar{\mu}_G) > \underline{q}$*), then reducing* τ_l *has no impact on total firm value, while increasing* b *strictly increases total firm value.*

Therefore, even among the policies that reward long-termism, the choice must be made carefully to avoid harming firm value inadvertently and to ensure increasing it.

3.4 Implications of Liquidity

Note that at the entry stage in the model, the activist decides whether to buy a stake or not, and if he buys, then at the exit stage, he decides whether to keep his stake or sell it. Recall that in the previous section, parameter η (λ) represents the looseness of the disclosure threshold at the entry (exit) stage. Importantly, since higher η (λ) represents a higher ability of the activist to camouflage his trade at the entry (exit) stage, it can also be interpreted as the *liquidity* at the entry (exit) stage. In fact, as I show in Online Appendix E, it is also possible to interpret higher η (λ) more directly as a measure of higher noise trading at the entry (exit) stage, without any change in the formal model. Indeed, the point that a tighter disclosure requirement is akin to a reduction in noise trading is previously made in the literature.[32]

Let us analyze implications of liquidity λ at the exit stage, starting with exogenous q or ρ (Section 3.3.1). In particular, Proposition 4 implies that V^* strictly decreases in λ. Intuitively, (a) for a given q, higher liquidity reduces the incentives of the activist to keep his stake, and in turn, to exert effort, and similarly, (b) for a given ρ, higher liquidity increases the profit that the value-destroying activist can make from intervening and then selling his stake, increasing the number of such value-destroying interventions in equilibrium.

Next, let us focus on the case of endogenous q and ρ (Section 3.3.2). Recall from part (ii)(c) of Proposition 7 that increasing λ strictly increases total firm value under endogenous q^* if $q^* \in (0, \underline{q})$. This implies that higher liquidity at the exit stage can strictly increase total firm value even though the activist *never* increases his stake as a result but utilizes this higher liquidity only to *sell* his shares. The reason is the same with that explained in Section 3.3.2: As λ increases, even though V^* decreases for a given q, in response q^* increases relatively more, and hence the dominant effect is the increase in the probability that the value-creating activist

[32] See, e.g., Section 7 in Back et al. (2018).

keeps his stake and exerts effort. In other words, as liquidity at the exit stage increases, the number of value-destroying activists increases so much that the latter dominates and results in a higher total firm value due to the higher effort exerted by the value-creating activists.

4 Extensions

In this section, to demonstrate the robustness of the results, I discuss several extensions of the model.

4.1 The Activist's Effort

Micro-foundation of effort as learning about projects. I begin this section with offering a micro-foundation for the effort dimension of the model. It was assumed both in the baseline model and in the main model that the activist exerts effort at the same time as he intervenes. Here, a simple interpretation is that effort increases the quality of the activist's intervention (e.g., better implementation). However, an alternative interpretation is that effort represents the activist's search and learning among different possible projects before deciding which one to implement. One way to model this micro-foundation is to assume that the activist faces several projects, where each project has an NPV of $\Delta_H > 0$ with some probability and $\Delta_{VL} < 0$ otherwise, where each project has an expected NPV of $\Delta_L \in (\Delta_{VL}, 0)$. The firm can implement only one of these projects, or none. With some probability, the activist gets a costless signal, which helps him to eliminate some of the projects that has NPV of Δ_{VL} such that the expected NPV of any of the remaining projects is $\Delta_M \in (0, \Delta_H)$. If he exerts effort (e.g., analyzes these remaining projects carefully), then he successfully identifies at least one project that has an NPV of Δ_H.[33] Moreover, if the joint distribution of the NPVs is symmetric, then conditional on project implementation, the market maker cannot infer anything about the NPV from the identity of the project itself, and hence the model would reduce to the original model. Moreover, the activist can do this search among the projects after or before he purchases his shares in the firm.

The ability of the activist with a bad intervention to exert effort. In the original model, it was assumed that if the activist has a bad intervention, he cannot improve the value

[33] An easy way to achieve these properties is to assume that the NPV distribution of the projects is such that there is always the same number of projects with NPVs of Δ_H. While this would imply that the distribution of projects is correlated, it does not prevent the joint distribution from being symmetric.

of the intervention by exerting effort. Let us relax this assumption, and suppose that he can exert effort and increase the value of the intervention from Δ_L to Δ_H. Then, it is reasonable to assume that then his cost of effort to do so will be larger than the cost of exerting of effort when the intervention is good (because the latter improves the NPV of the intervention from Δ_M to Δ_H, where $\Delta_H - \Delta_M > \Delta_H - \Delta_L$). Then, for any $q \leq \underline{q}$, the activist with a bad intervention would never exert effort. That is because in this region, the activist with a good intervention is indifferent between shirking (and exiting) and exerting effort (and keeping his stake unless hit with a liquidity shock), and hence, the activist with a bad intervention strictly prefers to shirk. Therefore, all of the results from the original model would continue to hold in this region, which is the region that yielded the main results in the paper.

4.2 Implementation of the intervention

The activist's ability to implement interventions unilaterally. Note that it was assumed in the original model that the activist can always intervene if he desires to. The justification behind this assumption is that the expected value of the intervention V^* is always positive in equilibrium under endogenous entry, as I show in Lemma 8 in Online Appendix D.1. This is because if $V^* \leq 0$ then the value-destroying activist makes a loss from intervening, hence contradicting with $V^* \leq 0$. In turn, $V^* > 0$ implies that in equilibrium, even if the activist needs to convince the management of the firm or gather the support of the shareholders to implement the intervention, he can do so if the other parties involved are not informed about the NPV of the intervention.

The management's information about the intervention. An aspect that is also related to the previous point is how the results would be impacted if the management of the firm is informed about the value of the activist's intervention (i.e., the management is competent). When this is the case, the likely scenario is that the management already knows about the project that is the subject of the intervention and its NPV before the activist arrives at the firm. Therefore, if the management is maximizing shareholder value, it would implement the project when it is good even before the activist arrives, and therefore the activist would never show up (because the market would perfectly infer that the project is bad since the management did not implement it in the first place). However, it might be that the management *prefers* to keep status quo, for example, due to some private benefits, and hence is opposed to implementing the project even if it is good. Then, the management would try to prevent the implementation of a bad intervention as well as a good intervention, and therefore the management's attempts to convince the shareholders that this intervention should not be implemented would not be heeded by the shareholders.[34] As a result, the activist would be able to implement the intervention with the support of the shareholders, if needed with a proxy fight (in this case, κ would include the activist's cost of a proxy fight). Overall, these arguments imply that the results in the original model would continue to hold as long as the management is entrenched or incompetent.

4.3 Allowing for Partial Sale

Note that in the original model, it was assumed that at the exit stage, the activist decides between selling all of his stake and selling none of it. However, even if this assumption is relaxed completely and the activist is allowed to sell as much of his stake as he would like, the equilibrium described in Section 2 continues to exist, as well as the equilibrium described in Section 3 if the short-term tax rate τ_s is not too high compared to τ_l and the additional perks b for holding the shares is not too high. The reason is that in these equilibria, (a) in the region $q \leq \underline{q}$, the exit price always satisfies $P^* \in (\Delta_M, \Delta_H)$,[35] and hence the activist strictly prefers to keep all of his stake if he has has implemented a good intervention and exerted effort, and he strictly prefers to sell all of his stake otherwise, and (b) in the region $q \in (\underline{q}, 1)$, the exit price satisfies $P^* \in (\Delta_L, \Delta_H)$, and hence the activist strictly prefers to keep all of his stake if he has implemented a good intervention (in which case he always exerts effort because $q > \underline{q}$) and strictly prefers to sell all of his stake if he has implemented a bad intervention.

4.4 Dynamic Disclosure and the Stake Size

One of the assumptions made in Section 3 was that when the activist is buying (or selling) shares, he makes disclosure only once. However, in reality, the activist might have to make back to back disclosures. For example, the current SEC rules require activists to amend their 13D filing in two business days, which implies that an activist might have to make such a dynamic disclosure. Nevertheless, even if the disclosure is modeled in this way, the equilibria described in Sections 2 and 3 would continue to exist. This is because in these equilibria, the additional

[34] Note that the management may not be able to produce verifiable evidence that the activist's intervention is bad. Moreover, even if the management can come up with such hard evidence, it may not be able to disclose it to the shareholders because disclosing such information may harm firm value (e.g., by giving an informational advantage to the competitors of the firm) and in turn may result in the breach of fiduciary duty of the management.

[35] This result is formally shown in the proofs of Propositions 1 and 3.

disclosure would reveal no additional information: Once the activist makes his first disclosure when buying (selling) shares, the market fully anticipates that he will continue buying until amassing a stake of α (continue selling until he sells all of stake α).

Another assumption that was made in Section 3 is that if the activist decides to buy a stake, the size α of the stake he buys is exogenous and constant. Here, α can represent the size of the stake that the activist prefers due to reasons not included in the full model, for example, in order to have sufficient voting power but without making his portfolio too concentrated on this firm (for diversification reasons). Moreover, also note that if α is endogenized, then the activist with a bad intervention always prefers to buy the same number of shares that activist with a good intervention does, because otherwise the market would infer the intervention quality in equilibrium, making it impossible for the value-destroying activist to make a profit on his stake.

To demonstrate the robustness of the results to endogenous α, suppose that the activist has alotted a certain capital K to buy shares in this firm. Then, α^* decreases with the expected value V^* that the activist creates in the firm, because the activist pays more on average to buy a stake if V^* is higher. All else equal, this reduces the incentives of the activist with a good intervention to exert effort, because c/α^* (i.e., his cost of effort per share he owns) increases. Since the main driver of the results in the paper is that $V^*(q)$ increases in q if $q < \underline{q}$ (due to higher effort of the activist), a particular concern that might arise is whether this result will not hold anymore under endogenous α. However, the reduction in α^* can never be so much that V^* does not increase with q if $q < \underline{q}$. This is because if V^* did not increase with q at any point $q = q' < \underline{q}$, then it would mean that the shares don't become more expensive as q increases at $q = q'$ and hence α^* would not decrease with q at $q = q'$, in turn yielding a contradiction with the result that V^* does not increase with q at $q = q'$.

4.5 Proxy Access

Even though the model in Section 3 assumes that the activist can enjoy the extra benefit b per share whenever he keeps his stake longer, in reality some of the policies of this kind might require in addition that the activist does not intend to influence the firm. For example, proxy access has this requirement in some firms. When this is the case, an activist that has intervened may not be able to utilize proxy access.[36] This has important implications for the

[36]For example, in 2016, the first ever attempt to utilize proxy access was denied on the grounds that the blockholder had filed 13D, which violated the proxy access' requirement that "the nominating shareholder acquired the shares in the ordinary course of business and not with the intent to change or influence control

effect of increasing b (e.g., introducing proxy access) on total firm value. In particular, if some value-destroying activists are intervening (i.e., $q^* > 0$), then as b increases, it has no impact on the value-creating activist's incentives to exert effort, because they always strictly prefer to intervene and hence are ineligible for these benefits.[37] On the other hand, increasing b strictly reduces the number of value-destroying activists that are intervening (i.e., μ_B^*), because it incentivizes them to switch to not intervening and keeping their stake so that they can enjoy these benefits b. This implies that q^* strictly decreases. As a result, in the region where $q^* \in (0, \underline{q})$, increasing b results in a decrease in V^* as well. Therefore, total firm value strictly decreases with b in this region. Moreover, in the region where $q^* = 0$, increasing b can also decrease total firm value, since in that region it can incentivize the value-creating activists to switch to not intervening in order to enjoy these benefits.

Overall, these arguments imply that proxy access is more likely to destroy firm value compared to other policies that reward long-termism. More generally, the result of the original model that policies that reward long-termism is less likely to destroy total firm value than all other kinds of policies studied in this paper (including policies that punish short-termism) continues to hold only if the policy that rewards long-termism does not put a direct restraint on the activist's incentives to intervene.

4.6 Career Concerns and the Activist's "Skill"

While the arguments in the paper abstract from career concerns, it is worthwhile to mention that the results would not get weaker and would instead get stronger if career concerns were incorporated in my model as well. First of all, it is important to note the modeling of the activist in this paper captures more than the skillfulness of the activist. In particular, I do not make any assumption about the time-series correlation of the quality of an activist's intervention. If the quality of the activists' interventions now and in the future are independent, then incorporating career concerns would have no impact on the activists' incentives, since the market wouldn't make any inference from the NPV of the activist's current intervention about the NPV of his future interventions (for simplicity, my model consists of a single stage where an activist intervenes only once, but it could be easily extended to a repeated game setting where the activist can implement different interventions over time, and the results would continue

of the Corporation." The blockholder, GAMCO, decided not to pursue proxy access as a result of this denial. For details, see, e.g. HLS Forum on Corporate Governance and Financial Regulation, "The Latest on Proxy Access," 02/01/2019.

[37] Specifically, the value-creating activists always strictly prefer to intervene in this region ($q^* > 0$) because their payoff from doing so is strictly larger than the value-destroying activists' payoff from doing so.

to hold). In contrast, if there is positive correlation between the activist's interventions over time (that is, the activist has a level of "skill"), then adding career concerns on top of my model would make the results of the model stronger. This is because as the probability that the activist is implementing a bad intervention increases, the incentives of the "skilled" activist to keep his stake would increase so that he can separate from the "unskilled" activist, in turn strengthening his incentives to exert effort.

Moreover, if there is such a skill component, then another implication of the model is that an activist that has proven his skill over time might start creating less value compared to an activist that might be unskilled, even if there are no career concerns involved. This is because as q (i.e., the probability that the activist at a firm is value-destroying) decreases, the expected value of the firm can decrease, as shown in Propositions 2 and 3.

4.7 Choosing Between Activists

The baseline model (Propositions 2 and 3 in particular) also has interesting implications for the case when the firm's shareholders or board of directors are choosing between activists. First, if the shareholders or the board is facing two different activists' proposals at the same time, it might be optimal to follow the proposal by the activist who is more likely to destroy value.

Second, it might be also optimal for the shareholders and the board to be *not* good at identifying whether the activist has a positive vs. negative NPV intervention (or alternatively, whether the activist is skilled vs. unskilled, if there is a skill component). This is because if they can perfectly distinguish between these activists, then they always prefer the activist that creates value. However, then the market correctly infers that the probability that the activist is destroying value is zero, and hence such an activist might create less value compared to an activist that destroys value with some probability (where this probability is denoted by q in the model), as again implied by Propositions 2 and 3.

4.8 Implications for Entrepreneurship and Venture Capital

The short-termism problem studied in this paper applies to other kinds of blockholders who are subject to short-termism, such as venture capitalists or entrepreneurs. For example, it might take sufficiently long time for the effect of a venture capitalist's intervention in a start-up to be revealed (e.g., at pharmaceutical or tech start-ups) such that (a) the VC with a bad intervention might contemplate whether to intervene in the start-up and then sell his stake (e.g., to a different investor in a private transaction, or via an IPO)[38] before the NPV of his intervention is revealed, while (b) the VC with a good intervention might decide whether to shirk and sell his stake before the NPV is revealed, or to exert effort and keep his stake until the NPV is revealed. Since this is the same setup with the baseline model, all of the implications of the baseline model continue to hold, including those in Sections 4.6 and Section 4.7 (where, in the latter case, an entrepreneur is deciding between venture capitalists to ask for financing).

Similarly, the baseline model also applies to an entrepreneur even in the absence of any venture capitalist involvement, where the entrepreneur's "intervention" represents any project that he can implement in his own firm (since he is a blockholder in his firm). Here, "exiting" would correspond to selling his stake (e.g., via or after an IPO) before the NPV of the project is revealed. Moreover, the interpretation of the "project" can be very broad: Indeed, in the case of an entrepreneur, the project can even represent the action of starting the firm itself, where the revelation of the NPV would be the realization of how profitable this firm will be (which would be revealed after the IPO if the firm goes public quickly). Therefore, the implications of the baseline model again hold in the case, as well as those in Sections 4.6 and 4.7 (where, in the latter case, a venture capitalist is choosing between entrepreneurs to finance).

5 Conclusion

Activist short-termism is a major problem that draws the attention of many policymakers, academics, and practitioners. An important reason why this short-termism problem arises is that it takes time for the long-term effects of activist interventions to be realized, giving activists the opportunity to sell their stake in their targets before the consequences of their interventions for firm value are revealed. In particular, two channels through which this short-termism problem can manifest itself are (*i*) moral hazard, where the activist can improve the effect of his intervention on firm value by exerting hidden effort, and (*ii*) adverse selection, where some interventions destroy firm value rather than creating value.

In this paper, I study a model of activist short-termism that combines both of these channels to analyze the implications of liquidity and the policies aimed at fighting activist short-termism. I find that making the activist's exit more difficult (e.g., higher short-term capital gains taxes, tighter disclosure thresholds for selling, or lower liquidity when the activist is deciding whether

[38] Even though there might be lock-up periods involved for a VC to sell his stake after the IPO, again it can take a lot of time for the intervention's NPV to be revealed, and hence the VC might be able to sell his stake before the NPV is revealed if he wishes to do so.

to sell his stake or keep it) can *increase* firm value if there is only moral hazard or only adverse selection. However, such changes in policies or liquidity *destroys* total firm value when both moral hazard and adverse selection are present simultaneously, even if they eliminate only value-destroying interventions and does not change the number of value-creating interventions. Moreover, even policies that reward long-termism (e.g., reducing long-term capital gains taxes, or giving activists additional perks for holding their stake long-term such as additional voting rights or a higher dividend rate) can destroy total firm value. Nevertheless, among many different policies that I compare (including some other proposed policies in addition to those above), I find that policies rewarding long-termism are least likely to result in a lower total firm value, even though they may *increase* the number of value-destroying interventions. Overall, these results point out that there is a difference between creating value in the economy and eliminating activists who destroy value. Indeed, as this model shows, when formulating policies, targeting the latter may contradict with the former.

The reason behind the implications above is that when the moral hazard problem is binding (i.e., the activist with a value-creating intervention sometimes shirks and sometimes exerts effort), average firm value *increases* with the probability that the activist is implementing a *value-destroying* intervention, because it results in a significantly higher probability of effort by the value-creating activist, and this effect dominates. Beyond shareholder activism, this result implies the optimality of value destruction in other settings that involve short-termism, such as entrepreneurship and venture capital. Therefore, a possible path for future research is to model the other kinds of blockholders in more detail to do policy comparisons for them as well.

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A Proofs of the Baseline Model (Section 2)

Proof of Lemma 1. The proof of this lemma provided right after the lemma in the main text. ■

Proof of Lemma 2. First, I prove part (i). Note that conditional on implementing the intervention, the payoff of the activist with a good intervention from exerting effort is given by

$$\alpha[\phi P + (1-\phi)\max\{P, \Delta_H\}] - c,$$

while his payoff from not exerting effort is given by

$$\alpha[\phi P + (1-\phi)\max\{P, \Delta_M\}],$$

Therefore, the activist exerts effort only if

$$\begin{aligned} \alpha[\phi P + (1-\phi)\max\{P, \Delta_H\}] - c &\geq \alpha[\phi P + (1-\phi)\max\{P, \Delta_M\}] \\ &\Leftrightarrow \frac{1}{1-\phi}\frac{c}{\alpha} \leq \max\{P, \Delta_H\} - \max\{P, \Delta_M\}, \end{aligned} \tag{21}$$

and the activist exerts effort if the inequality in (21) holds strictly.

Suppose that $c > \bar{c}$. Since the RHS of (21) is weakly smaller than $\Delta_H - \Delta_M$, then (1) and $c > \bar{c}$ imply that (21) is never satisfied. As a result, the activist never exerts effort.

Suppose that $c < \bar{c}$. There are two cases to consider: If $P \leq \Delta_M$, then (21) implies that the activist with a good intervention always exerts effort. In contrast, if $P > \Delta_M$, then it must be that the activist with a good intervention is exerting effort with positive probability (because if he never exerts effort, $P \leq \Delta_M$ would have to be satisfied). This completes the proof of part (i).

Second, I prove part (ii) by showing that for any $c > 0$, it must be that $\rho^* < 1$ if $q = 0$. Suppose that instead, there exists $c > 0$ such that $\rho^* = 1$ if $q = 0$. However, then it must be that $P^* = \Delta_H$, and hence the activist with a good intervention strictly prefers to deviate to shirking and exiting (since doing so increases his payoff by c), yielding a contradiction with $\rho^* = 1$. ■

Proof of Lemma 3. First, note that the activist with a bad intervention always exits, because $P^* > \Delta_L$ whenever $q < 1$ since $\phi > 0$. Even in the corner case of $q = 1$, he weakly prefers to exit since $P^* = \Delta_L$.

Second, I show that the activist with a good intervention that has exerted effort does not exit unless hit by a liquidity shock. Note that for this result, it is sufficient to show that $P^* < \Delta_H$. Suppose this is not the case. Then, it must be that $P^* = \Delta_H$, and hence, $\rho^* = 1$. However, then the activist with a good intervention strictly prefers to deviate to shirking and exiting, yielding a contradiction with $P^* = \Delta_H$.

Third, I show that the activist with a good intervention always exits if he has shirked. There are two cases to consider. First, suppose that $P^* > \Delta_M$. Then, the activist that has shirked strictly prefers exiting over not exiting. Second, suppose that $P^* \leq \Delta_M$. However, then the maximum expected payoff that the activist with a good intervention can achieve is given by $\alpha \left[\phi P^* + (1-\phi) \max \left\{P^*, \Delta_M\right\}\right]$ if he shirks. In contrast, if he exerts effort and does not exit unless hit by a liquidity shock, then he gets an expected payoff of $\alpha \left[\phi P^* + (1-\phi)\Delta_H - c\right]$. Due to (1), $c < \bar{c}$ implies that the latter of these two payoffs is strictly larger, implying that there cannot be any activist with a good intervention that shirks in this equilibrium. ■

Proof of Proposition 1. Note that $\underline{q} \in (0,1)$ because due to (1), $c < \bar{c}$ implies that $0 < \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} - \Delta_L$.

Note that by Lemma 3, in any equilibrium if the activist with a good intervention exerts effort, then he never exits (unless he has to, that is, unless he is hit by a liquidity shock) and gets a payoff of $\alpha \left[\phi P^* + (1-\phi)\Delta_H\right] - c$, and in contrast if he does not exert effort, then he gets a payoff of αP^* (because he always exists). Therefore, the best response of the activist with a good intervention (that is, the probability ρ that he exerts effort) as a function of the exit price P is given by

$$\rho(P) = \begin{cases} 0, & \text{if } P > \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}, \\ \text{any } \rho \in [0,1], & \text{if } P = \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}, \\ 1, & \text{if } P < \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}. \end{cases} \tag{22}$$

Suppose that $q < \underline{q}$. Then, $P^* > \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* = 1$ (due to (3)), and $P^* \leq \Delta_M < \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* = 0$ (where the latter inequality follows from $c < \bar{c}$). Therefore, it must be that $\rho^* \in (0,1)$, and hence $P^* = \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$. In turn, this yields (6), since plugging (3) in P^* yields

$$\begin{aligned}\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} &= P^* = \frac{(1-q)\,\rho\,[\phi\Delta_H - \Delta_M] + (1-q)\,\Delta_M + q\Delta_L}{1-(1-q)\,\rho\,(1-\phi)} \\ &\Leftrightarrow \rho^* = \frac{1}{1-q}\left(1 + \frac{q\,(\Delta_M - \Delta_L) - \frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \frac{c}{\alpha} - \Delta_M}\right)\end{aligned}$$

Note that $\rho^* > 0$ since

$$0 < 1 + \frac{q\,(\Delta_M - \Delta_L) - \frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \frac{c}{\alpha} - \Delta_M} \Leftrightarrow 0 < \frac{\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} - \Delta_M + q\,(\Delta_M - \Delta_L)}{\Delta_H - \frac{c}{\alpha} - \Delta_M},$$

which holds since $\Delta_M > \Delta_L$ and $\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} > \Delta_M$ (where the latter follows from $c < \bar{c}$ due to (1)). Moreover, also note that $\rho^* < 1$ if $q < \underline{q}$, because

$$\rho^* < 1 \Leftrightarrow \frac{1}{1-q}\left(1 + \frac{q\,(\Delta_M - \Delta_L) - \frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \frac{c}{\alpha} - \Delta_M}\right) < 1 \Leftrightarrow q < \frac{\frac{\phi}{1-\phi}\frac{c}{\alpha}}{\Delta_H - \frac{c}{\alpha} - \Delta_L} = \underline{q}$$

Moreover, also note that: (a) ρ^* is strictly increasing in q (because $\Delta_H - \frac{c}{\alpha} - \Delta_M > 0$ and $\rho^* > 0$), (b) $\rho^* \to 1$ as $q \to \underline{q}$, and (c) $\underline{q} \in (0,1)$ as shown at the beginning of the proof.

Suppose that $q \geq \underline{q}$. Then, to show that in equilibrium it must be $\rho^* = 1$, (22) implies that it is sufficient to show that $P^* < \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* < 1$, and $P^* \leq \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* = 1$. In turn, since $\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} > \Delta_M$ (due to $c < \bar{c}$), it is sufficient to show that $P^* \leq \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* = 1$, and P^* (which is given by (3)) is strictly increasing in ρ if $P^* \geq \Delta_M$. To see that $P^* \leq \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}$ if $\rho^* = 1$, note that

$$\begin{aligned} P^* &= \frac{(1-q)\,\phi\Delta_H + q\Delta_L}{1-(1-q)\,(1-\phi)} \leq \Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} \\ &\Leftrightarrow (1-q)\,(1-\phi)\left(\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}\right) + (1-q)\,\phi\Delta_H + q\Delta_L \leq \left(\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha}\right) \\ &\Leftrightarrow q \geq \underline{q}.\end{aligned}$$

And finally, to see that P^* is strictly increasing in ρ if $P^* \geq \Delta_M$, note that

$$\begin{aligned}
0 &< \frac{\partial P}{\partial \rho} \Leftrightarrow 0 < \frac{\partial}{\partial \rho} \frac{(1-q)\,\rho\phi\Delta_H + (1-q)\,(1-\rho)\,\Delta_M + q\Delta_L}{1-(1-q)\,\rho\,(1-\phi)} \\
&\Leftrightarrow \frac{(1-q)\,\rho\phi\Delta_H + (1-q)\,(1-\rho)\,\Delta_M + q\Delta_L}{1-(1-q)\,\rho\,(1-\phi)} + \frac{(1-q)\,\rho\phi\Delta_H + (1-q)\,(1-\rho)\,\Delta_M + q\Delta_L}{[1-(1-q)\,\rho\,(1-\phi)]^2}\,(1-q)\,(1- \\
&\Leftrightarrow 0 < \frac{(1-q)\,\phi\Delta_H - (1-q)\,\Delta_M}{1-(1-q)\,\rho\,(1-\phi)} + \frac{P^*}{1-(1-q)\,\rho\,(1-\phi)}\,(1-q)\,(1-\phi) \\
&\Leftrightarrow \Delta_M < \phi\Delta_H + (1-\phi)\,P^*,
\end{aligned}$$

which always holds if $P^* \geq \Delta_M$. ■

Proof of Proposition 2. First, I show that $(1-q)\rho^*$ is strictly increasing in q if $q < \underline{q}$. Note that since P^* is given by (3) and P^* does not change with q if $q < \bar{q}$ by Proposition 1(i), applying Implicit Function Theorem (IFT) on (3) w.r.t. q yields

$$\begin{aligned}
0 &= \frac{d}{dq}\frac{(1-q)\,\rho^*\,[\phi\Delta_H - \Delta_M] + (1-q)\,\Delta_M + q\Delta_L}{1-(1-q)\,\rho^*\,(1-\phi)} \\
0 &= [(1-q)\,\rho^*\,[\phi\Delta_H - \Delta_M] + (1-q)\,\Delta_M + q\Delta_L]\,(1-\phi)\,\frac{d\,(1-q)\,\rho^*}{dq} \\
&\quad + \left\{\frac{d\,(1-q)\,\rho^*}{dq}\,[\phi\Delta_H - \Delta_M] - \Delta_M + \Delta_L\right\}[1-(1-q)\,\rho^*\,(1-\phi)] \\
\frac{d\,(1-q)\,\rho^*}{dq} &= \frac{(\Delta_M - \Delta_L)\,[1-(1-q)\,\rho^*\,(1-\phi)]}{\phi\Delta_H + q\,(1-\phi)\,\Delta_L - [1-(1-q)\,(1-\phi)]\,\Delta_M}. \qquad (23)
\end{aligned}$$

Note that $\Delta_L < \Delta_M$ implies that the numerator of the RHS is positive, therefore it remains to show that the denominator is positive as well. To see this, note that for any $q < \underline{q}$, Proposition 1 implies that $\rho^* < 1$ and hence that $P^* > \Delta_M$, because if $P^* \leq \Delta_M$ then $c < \bar{c}$ (due to (1)) would imply that the activist with a good intervention would strictly prefer to exert effort and receive a payoff of $\alpha\,[\phi P^* + (1-\phi)\Delta_H] - c$ rather than to shirk and receive a payoff of $\alpha\,[\phi P^* + (1-\phi)\max\{P^*, \Delta_M\}]$. Moreover, since Proposition 1 implies that P^* is continuous w.r.t. q (note that plugging (5) in (8) yields that $P^*(\underline{q})$ is equal to (7)), this also implies that $P^* \geq \Delta_M$ when $q = \underline{q}$. Since the denominator of the RHS in (23) is strictly decreasing in q (because $\Delta_L < \Delta_M$), it is sufficient to show that it is positive for $q = \underline{q}$. Plugging P^* for $q = \underline{q}$

from Proposition 1 yields

$$\Delta_M \le P^* \Leftrightarrow \Delta_M \le \frac{(1-\underline{q})\phi\Delta_H + \underline{q}\Delta_L}{1-(1-\underline{q})(1-\phi)} \Leftrightarrow 0 \le (1-\underline{q})\phi\Delta_H + \underline{q}\Delta_L - \Delta_M\left[1-(1-\underline{q})(1-\phi)\right].$$

Since $\underline{q} \in (0,1)$ by Proposition 1, this implies that the denominator of the RHS in (23) is positive, concluding that $(1-q)\rho^*$ is strictly increasing in q if $q < \underline{q}$. Moreover, note that by Proposition 1, ρ^* and P^* are continuous w.r.t. q at $q = \underline{q}$, and hence going through the same steps above yields that $(1-q)\rho^*$ is also strictly increasing in q from below at $q = \underline{q}$.

Second, I show that V^* is strictly increasing in q if $q < \underline{q}$. Note that (9) and (3) imply that for all $q \in [0,1]$,

$$V^* = \Delta_H - (\Delta_H - P^*)\left[1-(1-q)\rho^*(1-\phi)\right]. \tag{24}$$

Note that P^* is constant and $P^* < \Delta_H$ for all $q \le \underline{q}$ by Proposition 1. Therefore, since $(1-q)\rho^*$ is strictly increasing in q if $q < \underline{q}$ (and also strictly increasing in q from below at $q = \underline{q}$), so is V^*.

Third, if $q \ge \underline{q}$, then $\rho^* = 1$ by Proposition 1, and hence (9) and $\Delta_L < \Delta_H$ imply that V^* strictly decreases as q increases. Due to the previous step, this implies that V^* attains its unique maximum at $q = \underline{q}$. ■

Proof of Corollary 1. Note that both of results that (a) $\underline{q}$ strictly decreases as Δ_L decreases and (b) $\underline{q}$ strictly increases with c directly follow from the the definition of $\underline{q}$ in (5).

Therefore, it remains to show that $V^*(\underline{q})$ strictly increases as Δ_L decreases. To see this result, note that plugging (5) in (8) yields that $P^*(\underline{q})$ is given by (7) and does not change w.r.t. Δ_L. Proposition 1 also implies that $\rho^* = 1$ if $q = \underline{q}$, and hence (24) and $P^* < \Delta_H$ imply that $V^*(\underline{q})$ is strictly larger if $\underline{q}$ is smaller. Since $\underline{q}$ strictly decreases as Δ_L decreases, this implies that $V^*(\underline{q})$ strictly increases as Δ_L decreases, concluding the proof. ■

INTERNET APPENDIX

B Online Appendix: Mapping and Proofs for Section 3.2

This section shows that all the results from the baseline model (i.e., Section 2.3) continue to hold under the model extended by Section 3.1. The proofs are provided in Section B.1. The mapping of the results in this section to their counterparts in the baseline are as follows (the formal results are provided below as well, right after the mapping):

(*I*) In Lemmas 5 and 6 below, I show that Lemmas 2 and 3 from the baseline model continue to hold, with the following exceptions:

  (*a*) First, (1) in Lemma 2 is replaced with (11). This is because if the activist with a good intervention keeps his stake with probability $1-\phi$ (that is, whenever he is not hit with a liquidity shock), the benefit he realizes from exerting effort is now $\alpha(1-\tau_l)(1-\phi)(\Delta_H-\Delta_M)$ rather than $\alpha(1-\phi)(\Delta_H-\Delta_M)$ due to the existence of taxes. Note that $\tau_s < \bar{\tau}_s$ is assumed (where τ_s is given by (26)) due to parameter restriction made in Section 3.1, where restriction is made due to part (ii) of Lemma 5, as it was explained in that section.

  (*b*) Second, Lemma 3 continue to hold as well, with the exception that part (i) (i.e., the result that the activist that has implemented a bad intervention always exits) is proven by a different lemma later on (by Lemma 8 in the Online Appendix D.1), since this particular result now utilizes endogenous entry.

(*II*) Proposition 3 in Section 3.2 shows that all of the other results from the baseline model (Propositions 1 and 2 and Corollary 1) continue to hold as well, with the exception that it no longer specifies the closed form solutions for P^*, and for ρ^* when $q < \underline{q}$.

Lemma 5 *In any equilibrium for any $q \in [0,1)$, conditional on implementing the intervention:*

(*i*) *The agent with a good intervention never exerts effort if $c > \bar{c}$ and exerts effort with positive probability if*

$$c < \bar{c} \equiv \alpha(1-\tau_l)(1-\phi)(\Delta_H-\Delta_M). \tag{25}$$

(*ii*) *If $c < \bar{c}$, then the agent with a good intervention always exerts effort if $\tau_s \geq \bar{\tau}_s$, where*

$$\bar{\tau}_s \equiv \tau_l + \frac{c-(1-\phi)\alpha b}{\eta(1-\phi)\alpha\Delta_H} \tag{26}$$

Lemma 6 *In any equilibrium for any $q \in [0,1)$, conditional on implementing the intervention:*

(i) If the activist with a good intervention exerts effort, then he does not exit unless hit by a liquidity shock.

(ii) If the activist with a good intervention shirks, he always exits.

(iii) The activist with good intervention shirks with positive probability (i.e., $\rho^* < 1$) if no bad intervention is implemented (i.e., $q = 0$).

B.1 Proofs

This section provides the proofs of all the results listed in Sections 3.2 and B. An auxiliary result (i.e., Lemma 7) has been relegated to the end of this section, to Section B.1.1 ("Supplemental results for Section B.1"). I refer to these results in some of the proofs below.

Proof of Lemma 5.

I start by proving part (i). Note that the payoff of the activist with a good intervention from exerting effort is given by

$$\begin{aligned}&\phi\alpha(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right]\\&+(1-\phi)\alpha\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_H-(1-\eta)V\right]+b\right\}-\kappa_G-c,\end{aligned}$$

while his payoff from shirking is given by

$$\begin{aligned}&\phi\alpha(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right]\\&+(1-\phi)\alpha\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_M-(1-\eta)V\right]+b\right\}-\kappa_G,\end{aligned}$$

where κ_G is the activist's cost of implementing the intervention. Therefore, the activist exerts effort only if

$$\begin{aligned}&(1-\phi)\alpha\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_H-(1-\eta)V\right]+b\right\}-c\\\geq\ &(1-\phi)\alpha\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_M-(1-\eta)V\right]+b\right\},\end{aligned}$$

or, equivalently,

$$\begin{aligned}\frac{1}{1-\phi}\frac{c}{\alpha}\ \leq\ &\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_H-(1-\eta)V\right]+b\right\}\\&-\max\left\{(1-\tau_s)\left[\lambda V+(1-\lambda)P-(1-\eta)V\right],(1-\tau_l)\left[\Delta_M-(1-\eta)V\right]+b\right\}\end{aligned}\quad(27)$$

and the activist exerts effort if the inequality in (27) holds strictly.

Suppose that $c > \bar{c}$. Since the RHS of the inequality in (27) is weakly smaller than $(1-\tau_l)(\Delta_H - \Delta_M)$, (25) implies that (27) is never satisfied. As a result, the activist never exerts effort.

Suppose that $c < \bar{c}$. It is sufficient to prove the activist does not always shirk in equilibrium. Suppose he does. Then, $V^* \leq \Delta_M$ and $P^* \leq \Delta_M$. Therefore, $\lambda V^* + (1-\lambda)P^* \leq \Delta_M$ and

$\Delta_M-(1-\eta)V^* \geq 0$. Hence, combining with $\tau_s \geq \tau_l$, (27) reduces to $\frac{1}{1-\phi}\frac{c}{\alpha} \leq (1-\tau_l)(\Delta_H-\Delta_M)$. However, this inequality holds strictly (because $c < \bar{c}$), yielding a contradiction with the activist always shirking in equilibrium.

Next, I prove part (ii). There are two steps: If $c < \bar{c}$ and $\tau_s \geq \bar{\tau}_s$, then (a) $\rho^* < 1$ cannot be an equilibrium, and (b) $\rho^* = 1$ is always an equilibrium (recall that ρ is the likelihood that the activist with a good intervention exerts effort). I start with (a). Suppose that $\rho^* < 1$ is an equilibrium. We already know from part (i) that $\rho^* > 0$. Hence, it must be that $\rho^* \in (0,1)$, that is, the activist is indifferent between exerting effort and not. This implies that in equilibrium, (27) must be holding with equality:

$$\begin{aligned}\frac{1}{1-\phi}\frac{c}{\alpha} &= \max\left\{(1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right], (1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] + b\right\} \\ &\quad - \max\left\{(1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right], (1-\tau_l)\left[\Delta_M - (1-\eta)V^*\right] + b\right\}\end{aligned} \tag{28}$$

Then, it must be that

$$(1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] + b > (1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right],$$

because otherwise the RHS of (28) equals 0, violating the equality (28) itself. Moreover, it also must be that

$$(1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right] > (1-\tau_l)\left[\Delta_M - (1-\eta)V^*\right] + b,$$

because otherwise (28) cannot be satisfied due to $c < \bar{c}$. Therefore, (28) reduces to

$$\frac{1}{1-\phi}\frac{c}{\alpha} = (1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] + b - (1-\tau_s)\left[\lambda V^* + (1-\lambda)P^* - (1-\eta)V^*\right], \tag{29}$$

or, equivalently,

$$\frac{1}{1-\phi}\frac{c}{\alpha} - b = (1-\tau_l)\Delta_H - (\tau_s-\tau_l)\left[(1-\eta)V^*\right] - (1-\tau_s)\left[\lambda V^* + (1-\lambda)P^*\right] \tag{30}$$

Note that the RHS in (30) is decreasing in P and V since $\tau_s \in [\tau_l, 1)$. Moreover, since $P^* < \Delta_H$ and $V^* < \Delta_H$ (because $\rho^* < 1$), (29) implies that

$$\frac{1}{1-\phi}\frac{c}{\alpha} - b > (1-\tau_l)\left[\Delta_H - (1-\eta)\Delta_H\right] - (1-\tau_s)\left[\lambda\Delta_H + (1-\lambda)\Delta_H - (1-\eta)\Delta_H\right],$$

or equivalently $\tau_s < \bar{\tau}_s$, yielding a contradiction.

Finally, I prove (b), that is, $\rho^* = 1$ is an equilibrium if $c < \bar{c}$ and $\tau_s \geq \bar{\tau}_s$. For this, it is sufficient to show that the activist weakly prefers exerting effort over shirking in this equilibrium. In turn, it is sufficient to show that (27) holds in this equilibrium. For this to hold, it must be

$$(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b>(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right],$$

because otherwise the RHS of the inequality in (27) equals 0, violating the equality (27) itself. Therefore, (27) reduces to

$$\begin{aligned}\frac{1}{1-\phi}\frac{c}{\alpha} \leq\ & (1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b \\ & -\max\left\{(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right],(1-\tau_l)\left[\Delta_M-(1-\eta)V^*\right]+b\right\}\end{aligned} \tag{31}$$

There are two cases to consider. First, suppose that

$$(1-\tau_l)\left[\Delta_M-(1-\eta)V^*\right]+b\geq(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right]. \tag{32}$$

Then, (31) reduces to $\frac{1}{1-\phi}\frac{c}{\alpha}\leq(1-\tau_l)(\Delta_H-\Delta_M)$, which holds due to $c<\bar{c}$. Second, suppose that (32) does not hold. Then, (31) reduces to

$$\frac{1}{1-\phi}\frac{c}{\alpha}-b\leq(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]-(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right]. \tag{33}$$

Note that the RHS in (33) is decreasing in P and V, since $\tau_s\in[\tau_l,1)$. Moreover, $P^*\leq\Delta_H$ and $V^*\leq\Delta_H$. Therefore, it is sufficient to show that (33) holds for $P^*=\Delta_H$ and $V^*=\Delta_H$, that is,

$$\frac{1}{1-\phi}\frac{c}{\alpha}-b\leq(1-\tau_l)\left[\Delta_H-(1-\eta)\Delta_H\right]-(1-\tau_s)\left[\lambda\Delta_H+(1-\lambda)\Delta_H-(1-\eta)\Delta_H\right],$$

which is equivalent to $\tau_s\geq\bar{\tau}_s$ and therefore holds by assumption. ■

Proof of Lemma 6. First, I prove part (i). To that end, note that for any V^* and P^*, if the activist with a good intervention exerts effort, then it must be that he weakly prefers exerting effort, that is (27) holds as explained in the proof of Lemma 5. Then, it must be that

$$(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b>(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right], \tag{34}$$

because otherwise the RHS of (27) is not positive, violating the equality (27) itself. In turn, (34) implies that for the activist that has intervened and exerted effort, his payoff from keeping his stake is strictly larger than his payoff from exiting.

Second, I prove part (ii). To prove by contradiction, suppose that with positive probability, the activist with a good intervention implements the intervention, shirks, and keeps his stake. Then, his payoff from keeping his stake must be weakly larger than exiting, that is:

$$(1-\tau_l)\left[\Delta_M-(1-\eta)V^*\right]+b\geq(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right]. \tag{35}$$

However, $c < \bar{c}$ implies that

$$(1-\phi)\,\alpha\left\{(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b\right\}-c>(1-\phi)\,\alpha\left\{(1-\tau_l)\left[\Delta_M-(1-\eta)V^*\right]+b\right\},$$

and combining this with (35) yields

$$\begin{aligned} & \phi\alpha(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right] \\ & +(1-\phi)\,\alpha\left\{(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b\right\}-c \\ > \; & \phi\alpha(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right] \\ & +(1-\phi)\,\alpha\left\{(1-\tau_l)\left[\Delta_M-(1-\eta)V^*\right]+b\right\}, \end{aligned} \tag{36}$$

Note that due to (35), the whole expression after the inequality sign in (36) represents the maximum payoff of the activist if he shirks. Therefore, conditional on implementing the intervention, the activist's payoff from exerting effort and keeping his stake (unless hit by a liquidity shock) is strictly larger than his payoff from shirking (regardless of whether he keeps his stake or not after shirking). This yields a contradiction with the starting assumption that the activist with a good intervention implements the intervention and shirks with positive probability.

Finally, I prove part (iii). To see this part, suppose that $q = 0$ and $\rho^* = 1$. Then, $V^* = P^* = \Delta_H$. Therefore, conditional on implementing the intervention, the activist's profit from shirking and exiting is

$$\alpha(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right]=\alpha(1-\tau_s)\eta\Delta_H, \tag{37}$$

while his profit from exerting effort is given by (recall that due to part (ii), he must be keeping his stake unless hit by a liquidity shock)

$$\begin{aligned} & \phi\alpha(1-\tau_s)\left[\lambda V^*+(1-\lambda)P^*-(1-\eta)V^*\right] \\ & +(1-\phi)\alpha\left\{(1-\tau_l)\left[\Delta_H-(1-\eta)V^*\right]+b\right\}-c \\ = \; & \alpha\left[\phi(1-\tau_s)+(1-\phi)(1-\tau_l)\right]\eta\Delta_H-c+(1-\phi)\alpha b \end{aligned} \tag{38}$$

Between these two payoffs (37) and (38), $\tau_s < \bar{\tau}_s$ implies that the former is strictly larger. Therefore, the activist with a good intervention strictly prefers shirking over exerting effort, yielding a contradiction with $\rho^* = 1$. This concludes the proof. ■

Proof of Proposition 3. Note that an activist intervenes only if he has acquired a stake in the target, because otherwise his payoff is weakly smaller than $-\kappa_i$, where κ_i is the activist's cost of intervening for intervention $i \in \{G, B\}$ (that is, $\kappa_G = \kappa$ and $\kappa_B = \kappa + \Delta\kappa$). Throughout the proof, I assume that all of the activists who purchase a stake and make a disclosure always intervene, which is a result that I confirm in Lemma 8 in Online Appendix D.1. Therefore, an activist intervenes if and only if he buys a stake in the target and makes a disclosure. I also assume that if an activist with a bad intervention purchases a stake in a target and intervenes, then he always exits, which is also a result that I confirm in Lemma 8 in Online Appendix D.1. Therefore, combining with Lemma 6, the average firm value conditional on the exit by the activist following intervention is given by $Q_0 + P^*$, where $P^*(\rho; q)$ is given by (3), and the average firm value conditional on stake disclosure by the activist is given by $Q_0 + V^*$, where $V^*(\rho; q)$ is given by (9). Note that whenever the notation $V^*(q)$ ($P^*(q)$) is used, it represents $V^*(\rho^*; q)$ ($P^*(\rho^*; q)$).

Mapping of the proof. The proof consists of several steps. Among these, steps 6, 8, and 10 prove that for any $q \in [0, 1)$, an equilibrium ρ^* always exists and it is unique, steps 8 and 10 prove that ρ^* is continuous in q for all $q \in [0, 1)$, step 5 proves that $\underline{q} \in (0, 1)$, steps 8 and 10 prove part (i), steps 12 and 13 prove part (ii), step 11 proves that $\underline{q}$ strictly increases with c, step 14 proves that $\underline{q}$ strictly increases with Δ_L, and step 15 proves that $V^*(\underline{q})$ strictly increases as Δ_L decreases. The rest of the steps are intermediary steps. The steps are as follows.

Step 1: Note that any equilibrium with $\rho^* \in (0, 1)$ has to satisfy

$$(1 - \tau_s) [\lambda V^* + (1 - \lambda) P^* - (1 - \eta)V^*] = (1 - \tau_l) [\Delta_H - (1 - \eta)V^*] - \frac{1}{1 - \phi} \frac{c}{\alpha} + b, \quad (39)$$

which is equivalent to

$$(1 - \tau_s) \lambda V^* + (\tau_s - \tau_l)(1 - \eta)V^* + (1 - \tau_s) (1 - \lambda)P^* = (1 - \tau_l)\Delta_H - \frac{1}{1 - \phi} \frac{c}{\alpha} + b. \quad (40)$$

Note that this directly follows from Lemma 6, as this lemma implies that for an activist with a good intervention, in any equilibrium his payoff from shirking is given by

$$\alpha(1 - \tau_s) [\lambda V^* + (1 - \lambda)P^* - (1 - \eta)V^*] - \kappa_G, \quad (41)$$

because he always exits, while his payoff from exerting effort is given by

$$\begin{aligned} &\phi\alpha(1 - \tau_s) [\lambda V^* + (1 - \lambda)P^* - (1 - \eta)V^*] \\ &+(1 - \phi)\alpha \{(1 - \tau_l) [\Delta_H - (1 - \eta)V^*] + b\} - \kappa_G - c, \end{aligned} \quad (42)$$

because he does not exit unless hit by a liquidity shock. Since $\rho^* \in (0, 1)$ implies that (41) is equal to (42), this yields (39).

Step 2: I show that for any given $q \in [0, 1)$, if there exists any $\rho' \in [0, 1]$ that satisfies (39), then $\rho^* = \rho'$ is the unique equilibrium for q, and it must be that $P^*(\rho'; q) > \Delta_M$ (recall that both $V^*(\rho; q)$ and $P^*(\rho; q)$ are functions of ρ). To see this, note that (39) is equivalent to (40) by step 1. Then, it must be that $P^*(\rho'; q) > \Delta_M$, because if $P^*(\rho'; q) \leq \Delta_M$ then $c < \hat{c}$ implies that (40) is violated, because then $V^* \leq \Delta_H$ and hence

$$\begin{aligned} & (1-\tau_s)\,\lambda V^* + (\tau_s-\tau_l)(1-\eta)V^* + (1-\tau_s)\,(1-\lambda)P^* \\ \leq\ & (1-\tau_s)\,\lambda \Delta_H + (\tau_s-\tau_l)(1-\eta)\Delta_H + (1-\tau_s)\,(1-\lambda)\Delta_M \\ <\ & (1-\tau_l)\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} + b, \end{aligned}$$

where the last inequality follows from $c < \hat{c}$, where $\hat{c}$ is given in (12). This establishes that $P^*(\rho';q) > \Delta_M$. In turn, since $P^*(\rho;q)$ is strictly increasing in ρ if $P^*(\rho;q) > \Delta_M$ (by the proof of Proposition 1), it must be that $P^*(\rho;q) > P^*(\rho';q)$ for any $\rho > \rho'$ and $P^*(\rho;q) < P^*(\rho';q)$ for any $\rho < \rho'$. Moreover, $V^*(\rho;q)$ is strictly increasing in ρ. Therefore, it follows that for any $\rho > \rho'$, the LHS of (39) is strictly larger than the RHS (as can be seen more clearly from (40)), and hence (41) and (42) imply that an activist with a good intervention strictly prefers to shirk, yielding a contradiction with $\rho > \rho' \geq 0$. Similarly, for any $\rho < \rho'$, the LHS of (39) is strictly smaller than the RHS, and hence (41) and (42) imply that an activist with a good intervention strictly prefers to exert effort, yielding a contradiction with $\rho < \rho' \leq 1$. In contrast, $\rho = \rho'$ is indeed an equilibrium because (39) is satisfied for $\rho = \rho'$, and (39) implies that an activist with a good intervention is indifferent between shirking and exerting effort.

Step 3: I show that if $q = 0$, then it must be that $\rho^* \in (0,1)$. To see this, suppose $\rho^* = 1$. Then, $V^* = P^* = \Delta_H$ and hence

$$\begin{aligned} & (1-\tau_s)\,\lambda V^* + (\tau_s-\tau_l)(1-\eta)V^* + (1-\tau_s)\,(1-\lambda)P^* \\ =\ & (1-\tau_s)\,\lambda \Delta_H + (\tau_s-\tau_l)(1-\eta)\Delta_H + (1-\tau_s)\,(1-\lambda)\Delta_H \\ >\ & (1-\tau_l)\Delta_H - \frac{1}{1-\phi}\frac{c}{\alpha} + b, \end{aligned} \tag{43}$$

where the last inequality follows from $\tau_s < \bar{\tau}_s$ (where $\bar{\tau}_s$ is given by (26)). Therefore, (41) and (42) imply that an activist with a good intervention strictly prefers to shirk, yielding a contradiction with $\rho^* = 1$. Next, supppose that $\rho^* = 0$. Then, $V^* = P^* = \Delta_M$. However, then (41) and (42) imply that the activist with a good intervention strictly prefers to exert effort (yielding a contradiction with $\rho^* = 0$), because

$$(1-\tau_s)\left[\lambda V^* + (1-\lambda)\,P^* - (1-\eta)V^*\right] < (1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] - \frac{1}{1-\phi}\frac{c}{\alpha} + b,$$

which is satisfied because

$$\begin{aligned} \frac{1}{1-\phi}\frac{c}{\alpha} - b\ &<\ (1-\tau_l)\Delta_H - (1-\tau_s)\,\lambda\Delta_M - (\tau_s-\tau_l)(1-\eta)\Delta_M - (1-\tau_s)\,(1-\lambda)\Delta_M \\ \Leftrightarrow\ & \frac{1}{1-\phi}\frac{c}{\alpha} - b < (\tau_s-\tau_l)\eta\Delta_M + (1-\tau_l)\,(\Delta_H - \Delta_M) \end{aligned} \tag{44}$$

which holds due to $c < \bar{c}$ (where $\bar{c}$ is given by (25)).

Step 4: I show that if $V^*(\rho; q) \leq 0$ then the LHS of (40) is strictly smaller than the RHS. This is because the RHS is positive due to $c < \bar{c}$ (where $\bar{c}$ is given by (25)), and the LHS is not positive because $V^*(\rho; q) \leq 0$ implies that $P^*(\rho; q) \leq 0$, which in turn satisfied because (3) and (9) imply that

$$P^*(\rho; q) = \Delta_H - \frac{\Delta_H - V^*(\rho; q)}{1 - (1-q)\rho(1-\phi)} \tag{45}$$

for any ρ and q.

Step 5: Letting $\underline{q}$ be the value of q such that (40) holds for $\rho = 1$, I show that $\underline{q}$ is unique and $\underline{q} \in (0, 1)$. Note that $\underline{q}$ is unique because (a) $P^*(1; q)$ is strictly decreasing in q (as shown in the proof of Lemma 7), and (b) $V^*(1; q)$ is strictly decreasing in q due to (9). Moreover, $\underline{q} \in (0, 1)$ because (a) $P^*(1; q)$ and $V^*(1; q)$ are continuous in q, (b) if $q = 0$ and $\rho = 1$, then (43) implies that the LHS of (40) is strictly larger than the RHS, and (c) if $\rho = 1$ and $q \uparrow 1$, then $P^*(1; q)$ and $V^*(1; q)$ become negative, implying that the LHS of (40) becomes strictly smaller than the RHS, because the LHS becomes negative and the RHS is positive due to $c < \bar{c}$ (where $\bar{c}$ is given by (25)).

Step 6: Note that due to step 2 above, step 5 implies that for $q = \underline{q}$, $\rho^* = 1$ is the unique equilibrium and satisfies (40), and $P^*(1; \underline{q}) > \Delta_M$.

Step 7: I show that for any $q < \underline{q}$, there exists $\rho' \in (0, 1)$ such that (40) holds for $\rho = \rho'$. To see this, note that for any $q < \underline{q}$, if $\rho = 1$ then the LHS in (40) is strictly larger than the RHS, because as implied by step 5, (a) (40) holds with equality if $q = \underline{q}$ and $\rho = 1$, and (b) holding $\rho = 1$ constant, $P^*(1; q)$ and $V^*(1; q)$ strictly decrease in q for all $q \leq \underline{q}$. On the other hand, for any $q < \underline{q}$, if $\rho = 0$ then $V \leq \Delta_M$ and $P^* \leq \Delta_M$, which in turn imply that the LHS in (40) is strictly smaller than the RHS due to (44). Therefore, due to continuity of the LHS (40) w.r.t. ρ, there has to be some $\rho' \in (0, 1)$ such that this equation holds.

Step 8: Due to step 2, step 7 implies that for any $q < \underline{q}$, the equilibrium is unique, it satisfies $\rho^* \in (0, 1)$ and (40), and it must be $P^* > \Delta_M$ in this equilibrium. Moreover, ρ^* is strictly increasing and continuous for all $q < \underline{q}$ (including when q is approaching $\underline{q}$ from below). To see this latter argument, note that as q decreases and ρ is fixed, (a) $V^*(\rho; q)$ strictly increases (as implied by (9)), (b) $P^*(\rho; q)$ strictly increases (as shown in the proof of Lemma 7), and (c) $V^*(\rho; q)$ and $P^*(\rho; q)$ are continuous. Similarly, as ρ increases and q is fixed, (a) $V^*(\rho; q)$ strictly increases (as again implied by (9)), (b) $P^*(\rho; q)$ strictly increases (as shown in the proof of Proposition 1, because $P^* > \Delta_M$), and (c) $V^*(\rho; q)$ and $P^*(\rho; q)$ are continuous. Since (40) must be satisfied in equilibrium for all $q \leq \underline{q}$ (note that this is implied for $q = \underline{q}$ due to step 6), it indeed follows that ρ^* is strictly increasing and continuous for all $q < \underline{q}$ (including when q is approaching $\underline{q}$ from below).

Step 9: Note that $P^*(q)$ and $V^*(q)$ are continuous in q for all $q \in [0, 1)$ (because, by definition, $P^*(q) = P^*(\rho^*(q); q)$ and $V^*(q) = V^*(\rho^*(q); q)$). In particular, for $q \leq \underline{q}$, the continuity is implied by step 8, and for $q > \underline{q}$, it follows trivially since $\rho^* = 1$, which is shown in the next step below.

Step 10: I show that for all $q \geq \underline{q}$, the equilibrium is unique and given by $\rho^* = 1$. For $q = \underline{q}$, this result follows directly from step 6 above. Therefore, it remains to show this result for any $q > \underline{q}$. To that end, note that for any $q > \underline{q}$ and $\rho \in [0, 1]$, the LHS in (40) is strictly smaller than the RHS, because (a) (40) holds for $q = \underline{q}$ and $\rho = 1$ by step 5, (b) $P^*(1; \underline{q}) > \Delta_M$ by step 6, (c) $P^*(\rho; q)$ strictly increases in ρ if $P^*(\rho; q) > \Delta_M$ (as shown in the proof of Proposition 1), and it also strictly decreases in q (as shown in the proof of Lemma 7), and (d) $V^*(\rho; q)$ strictly decreases in q and strictly increases in ρ. In turn, since the LHS in (40) is strictly smaller than the RHS, (41) and (42) imply that an activist with a good intervention strictly prefers to exert effort, establishing the unique equilibrium as $\rho^* = 1$.

Step 11: I show that $\underline{q}$ strictly increases with c. To see this, compare any two possible values of c, denoted by c_1 and c_2, where $c_2 > c_1$. Recall that if $q = \underline{q}$, then (40) holds for $\rho = 1$ by step 5, and in equilibrium $\rho^* = 1$ by step 6. However, holding $\rho = 1$ and $q = \underline{q}(c_1)$ constant, if c increases from c_1 to c_2, the LHS of (40) becomes strictly larger than the RHS. Moreover, given $c = c_2$ and $\rho = 1$, the LHS of (40) is strictly larger than the RHS for any $q < \underline{q}(c_1)$ as well. This is because $P^*(1; q)$ and $V^*(1; q)$ strictly decrease w.r.t. q for all $q < \underline{q}(c_1)$. Since (40) has to be satisfied for $q = \underline{q}$, this implies that $\underline{q}(c_2) > \underline{q}(c_1)$.

Step 12: I show that $V^*(q)$ is strictly increasing in q if $q < \underline{q}$. To see this, consider any q_1 and q_2 such that $0 \leq q_1 < q_2 < \underline{q}$. Suppose that $V^*(q_2) \leq V^*(q_1)$. However, then $P^*(q_2) < P^*(q_1)$, which follows from combining Lemma 7 with $P^*(q_1) > \Delta_M$ and $P^*(q_2) > \Delta_M$, where the latter inequalities are in turn implied by step 8. Therefore, at the equilibrium $\rho^*(q)$, the LHS of (40) evaluated at $q = q_2$ is strictly smaller than the LHS of (40) evaluated at $q = q_1$, yielding a contradiction with the fact that at the equilibrium $\rho^*(q)$, (40) is satisfied for both $q = q_1$ and $q = q_2$ (by step 8).

Step 13: I show that $V^*(q)$ attains its unique maximum at $q = \underline{q}$, and it is strictly decreasing in q if $q \geq \underline{q}$. Because of step 12, as well as because $V^*(q)$ is continuous for all q (by step 9), it is sufficient to show that if $q \geq \underline{q}$, then $V^*(q)$ is strictly decreasing in q. In turn, this holds because $\rho^* = 1$ for all $q \geq \underline{q}$ (by step 10) and $V^*(\rho; q)$ is given by (9).

Step 14: I show that $\underline{q}$ strictly increases with Δ_L. To prove by contradiction, consider any two values Δ_L' and Δ_L'' that Δ_L can take, and suppose that $\Delta_L'' < \Delta_L'$ and $\underline{q}(\Delta_L'') \geq \underline{q}(\Delta_L')$. Note that step 6 implies that for $q = \underline{q}$, $\rho^* = 1$ is the unique equilibrium and satisfies (40). However, the LHS of (40) evaluated at equilibrium (i.e., at $\rho^* = 1$) for $\Delta_L = \Delta_L''$ and $q = \underline{q}(\Delta_L'')$ is strictly smaller than the LHS of (40) evaluated at equilibrium (i.e., at $\rho^* = 1$) for $\Delta_L = \Delta_L'$ and $q = \underline{q}(\Delta_L')$ (because $V^*(\rho; q)$ and $P^*(\rho; q)$ strictly decrease in q and strictly increase in Δ_L), which yields a contradiction with the fact that (40) is satisfied in equilibrium for both $(q, \Delta_L) = (\underline{q}(\Delta_L'), \Delta_L')$ and $(q, \Delta_L) = (\underline{q}(\Delta_L''), \Delta_L'')$ (again, by step 6).

Step 15: I show that $V^*(\underline{q})$ strictly increases as Δ_L decreases. To see this, again consider any two values Δ_L' and Δ_L'' that Δ_L can take such that $\Delta_L'' < \Delta_L'$. To prove by contradiction, suppose that $V^*(\underline{q}(\Delta_L')|\Delta_L = \Delta_L') \geq V^*(\underline{q}(\Delta_L'')|\Delta_L = \Delta_L'')$. Then, since (40) is satisfied at equilibrium (i.e., for $\rho^* = 1$) for both $(q, \Delta_L) = (\underline{q}(\Delta_L'), \Delta_L')$ and $(q, \Delta_L) = (\underline{q}(\Delta_L''), \Delta_L'')$, it follows that $P^*(q(\Delta_L')|\Delta_L = \Delta_L') \leq P^*(\underline{q}(\Delta_L'')|\Delta_L = \Delta_L'') < \Delta_H$ (where the last inequality follows from $\underline{q} > 0$, which was shown in step 5). However, then it must be that $V^*(q(\Delta_L')|\Delta_L =$

$\Delta_L') < V^*(\underline{q}(\Delta_L'')|\Delta_L = \Delta_L'')$, because: (a) (9) and (3) imply that for all $q \in [0,1]$,

$$V^*(\rho; q) = \Delta_H - (\Delta_H - P^*(\rho; q)) \left[1 - (1-q)\rho(1-\phi)\right],$$

(b) $\underline{q}(\Delta_L'') < \underline{q}(\Delta_L')$ by step 14, and (c) $\rho^*(\underline{q}(\Delta_L)|\Delta_L) = 1$ for any Δ_L due to step 6. ■

B.1.1 Supplemental results for Section B.1

This section provides the supplemental results that are utilized by some of the proofs in Section B.1.

Lemma 7 *Let $q_1, q_2 \in [0,1)$ such that $q_2 > q_1$. Suppose that if an activist with a bad intervention implements his intervention, then he always exits. Also suppose that for both $q = q_1$ and $q = q_2$, $P^*(q) \geq \Delta_M$ in equilibrium. Then, $P^*(q_2) \geq P^*(q_1)$ implies that $V^*(q_2) > V^*(q_1)$.*

Proof of Lemma 7. Recall that $V^*(\rho; q)$ is given by (9). Moreover, Lemma 6 and the assumption that if an activist with a bad intervention always exits after intervening imply that in any equilibrium, $P^*(\rho; q)$ is given by (3). Throughout the proof, I denote the equilibrium variables by (ρ_1, V_1, P_1) and (ρ_2, V_2, P_2) for $q = q_1$ and $q = q_2$ respectively.

First, I show that for a given ρ, $P^*(\rho; q)$ is strictly decreasing in q for any $q \in [0,1)$. Indeed, plugging in (3) for P^* yields

$$\begin{aligned} 0 &> \frac{\partial P^*}{\partial q} \Leftrightarrow 0 > \frac{-\rho\phi\Delta_H - (1-\rho)\,\Delta_M + \Delta_L}{1-(1-q)\,\rho\,(1-\phi)} - \frac{(1-q)\,\rho\phi\Delta_H + (1-q)\,(1-\rho)\,\Delta_M + q\Delta_L}{\left[1-(1-q)\,\rho\,(1-\phi)\right]^2}\rho\,(1-\phi) \\ &\Leftrightarrow \rho\phi(\Delta_H - \Delta_L) + (1-\rho)\,(\Delta_M - \Delta_L) > 0, \end{aligned}$$

which always holds.

Second, note that $P_2 \geq P_1 \geq \Delta_M$ implies that $\rho_2 > \rho_1$. This is because as shown in the proof of Proposition 1, if $P(\rho; q) \geq \Delta_M$ for any given q, then $P(\rho; q)$ is strictly increasing in ρ. Combining this with the first step above yields the desired result.

Third, I show that $P_2 \geq P_1 \geq \Delta_M$ implies that $V_2 > V_1$. The proof consists of two substeps. First, I show that for $P_2 \geq P_1 \geq \Delta_M$ to hold, it must be that $(1-q_2)\rho_2 > (1-q_1)\rho_1$. Suppose that $(1-q_2)\rho_2 \leq (1-q_1)\rho_1$. However, then

$$(1-q_2)\rho_2\phi\Delta_H \leq (1-q_1)\rho_1\phi\Delta_H. \tag{46}$$

Moreover, $\rho_2 > \rho_1$ and $q_2 > q_1$ also imply that

$$(1-q_2)(1-\rho_2)\Delta_M \;\leq\; (1-q_1)(1-\rho_1)\Delta_M, \tag{47}$$

$$q_2\Delta_L \;<\; q_1\Delta_L \tag{48}$$

However, since P^* is given by (3), combining $(1-q_2)\rho_2 \leq (1-q_1)\rho_1$ and (46)-(48) with $P_2 \geq \Delta_M \geq 0$ and $P_1 \geq \Delta_M \geq 0$ yields $P_2 < P_1$, yielding a contradiction.

Second, I complete the proof by showing that it must be $V_2 > V_1$. Indeed, note that (3) and (9) imply that in equilibrium, P^* can also be expressed as

$$P^* = \Delta_H - \frac{\Delta_H - V^*}{1-(1-q)\rho^*(1-\phi)} \tag{49}$$

Also note that $V_1 < \Delta_H$ and $V_2 < \Delta_H$ since $\rho_1 < 1$ (because $\rho_1 < \rho_2 \leq 1$) and $q_2 > 0$. Combining this with $(1-q_2)\rho_2 > (1-q_1)\rho_1$, $P_2 \geq P_1$, and (49) implies that $V_2 > V_1$, concluding the proof. ∎

C Online Appendix: Proofs for Policy Implications Under Exogenous q or ρ (Section 3.3.1)

Proof of Proposition 4. Note that it is sufficient to show the comparative statics stated for ρ^*, since those stated for V^* follow immediately due to (9).

First, I prove the proposition for $q < \underline{q}$. Note that by part (i) of Proposition 3, $\rho^* \in (0,1)$. In turn, step 1 in the proof of Proposition 3 implies that (39), which is equivalent to (40), must be both satisfied in equilibrium. Note that holding ρ^* and q constant, the LHS in (39) becomes strictly smaller than the RHS as τ_s and b increases or as τ_l and λ decreases (because $\lambda V^* + (1-\lambda) P^* - (1-\eta)V^* > 0$, $V^* < \Delta_H$ (due to $\rho^* < 1$), and also $P^* < V^*$ because

$$P^*(\rho^*; q) = \Delta_H - \frac{\Delta_H - V^*(\rho^*; q)}{1-(1-q)\rho^*(1-\phi)} \tag{50}$$

due to (3) and (9)). In turn, this implies that holding ρ^* and q constant, the LHS in (40) becomes strictly smaller than the RHS as τ_s and b increases, or as τ_l and λ decreases. As a result, ρ^* has to strictly increase so that (40) is established again, because (a) $V^*(\rho; q)$ strictly increases in ρ, and moreover, (b) $P^*(\rho^*; q) > \Delta_M$ due to step 8 in the proof of Proposition 3 and $P^*(\rho; q)$ is strictly increasing in ρ if $P^*(\rho; q) > \Delta_M$ (by the proof of Proposition 1).

Second, I prove the proposition for $q > \underline{q}$. Note that by part (i) of Proposition 3, $\rho^* = 1$. Moreover, note that by step 10 in the proof of Proposition 3, for any $q > \underline{q}$ and $\rho \in [0,1]$, the LHS in (40) is strictly smaller than the RHS. Therefore, in turn (41) and (42) imply that an activist with a good intervention strictly prefers to exert effort, implying that the equilibrium remains to be unique and satisfy $\rho^* = 1$.

Finally, I prove the proposition for $q = \underline{q}$. Note that again by part (i) of Proposition 3, $\rho^* = 1$. Moreover, step 6 in the proof of Proposition 3 implies that (40), which is equivalent to (39), are both satisfied in equilibrium. Note that holding $\rho^* = 1$ and q constant, the LHS in (39) becomes strictly smaller than the RHS as τ_s and b increases or as τ_l and λ decreases (because again, $\lambda V^* + (1-\lambda) P^* - (1-\eta)V^* > 0$, $V^* < \Delta_H$ (because $q = \underline{q}$, where $\underline{q} > 0$ due to Proposition 3), and $P^* < V^*$ due to (50)). There are two cases to consider:

(I) Suppose that τ_s and b increases or τ_l and λ decreases. Then, the LHS in (39) becomes strictly smaller than the RHS for all $\rho \in [0,1]$ (again, holding q constant), because (39) is equivalent to (40), and (a) $V^*(\rho; q)$ strictly increases in ρ, and moreover, (b) $P^*(1; \underline{q}) > \Delta_M$ due to step 6 in the proof of Proposition 3 and $P^*(\rho; q)$ is strictly increasing in ρ if $P^*(\rho; q) > \Delta_M$. Since the LHS in (40) becomes strictly smaller than the RHS for all $\rho \in [0,1]$, in turn (41) and (42) imply that an activist with a good intervention strictly prefers to exert effort, implying that the equilibrium remains to be unique and satisfy $\rho^* = 1$.

(II) Suppose that τ_s and b decreases or τ_l and λ increases. Then, holding $\rho^* = 1$ and q constant, the LHS in (40) becomes strictly larger than the RHS. However, since the LHS in (40) is never larger than the RHS in equilibrium (as implied by steps 6, 8, and 10 in the proof of Proposition 3), this implies that ρ^* must change in response. In particular, since $\rho^* = 1$, the only direction it can change is to decrease. ■

Proof of Proposition 5. The proof of this proposition is analogous to the proof of part (iii) of Proposition 7, and thus not repeated here. ■

D Online Appendix: Proofs for Policy Implications Under Endogenous q and ρ (Section 3.3.2)

Proof of Lemma 4. Note that whenever there exists an equilibrium where a positive measure of activists intervene, I select that equilibrium. Moreover, in any such equilibrium, (a) an activist never intervenes if he hasn't acquired a stake (because otherwise he incurs the cost of implementing the intervention but gets nothing), (b) if an activist has acquired a stake and made a disclosure, then he always intervenes (by part (vi) of Lemma 8), and (c) the average firm value conditional on stake disclosure by the activist is given by $Q_0 + V^*$, where $V^*(\rho; q)$ is given by (9), and the average firm value conditional on the exit by the activist following intervention is given by $Q_0 + P^*$, where $P^*(\rho; q)$ is given by (3). The latter is due to combining Lemma 6 with part (ii) of Lemma 8.

Also note that if any activist with a bad intervention enters in equilibrium, then the activists with good interventions strictly prefer to enter. This is because the entry cost of is $\kappa + \Delta\kappa$ for the activists with bad interventions, while it is κ for the activists with good interventions. On the other hand, after entering, the payoff of an activist with a good intervention is weakly larger than that of an activist with a bad intervention.

Therefore, to prove parts (i) and (ii), it is sufficient to show that if no activist with bad intervention enters, then no activist with good interventions enters if $\kappa > \bar{\kappa}$, and all of them enter if $\kappa < \bar{\kappa}$, where

$$\bar{\kappa} \equiv -\alpha(1-\phi)b + \alpha(1-\tau_s)[\lambda V^*(0) + (1-\lambda)\, P^*(0) - (1-\eta)V^*(0)], \tag{51}$$

where $V^*(0)$ and $P^*(0)$ are $V^*(q)$ and $P^*(q)$ evaluated at $q = 0$, respectively. To that end, note that Proposition 3 implies that $\rho^* \in (0,1)$, because $q = 0$. This implies that the activist with a good intervention is indifferent between exerting effort and shirking if he enters. Since Lemma 6 implies that an activist with a good intervention always exits if he shirks, it follows that this activist's payoff in equilibrium is given by

$$\alpha(1-\tau_s)[\lambda V^*(0) + (1-\lambda)\, P^*(0) - (1-\eta)V^*(0)] - \kappa. \tag{52}$$

when $q = 0$. On the other hand, if the activist buys a stake and does not intervene, then his payoff is $\alpha(1-\phi)b$. This is because part (vi) of Lemma 8 implies that he does not disclose his stake, hence he pays αQ_0 for his stake. Since he does not intervene, he keeps his stake (unless hit by a liquidity shock) and gets a value of $\alpha\,(Q_0 + b)$ out of his stake (note that if he exits, he gets a value of αQ_0 instead, so he strictly prefer to keep his stake). Comparing the payoff of $\alpha(1-\phi)b$ with the payoff of (52), no activist with good intervention enters if $\kappa > \bar{\kappa}$, and all of them enter if $\kappa < \bar{\kappa}$.

Finally, as the last step, I show that there exists $\bar{b} > 0$ such that $0 < \bar{\kappa}$ if $b < \bar{b}$ and $\eta > 1 - \max\{\lambda, \Delta_M/\Delta_H\}$. Since $\bar{\kappa}$ is linear in b, it is sufficient to show this result for $b = 0$: First, I show that if $\lambda > 1-\eta$, then $\bar{\kappa} > 0$. Note that this follows directly from (51), because $\rho^*(0) > 0$ (due to Proposition 3), which also implies $V^*(0) > \Delta_M \geq 0$ and $P^*(0) > \Delta_M \geq 0$. Second, I show that if $\eta > 1 - \frac{\Delta_M}{\Delta_H}$, then $\bar{\kappa} > 0$. Indeed, this holds because $\eta > 1 - \frac{\Delta_M}{\Delta_H}$ implies that $\Delta_M - (1-\eta)\,\Delta_H > 0$, which in turn implies that (51) is positive, because $P^*(0) \geq \Delta_M \geq 0$ and $V^*(0) \in [\Delta_M, \Delta_H]$, concluding the proof. ■

Proof of Proposition 6. Note that $\kappa < \bar{\kappa}$ by assumption, and therefore all of the $\bar{\mu}_G$ activists with good interventions enter due to Lemma 4. Also note that $\Delta_L < 0$, and $\pi^*_{exit}(q)$ is continuous in q for all q, because ρ^* is continuous w.r.t. q by Proposition 3. Here, $\pi^*_{exit}(q)$ denotes the gross profit (that is, the payoff excluding the intervention cost κ) of the activist with a bad intervention from entering, because by Lemma 8, he always exits after intervening.

First, I prove that $\pi^*_{exit}(q)$ is strictly decreasing with q if $\pi^*_{exit}(q) \geq 0$. In fact, I show a stronger result: I show that $\pi^*_{exit}(q)$ is strictly decreasing with q if $\pi^*_{exit}(q) > -\varepsilon_\pi$, where

$$\varepsilon_\pi \equiv \alpha(1-\tau_s)\eta\,(-\Delta_L) > 0.$$

I show this result in three substeps: (a) First, I show that $\pi^*_{exit}(q)$ is strictly decreasing with q if $q < \underline{q}$. Note that by steps 6 and 8 in the proof of Proposition 3, (40) is always satisfied in equilibrium. Since (40) is equivalent to (39) as explained in step 1 of the same proof, it is sufficient to show that $V^*(q)$ strictly increases in q. To prove by contradiction, suppose that there exist q' and q'' such that $0 \leq q' < q'' \leq \underline{q}$ and $V^*(q'') \leq V^*(q')$. Since $P^*(q') > \Delta_M$ and

$P^*(q'') > \Delta_M$ (due to steps 6 and 8 of Proposition 3), Lemma 7 implies that $P^*(q'') < P^*(q')$. However, then the LHS of (40) evaluated at $q = q''$ is strictly smaller than the LHS of (40) evaluated at $q = q'$, yielding a contradiction with the fact that at the equilibrium $\rho^*(q)$, (40) is satisfied for both $q = q'$ and $q = q''$.

(b) Second, note that due to the same arguments made in substep (a) above, $\pi^*_{exit}(q)$ is strictly decreasing in q as q converges $\underline{q}$ from below.

(c) Third, I show that $\pi^*_{exit}(q)$ is strictly decreasing in q if $q \geq \underline{q}$ and $\pi^*_{exit}(q) > -\varepsilon_\pi$. Note that $\rho^* = 1$ by Proposition 3. Therefore, both V^* and P^* strictly decrease with q (note that the latter was shown in the proof of Lemma 7). Also note that letting $\tilde{\eta} \equiv 1 - \frac{\eta}{1-\lambda}$,

$$\begin{aligned}\pi^*_{exit}(q) &= \alpha(1-\tau_s)\left[\lambda V^* + (1-\lambda)\,P^* - (1-\eta)V^*\right] \\ &= \alpha(1-\tau_s)\,(1-\lambda)\left[P^* - \tilde{\eta}V^*\right] \end{aligned} \tag{53}$$

There are two cases to consider: (I) Suppose $\tilde{\eta} \leq 0$. Note that as q increases, P^* and V^* strictly decrease. Therefore, (53) strictly decreases in q.

(II) Suppose $\tilde{\eta} > 0$. Note that $\tilde{\eta} < 1$ as well by the definition of $\tilde{\eta}$. Hence, $\tilde{\eta} \in (0,1)$. By the proof of Lemma 9, $P^* - \tilde{\eta}V^*$ is strictly decreasing in q if $P^* - \tilde{\eta}V^* \geq (1-\tilde{\eta})\Delta_L$. Therefore, due to (53), $\pi^*_{exit}(q)$ is strictly decreasing in q if $\pi^*_{exit}(q) \geq \alpha(1-\tau_s)\,(1-\lambda)\,(1-\tilde{\eta})\Delta_L$, or equivalently, $\pi^*_{exit}(q) \geq -\varepsilon_\pi$. This concludes the proof of the first result (that is, $\pi^*_{exit}(q)$ is strictly decreasing with q if $\pi^*_{exit}(q) > -\varepsilon_\pi$).

Second, I show that the equilibrium is unique and is given by

$$q^* = \begin{cases} 0, & \text{if } \kappa + \Delta\kappa \geq \bar{\kappa} = \pi^*_{exit}(0) - \alpha(1-\phi)b, \\ \frac{\bar{\mu}_B}{\bar{\mu}_B + \bar{\mu}_G}, & \text{if } \kappa + \Delta\kappa \leq \tilde{\kappa} \equiv \pi^*_{exit}\left(\frac{\bar{\mu}_B}{\bar{\mu}_B+\bar{\mu}_G}\right) - \alpha(1-\phi)b \\ q \in (0,1) \text{ such that } \pi^*_{exit}(q) = \kappa + \Delta\kappa + \alpha(1-\phi)b, & \text{otherwise,} \end{cases} \tag{54}$$

This directly follows from the results that (a) $\pi^*_{exit}(q)$ is strictly decreasing with q if $\pi^*_{exit}(q) > -\varepsilon_\pi$, (b) $\pi^*_{exit}(q)$ is continuous in q (as noted at the beginning of the proof), (c) all of the $\bar{\mu}_G$ activists with good interventions enter (as mentioned at the beginning of the proof), (d) for a given q, the payoff of an activist with a bad intervention from buying a stake and not intervening is $\alpha(1-\phi)b$ (as explained in the proof of Lemma 4), while his payoff from buying a stake and intervening is $\pi^*_{exit}(q) - (\kappa + \Delta\kappa)$ (by definition of $\pi^*_{exit}(q)$).

Fourth, note that $0 < \bar{\kappa}$ if and only if $b < \frac{1}{\alpha(1-\phi)}\pi^*_{exit}(0)$, where $0 < \pi^*_{exit}(0)$. Here, $0 < \pi^*_{exit}(0)$ holds because $0 < \kappa < \bar{\kappa}$ by assumption. By the definition (51) of $\bar{\kappa}$, this implies that $\bar{\kappa} + \alpha(1-\phi)b = \pi^*_{exit}(0) > 0$.

Fifth, I show that if $\bar{\mu}_B > (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$, then $q^* < \frac{\bar{\mu}_B}{\bar{\mu}_B+\bar{\mu}_G}$ in equilibrium (which is equivalent to $\mu^*_B < \bar{\mu}_B$, because $\mu^*_G = \bar{\mu}_G$ by Lemma 4). Note that $\bar{\mu}_B > \left(\frac{\Delta_H}{-\Delta_L}\right)\bar{\mu}_G$ implies that $0 > \frac{1}{\mu_F}\left(\bar{\mu}_B\Delta_L + \bar{\mu}_G\Delta_H\right)$, which in turn implies that $V^*(q) < 0$ if $q \geq \frac{\bar{\mu}_B}{\bar{\mu}_B+\bar{\mu}_G}$. However, since $V^*(q^*) > 0$ in equilibrium by Lemma 8, it follows that $q^* < \frac{\bar{\mu}_B}{\bar{\mu}_B+\bar{\mu}_G}$.

Sixth, note that the second and fifth steps together imply that if $\bar{\mu}_B > (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$, then the equilibrium is unique and given by (19). ■

Proof of Proposition 7. Note that unless noted otherwise, in almost all of the results below, the comparative statics of $(\mu_B^* + \mu_G^*)V^*$ immediately follow from the comparative statics of q^* and V^*, because $\mu_G^* = \bar{\mu}_G$ by Lemma 4. In the cases where the comparative statics of $(\mu_B^* + \mu_G^*)V^*$ does not immediately follow, it is proven explicitly.

First, note that $q^* < \frac{\bar{\mu}_B}{\bar{\mu}_B + \bar{\mu}_G}$ in equilibrium (which is equivalent to $\mu_B^* < \bar{\mu}_B$, because $\mu_G^* = \bar{\mu}_G$ by Lemma 4) by the fifth step in the proof of Proposition 6, because $\bar{\mu}_B > (\frac{\Delta_H}{-\Delta_L})\bar{\mu}_G$ by assumption.

Second, I prove part (i). To that end, suppose that $\kappa + \Delta\kappa > \bar{\kappa}$. Then, $q^* = 0$ by Proposition 6. Moreover, by steps 1 and 8 in the proof of Proposition 3, in equilibrium (39) (which is equivalent to (40)) is satisfied, and also $P^* > \Delta_M$. Note that for a given q, the LHS in (40) (and hence the LHS in (39) compared to its RHS) is strictly increasing in ρ around $\rho = \rho^*$, because $V^*(\rho; q)$ is strictly increasing in ρ, and $P^*(\rho; q)$ is also strictly increasing in ρ when $P^* > \Delta_M$ (as shown in the proof of Proposition 1). Therefore, the following arguments complete the proof for this step: (a) As $\kappa + \Delta\kappa$ increases, (39) continues to hold for the same level of ρ. Therefore, ρ^* does not change (recall by step 2 in the proof of Proposition 3, for any given q, if there exists any $\rho' \in [0, 1]$ that satisfies (39), then $\rho^* = \rho'$ is the unique equilibrium for q). (b) As τ_s increases, holding $\rho = \rho^*$ constant, the LHS in (39) becomes strictly smaller than the RHS, because the LHS in (39) is positive (this is because $\pi_{exit}^*(0) > 0$ by the fourth step in the proof of Proposition 6). As a result, ρ^* strictly increases. (c) If $\tau_s = \tau_l$, then as η increases, (40) continues to hold for the same level of ρ. Therefore, ρ^* does not change. (d) If $\tau_s > \tau_l$, then as η increases, the LHS in (40) becomes strictly smaller than the RHS, because $V^* > 0$ by Lemma 8. As a result, ρ^* strictly increases. (e) As τ_l increases, the RHS in (39) becomes strictly smaller than the LHS, because $\eta > 0$. As a result, ρ^* strictly decreases. (f) As λ increases, the LHS in (39) becomes strictly larger than the RHS. This is because $P^* < V^*$, which in turn holds because $\rho^* > 0$ (due to Proposition 3) and

$$P^*(\rho^*; q) = \Delta_H - \frac{\Delta_H - V^*(\rho^*; q)}{1 - (1 - q)\rho^*(1 - \phi)} \tag{55}$$

due to (3) and (9)). As a result, ρ^* strictly decreases. (h) As b increases, the RHS in (39) becomes strictly larger than than the LHS. As a result, ρ^* strictly increases.

Third, letting $\underline{\kappa} \equiv \pi_{exit}^*(\underline{q}) - \alpha(1 - \phi)b$, I prove that $\underline{\kappa} < \bar{\kappa}$. Note that $\pi_{exit}^*(q)$ is strictly decreasing in q if $\pi_{exit}^*(q) \geq 0$ (by Proposition 6), and $\pi_{exit}^*(0) > 0$ (by the fourth step in the proof of Proposition 6). Since $\bar{\kappa} = \pi_{exit}^*(0) - \alpha(1 - \phi)b$, this implies that $\pi_{exit}^*(\underline{q}) < \pi_{exit}^*(0)$, and hence $\underline{\kappa} < \bar{\kappa}$.

Fourth, I prove part (iii). To that end, suppose that $\kappa + \Delta\kappa < \underline{\kappa}$, where $\underline{\kappa} = \pi_{exit}^*(\underline{q}) - \alpha(1 - \phi)b$. Note that Proposition 6 implies that $\kappa + \Delta\kappa + \alpha(1 - \phi)b = \pi_{exit}^*(q^*)$ (because $\kappa + \Delta\kappa < \underline{\kappa} < \bar{\kappa}$) and $q^* > \underline{q}$ (because $\pi_{exit}^*(q)$ is strictly decreasing in q if $\pi_{exit}^*(q) \geq 0$). In turn, $q^* > \underline{q}$ implies that $\rho^* = 1$ due to Proposition 3. Recall that $\pi^*_{exit}(q)$ is given by (18). Also given that $\rho^* = 1$, $\pi^*_{exit}(q)$ is strictly decreasing in q if $\pi^*_{exit}(q) \geq 0$ (by substep (c) of the first step in the proof of Proposition 6), and $V^*(\rho^*; q) = \frac{1}{\mu_F}(\mu^*_B \Delta_L + \bar{\mu}_G \Delta_H) = \frac{1}{\mu_F}(\frac{q}{1-q}\bar{\mu}_G \Delta_L + \bar{\mu}_G \Delta_H)$ is strictly decreasing in q. Therefore, the following arguments complete the proof for this step: (a) As $\kappa + \Delta\kappa$ increases, q^* strictly decreases such that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ is established again. (b) As τ_s increases, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ strictly decreases (because $\pi^*_{exit}(q^*) > 0$). Therefore, q^* strictly decreases. (c) As λ increases, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ strictly increases, because $P^* < V^*$ (for the same reasons explained in substep (f) of the proof of part (i) above). As a result, q^* strictly increases. (d) As η increases, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ strictly increases, because $V^* > 0$ by Lemma 8. Therefore, q^* strictly increases. (e) As τ_l increases, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ does not change. As a result, q^* does not change either. (f) As b increases, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ does not change while $\alpha(1-\phi)b$ increases. Therefore, q^* strictly decreases such that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ is established again.

Fifth, I prove part (ii). To that end, suppose that $\kappa + \Delta\kappa \in (\underline{\kappa}, \bar{\kappa})$, that is, $\kappa + \Delta\kappa \in (\pi^*_{exit}(\underline{q}) - \alpha(1-\phi)b, \pi^*_{exit}(0) - \alpha(1-\phi)b)$. Note that Proposition 6 implies that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ and $q^* \in (0, \underline{q})$ (where the latter follows because $\pi^*_{exit}(q)$ is strictly decreasing in q if $\pi^*_{exit}(q) \geq 0$). In turn, $\rho^* \in (0,1)$ due to Proposition 3, and moreover, due to steps 1 and 8 in the proof of Proposition 3, in equilibrium (39) (which is equivalent to (40)) is satisfied, and also $P^* > \Delta_M$. Note that for a given q, the LHS in (40) (and hence the LHS in (39) compared to its RHS) is strictly increasing in ρ around $\rho = \rho^*$ due to the same arguments made in the proof of part (i) above. Moreover, (39) and $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ together imply that

$$\frac{\kappa + \Delta\kappa}{\alpha(1-\phi)} = (1-\tau_l)\left[\Delta_H - (1-\eta)V^*\right] - \frac{1}{1-\phi}\frac{c}{\alpha} \tag{56}$$

has to be satisfied in equilibrium as well. Therefore, the following arguments complete the proof for this step:

(a) As τ_s increases, holding $q = q^*$ and $\rho = \rho^*$ constant, the LHS in (39) strictly decreases while the RHS remains constant (because the LHS is positive due to $\pi^*_{exit}(q) = \kappa + \Delta\kappa + \alpha(1-\phi)b > 0$). Therefore, holding q constant: (I) ρ^* strictly increases until (39) is established again, (II) hence V^* strictly increases, (III) in turn, the RHS in (39) strictly decreases (and hence, so does the LHS in (39)), and (IV) hence, $\pi^*_{exit}(q)$ strictly decreases. Since $\pi^*_{exit}(q)$ strictly decreases, it follows that q^* (and hence μ^*_B) strictly decreases. Moreover, $V^*(q^*)$ does not change, because (56) continues to hold.

(b) As $\kappa + \Delta\kappa$ increases, holding $q = q^*$ constant, ρ^* does not change, because (39) continues to hold (recall from step 2 in the proof of Proposition 3 that for any given q, if there exists any $\rho' \in [0,1]$ that satisfies (39), then $\rho^* = \rho'$ is the unique equilibrium for q). In turn, this implies that holding $q = q^*$ constant, $\pi^*_{exit}(q)$ does not change and hence becomes strictly smaller than $\kappa + \Delta\kappa + \alpha(1-\phi)b$. Therefore, q^* strictly decreases such that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ is established again. Moreover, $V^*(q^*)$ strictly decreases due to (56).

(c) As λ increases, holding $q = q^*$ and $\rho = \rho^*$ constant, the LHS in (40) strictly increases while the RHS remains constant, because $P^* < V^*$ (for the same reasons explained in substep (f) of the proof of part (i) above). Therefore, holding q constant: (I) ρ^* strictly decreases until (40) is established again, (II) hence V^* strictly decreases, (III) in turn, the RHS in (39) strictly increases (and hence, so does the LHS in (39)), and (IV) hence, $\pi^*_{exit}(q)$ strictly increases. Since $\pi^*_{exit}(q)$ strictly increases, it follows that q^* (and hence μ^*_B) strictly increases. Moreover, $V^*(q^*)$ does not change, because (56) continues to hold.

(d) As τ_l increases, holding $q = q^*$ and $\rho = \rho^*$ constant, the RHS in (39) strictly decreases while the LHS remains constant, because $\eta > 0$. Therefore, holding q constant, ρ^* strictly decreases until (39) is established again. Note that while q^* responds as well, this response cannot reverse the decrease in ρ^*, since the sole reason of the change in q^* is the change in ρ^* (because τ_l does not appear in $\pi^*_{exit}(q^*)$). Moreover, $V^*(q^*)$ decreases due to (56). In addition, I also show that $(\mu^*_G(q^*) + \mu^*_B(q^*))V^*(q^*)$ strictly decreases. To see this, there are two cases to consider: (I) First, suppose that q^* (and hence μ^*_B) weakly increases. Then, $(\mu^*_G(q^*) + \mu^*_B(q^*))V^*(q^*) = [\bar{\mu}_G(\rho^*\Delta_H + (1-\rho^*)\Delta_M) + \frac{q^*}{1-q^*}\bar{\mu}_G\Delta_L]$ strictly decreases (because recall that ρ^* strictly decreases as well). (II) Second, suppose that q^* (and hence μ^*_B) strictly decreases. Then, $(\mu^*_G(q^*) + \mu^*_B(q^*))V^*(q^*)$ strictly decreases because both $V^*(q^*)$ and $\mu^*_B(q^*)$ strictly decrease, while $\mu^*_G(q^*)$ remains constant at $\mu^*_G(q^*) = \bar{\mu}_G$.

(e) Suppose $\tau_s = \tau_l$. As η increases, holding $q = q^*$ constant, ρ^* does not change, because (39) continues to hold. However, holding $q = q^*$ constant, even though ρ^* does not change, $\pi^*_{exit}(q)$ strictly increases, because $V^* > 0$ by Lemma 8. Therefore, q^* strictly increases such that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ is established again. Moreover, (56) implies that $V^*(q^*)$ strictly increases.

(f) Suppose $\tau_s > \tau_l$. As η increases, holding $q = q^*$ and $\rho = \rho^*$ constant, the LHS in (40) strictly decreases while the RHS remains constant, because $V^* > 0$. Therefore, holding q constant at $q = q^*$: (I) ρ^* strictly increases until (40) is established again, (II) hence V^* and P^* strictly increase (P^* strictly increases as shown in the proof of Proposition 1, because $P^* > \Delta_M$ as mentioned previously), and (III) $(1-\eta)V^*$ strictly decreases. To see why $(1-\eta)V^*$ strictly decreases, suppose that it weakly increases at η for some $\eta = \eta'$ (again, holding q constant). However, then, this implies that holding q constant, the LHS in (40) gets strictly larger (because V^* and P^* gets strictly larger), while the RHS remains constant, yielding a contradiction with (40). This establishes that holding q constant, $(1-\eta)V^*$ strictly decreases, and therefore the RHS in (39) and in turn the LHS in (39) strictly increases (because they have to be equal). Therefore, holding q constant at $q = q^*$, $\pi^*_{exit}(q)$ strictly increases. Hence, q^* strictly increases such that $\kappa + \Delta\kappa + \alpha(1-\phi)b = \pi^*_{exit}(q^*)$ is established again. Moreover, (56) implies that $V^*(q^*)$ has to strictly increase as well.

(g) As b increases, (56) implies that $V^*(q^*)$ does not change.

Even though the proof of the proposition is complete, in addition I also prove that V^* attains its global maximum w.r.t. $\kappa + \Delta\kappa \in [0, \infty)$ at $\kappa + \Delta\kappa = \max\{0, \underline{\kappa}\}$. To that end, note that $\pi^*_{exit}(q)$ is continuous w.r.t. q, because ρ^* is continuous w.r.t. q by Proposition 3. Since $\pi^*_{exit}(q)$ is also strictly decreasing in q if $\pi^*_{exit}(q) > -\varepsilon_\pi$ for some $\varepsilon_\pi > 0$ (as shown in the first step of Proposition 6), this implies that q^* is continuous and strictly decreasing w.r.t. $\kappa + \Delta\kappa$ for all $\kappa + \Delta\kappa \geq 0$ as well. Therefore, V^* is continuous w.r.t. κ as well, and combining with the results in parts (i)-(iii) about κ and $\Delta\kappa$ implies that V^* attains its global maximum w.r.t. $\kappa + \Delta\kappa \in [0, \infty)$ at $\kappa + \Delta\kappa = \max\{0, \underline{\kappa}\}$. ■

D.1 Supplemental results for Section D

Lemma 8 *Suppose that $\alpha b < \kappa + \Delta\kappa$. Under endogenous entry, in any equilibrium, if there is a positive measure of activists intervening, then:*

(i) *If an activist has not acquired a stake, then he never intervenes.*

(ii) *If an activist with a bad intervention acquires a stake in a target and intervenes, then he always exits.*

(iii) $q^* < 1$*, that is, if an intervention is implemented, it cannot be a bad intervention with probability 1.*

(iv) $\rho^* > 0$*, that is, if an activist with a good intervention intervenes, he exerts effort with positive probability.*

(v) $V^* > 0$ *and* $\lambda V^* + (1-\lambda)P^* > 0$*, where* P^* *and* V^* *are given by (3) and (9), respectively.*

(vi) *If an activist has acquired a stake and made a disclosure, then he always intervenes.*

Proof of Lemma 8. Throughout the proof, denote by $\gamma \in [0, 1]$ the fraction of activists who intervene after acquiring and disclosing their stake.

To see part (i), note that an activist never intervenes if he hasn't acquired a stake, because his payoff from doing so is negative due to the cost of intervention. Note that an implication of this result is that $\gamma^* > 0$. This is because: (a) a positive measure of activists intervene in equilibrium, (b) they implement only if they acquire a stake, and (c) when acquiring a stake, then they have to disclose it if they will intervene.

To prove part (ii), suppose that with positive probability, an activist with a bad intervention acquires a stake, intervenes, and keeps his stake. Note that his payoff from keeping his stake after acquiring a stake and intervening is given by

$$\alpha \left\{ (1 - \tau_l) \left[\Delta_L - (1 - \eta)\gamma^* V^* \right] + b \right\} - (\kappa + \Delta\kappa), \tag{57}$$

where V^* is given by (9). Note that (57) follows because once the activist discloses his stake, the market assigns probability γ that he will intervene, and if the intervention is not implemented, the market sees this (before the activist can sell his stake) and as a result the share price becomes Q_0. However, (57) is negative since $V^* \geq \Delta_L$ and $\alpha b < \kappa + \Delta\kappa$. In turn, it must be that this activist's payoff in equilibrium is negative, because when he exits after intervening, his payoff from doing so must be weakly smaller than (57), because he keeps his stake with positive probability. However, this yields a contradiction, because this activist strictly prefers not to buy any stake in the target and not to intervene, as this would give the activist a payoff of zero (which is strictly larger than (57)).

Before proceeding to the proof of remaining parts, note that in any equilibrium where intervention is on-the-equilibrium-path, due to parts (i), (ii), and Lemma 6, the average firm value conditional on the exit by the activist following intervention is given by $Q_0 + P^*$, where P^* is given by (3). We also already know that the average firm value conditional on intervention is given by $Q_0 + V^*$, where V^* is given by (9).

Next, I prove (iii), that is, $q^* < 1$. Recall that q denotes the probability that conditional on an intervention, it is a bad intervention. Suppose that $q^* = 1$. However, then $V^* = P^* = \Delta_L$, and hence the activist with a bad intervention always makes a loss by acquiring a stake and intervening. This is because if he exits, then his payoff is $\alpha (1 - \tau_s) [\lambda V^* + (1 - \lambda)P^* - (1 - \eta)\gamma^* V^*] - (\kappa + \Delta\kappa)$, which is negative, and if he keeps his stake then his payoff is given by (57), which is negative as well. However, then any activist with a bad intervention strictly prefers to not buy any stake and not to implement his intervention, because this gives him a payoff of zero. However, this yields a contradiction with $q^* = 1$.

Next, I prove part (iv), that is, $\rho^* > 0$. Recall that ρ denotes the probability of activist exerting effort conditional on intervention being implemented by an activist with a good intervention. Also recall from part (i) that an activist never implements his intervention if he hasn't acquired a stake, and note that by Lemma 6, if an activist with a good intervention shirks (exerts effort), then he always exits (keeps his stake unless hit by a liquidity shock). Then, for an activist with a good intervention, his expected payoff from acquiring a stake, exerting effort, and intervening is (because he keeps his stake unless hit by a liquidity shock)

$$\begin{aligned}&\phi\alpha (1 - \tau_s) [\lambda V^* + (1 - \lambda)P^* - (1 - \eta)\gamma^* V^*] \\ &+(1 - \phi)\alpha \{(1 - \tau_l) [\Delta_H - (1 - \eta)\gamma^* V^*] + b\} - \kappa - c,\end{aligned} \tag{58}$$

Similarly, his expected payoff from acquiring a stake, shirking, and intervening is (because then he always exits)

$$\alpha (1 - \tau_s) [\lambda V^* + (1 - \lambda)P^* - (1 - \eta)\gamma^* V^*] - \kappa. \tag{59}$$

To see $\rho^* > 0$, suppose that $\rho^* = 0$. However, then $P^* \leq \Delta_M$ and $V^* \leq \Delta_M$. Therefore, (58) is strictly larger than (59), because

$$(1 - \tau_s) [\lambda V^* + (1 - \lambda) P^* - (1 - \eta)\gamma^* V^*] < (1 - \tau_l) [\Delta_H - (1 - \eta)\gamma^* V^*] + b - \frac{c}{(1 - \phi) \alpha},$$

which is satisfied because

$$\frac{c}{(1-\phi)\,\alpha}-b \;<\; (1-\tau_l)\Delta_H-(1-\tau_s)\,\lambda\Delta_M-(\tau_s-\tau_l)(1-\eta)\Delta_M-(1-\tau_s)\,(1-\lambda)\Delta_M$$

$$\Leftrightarrow\;\; \frac{c-(1-\phi)\alpha b}{(1-\phi)\,\alpha}<(\tau_s-\tau_l)\eta\Delta_M+(1-\tau_l)\,(\Delta_H-\Delta_M)$$

which holds due to $c<\bar{c}$ (where $\bar{c}$ is given by (25)). However, since (58) is strictly larger than (59), any activist with a good intervention strictly prefers to exert effort when implementing the intervention, yielding a contradiction with $\rho^*=0$.

Next, I prove that $V^*>0$. Recall from part (i) that an activist never implements his intervention if he hasn't acquired a stake. Also note that any activist's payoff from acquiring a stake, implementing his intervention, and then exiting is

$$\alpha\,(1-\tau_s)\,[\lambda V^*+(1-\lambda)P^*-(1-\eta)\gamma^*V^*]-\kappa_i, \tag{60}$$

(where κ_i denotes the activist's cost of implementing intervention $i\in\{G,B\}$, that is, $\kappa_G=\kappa$ and $\kappa_B=\kappa+\Delta\kappa$), or equivalently,

$$\alpha\,(1-\tau_s)\,[(1-(1-\eta)\,\gamma^*)\,V^*+(1-\lambda)\,(P^*-V^*)]-\kappa_i. \tag{61}$$

However, if $V^*\leq 0$, then (61) is negative because $\lambda<1$, $\rho^*>0$, $q^*<1$, and $P^*<V^*$ (where the last inequality follows from

$$P^*=\Delta_H-\frac{\Delta_H-V^*}{1-(1-q^*)\rho^*(1-\phi)},$$

which in turn follows from (3) and (9)). In turn, since an activist with a bad intervention always exits if he implements the intervention (due to part (ii)), (61) being negative implies that an activist with a bad intervention would strictly prefer not acquiring any stake and not implementing the intervention over acquiring a stake and implementing the intervention. Because an activist never implements his intervention if he hasn't acquired a stake (by part (i)), this implies that all activists with a bad intervention strictly prefers not to implement their intervention, implying that $q^*=0$. However, combining this with $\rho^*>0$, then it cannot be $V^*\leq 0$, yielding a contradiction.

Next, I prove part (vi), that is, $\gamma^*=1$. For any activist that does not implement his intervention, his payoff from acquiring his stake and disclosing is $(1-\tau_s)\alpha\,(-(1-\eta)\gamma^*V^*)$ if the activist exits and $(1-\tau_l)\alpha\,(-(1-\eta)\gamma^*V^*+b)$ if he keeps his stake, and his payoff from acquiring his stake and not disclosing it is zero if the activist exits and αb if he keeps his stake (note that as also explained above, once the activist discloses his stake, the market assigns probability γ that he will implement the intervention, and once the intervention is not implemented, the market sees this before the activist can sell his stake and as a result, the share price becomes Q_0). Since $V^*>0$ and $\gamma^*>0$ (where the latter was explained at the end of the proof of part (i) above), it follows that $(1-\tau_s)\alpha\left(-(1-\eta)\gamma^* V^*\right) < 0$ and $(1-\tau_l)\alpha\left(-(1-\eta)\gamma^* V^* + b\right) < \alpha b$. Therefore, the activist strictly prefers not to disclose his stake if he will not implement the intervention. As a result, $\gamma^* = 1$.

Finally, I prove that $\lambda V^* + (1-\lambda)P^* > 0$. Suppose that $\lambda V^* + (1-\lambda)P^* \leq 0$. Recall from the proof of $V^* > 0$ above that any activist's payoff from acquiring a stake, implementing his intervention, and then exiting is given by (60), which is negative because $\lambda V^* + (1-\lambda)P^* \leq 0$, $V^* > 0$, and $\gamma^* > 0$. In turn, since an activist with a bad intervention always exits if he implements the intervention (due to part (ii)), this again implies that all activists with a bad intervention strictly prefers not to implement their intervention, implying that $q^* = 0$. However, combining this with $\rho^* > 0$, then it cannot be $P^* \leq 0$, yielding a contradiction with $V^* > 0$ and $\lambda V^* + (1-\lambda)P^* \leq 0$. ■

Lemma 9 *For any $\tilde{\eta} \in (0,1]$, if $q \geq \underline{q}$ and $P^*(q) - \tilde{\eta}V^*(q) > (1-\tilde{\eta})\Delta_L$, then $P^*(q) - \tilde{\eta}V^*(q)$ strictly decreases w.r.t. q.*

Proof of Lemma 9. Note that due to Proposition 3, $q^* \geq \underline{q}$ implies that $\rho^* = 1$. Hence,

$$P - \tilde{\eta}V = \frac{\phi\Delta_H + q\left[\Delta_L - \phi\Delta_H\right]}{1-(1-q)(1-\phi)} - \tilde{\eta}\left\{\Delta_H + q\left[\Delta_L - \Delta_H\right]\right\}$$

Therefore,

$$\begin{aligned}
0 &> \frac{d\left(P-\tilde{\eta}V\right)}{dq} \Leftrightarrow 0 > \frac{\left[\Delta_L - \phi\Delta_H\right]}{1-(1-q)(1-\phi)} - \frac{\phi\Delta_H + q\left[\Delta_L - \phi\Delta_H\right]}{\left[1-(1-q)(1-\phi)\right]^2}(1-\phi) - \tilde{\eta}\left\{\left[\Delta_L - \Delta_H\right]\right\} \\
&\Leftrightarrow \frac{\phi\Delta_H + q\left[\Delta_L - \phi\Delta_H\right]}{\left[1-(1-q)(1-\phi)\right]^2}(1-\phi) + \tilde{\eta}\left\{\left[\Delta_L - \Delta_H\right]\right\} > \frac{\left[\Delta_L - \phi\Delta_H\right]}{1-(1-q)(1-\phi)} \\
&\Leftrightarrow \left[\phi\Delta_H + q\left[\Delta_L - \phi\Delta_H\right]\right](1-\phi) + \tilde{\eta}\left\{\left[\Delta_L - \Delta_H\right]\right\}\left[1-(1-q)(1-\phi)\right]^2 \\
&> \left[\Delta_L - \phi\Delta_H\right]\left(1-(1-q)(1-\phi)\right) \\
&\Leftrightarrow \tilde{\eta}\left\{\left[\Delta_L - \Delta_H\right]\right\}\left[1-(1-q)(1-\phi)\right]^2 > \phi\left[\Delta_L - \Delta_H\right] \Leftrightarrow \tilde{\eta}\left[1-(1-q)(1-\phi)\right]^2 < \phi \\
&\Leftrightarrow q < q' \equiv 1 - \frac{1-\sqrt{\phi/\tilde{\eta}}}{1-\phi},
\end{aligned}$$

where the RHS is strictly larger than 0. Moreover, since $P^*(q) = V^*(q) = \Delta_L$ and hence $P^*(q) - \tilde{\eta}V^*(q) = (1-\tilde{\eta})\Delta_L$ if $q = 1$, this implies that $P^*(q) - \tilde{\eta}V^*(q)$ strictly decreases w.r.t q if $P^*(q) - \tilde{\eta}V^*(q) > (1-\tilde{\eta})\Delta_L$ and $q \geq \underline{q}$, concluding the proof. ■

E Online Appendix for Liquidity (Section 3.4)

As mentioned in Section 3.4, the parameters λ and η in Sections 3.1-3.3 can be more directly interpreted as the intensity of noise trading, without any change in the formal model. To see why, suppose that the activist is not required to make any disclosure. Instead, suppose that there is noise trading. In particular, suppose that at the exit (entry) stage, there are two possibilities: With probability λ (η), the intensity of noise trading is very high, and hence the market maker cannot make any inference about the activist's trade from the total order flow. With probability $1 - \lambda$ ($1 - \eta$), the intensity of noise trading is very low or non-existent, and hence the market maker can perfectly infer the activist's trade from the order flow.

Then, at the time when the activist decides to exert effort or not, he calculates his expected proceeds from selling his stake at the exit stage as $\alpha(Q_0 + \lambda V^* + (1 - \lambda)P^*)$, just like in the original model of Section 3.[39] Similarly, the activist pays $\alpha [Q_0 + (1 - \eta)V^*]$ to buy his stake if he intends to intervene.[40] Importantly, compared to the original interpretation of λ (η) in the model, the activist's expected payoff from selling (buying) remains unchanged. Similarly, his expected payoff from keeping his stake does not change either, since the NPV is perfectly revealed anyway if he keeps his stake. As a result, under this alternative interpretation of λ and η as the intensity of noise trading, the formal results derived in Sections 3.2 and 3.3 continue to hold without any change.

[39] In the paper, these proceeds were calculated in the discussion just before expression (14). Here, recall that α represents the activist's stake, Q_0 is the value of the firm in status quo (i.e., in the absence of implementation of the intervention), V^* is the expected NPV conditional on implementation of the intervention, and P^* is the expected NPV conditional on the activist's exit decision.

[40] Again, see the discussion before expression (14).