Anomaly Predictability with the Mean-Variance Portfolio[*]

Carlo A. Favero[†] Alessandro Melone[‡] Andrea Tamoni[§]

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Abstract

According to no-arbitrage, risk-adjusted returns should be unpredictable. Using several prominent factor models and a large cross-section of anomalies, we find that past pricing errors predict future risk-adjusted anomaly returns. We show that past pricing errors can be interpreted as deviations of an anomaly price from the mean-variance efficient portfolio. Price deviations constitute a novel anomaly-specific predictor, endogenous to the given heuristic mean-variance portfolio, thus providing direct evidence for conditional misspecification. A zero-cost investment strategy using price deviations generates positive alphas. Our findings suggest that cross-sectional models should incorporate the information in prices to capture the time-series dynamics of returns.

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[†] Bocconi University & CEPR. Address: Bocconi University, Department of Finance, via Roentgen 1, 20136 Milano, Italy. E-mail: carlo.favero@unibocconi.it.

[‡] The Ohio State University Fisher College of Business. Address: Department of Finance, 838 Fisher Hall, 2100 Neil Avenue, Columbus, OH 43210. E-mail: melone.11@osu.edu.

[§] Rutgers Business School. Address: Department of Finance, 1 Washington Pl, Newark, NJ 07102. E-mail: andrea.tamoni.research@gmail.com.

1 Introduction

In this paper, we document novel evidence of time-series anomaly return predictability that contradicts the standard no-arbitrage condition. Our proposed predictor has a unique feature: it relies solely on information about anomaly and factor price dynamics, without using data outside of the model such as sentiment, accounting signals, or macro-based state variables. We show that economic theory puts sign restrictions on the loading from regressing anomaly returns on our endogenous predictor. Violations of these restrictions represent direct evidence for the conditional misspecification of the candidate factor model. Thus, our work bridges two prominent research areas in finance—the literature on time-series return predictability and the one on cross-sectional factor models (see Karolyi and Van Nieuwerburgh, 2020, for a review)—by offering a coherent predictive framework to study whether a candidate set of factors captures the stochastic discount factor. Our portfolio predictability evidence points to systematic mispricing that vanishes over time, while the sign associated with our predictor is consistent with models featuring overreaction and slow adjustment of prices to news.

Our analysis starts from the expected return-beta representation which posits a linear relationship between expected returns of any asset and the expected return on the mean-variance efficient portfolio. When the model is correctly specified, no variable should forecast risk-adjusted returns. Nonetheless, the identification of an appropriate predictor is subject to data snooping (e.g. Lo and MacKinlay, 1990) and publication bias (e.g. McLean and Pontiff, 2016). To address these issues, we propose to construct a predictor that is endogenous to a given factor model: the difference between assets and factors long-term returns. We refer to this difference as price deviation. This choice builds on the notion that long-term returns convey information about possible model misspecification beyond one-period returns (Chernov, Lochstoer, and Lundeby, 2021) as they can capture persistence mispricing (see, e.g. Shiller, 1981).

We proceed to construct a simple test using our predictor. Consider regressing risk-adjusted returns on lagged price deviations. We show that if factors do not span the mean-variance efficient portfolio, the loading from this predictive regression should be negative. The intuition for the negative sign is straightforward: When an asset's price is above its intrinsic value as measured by long-run mean-variance portfolio returns, prices will revert back to their target level, leading to lower future expected returns. This return response to price deviations is consistent with a model where prices feature a permanent and transitory components (see Fama and French, 1988 and, more recently, Dong, Li, Rapach, and Zhou, 2021).

Our test is formulated in terms of implications for the conditional mean-variance efficient (CMVE) portfolio. As a benchmark case, to construct different heuristic CMVE portfolios, we use several versions of the Fama-French (2015, FF5) five-factor model. First, we employ a standard combination of the FF5 factors based on their unconditional first and second moments. Then, to account for conditioning information about the factors' mean and volatility in the construction of the mean-variance efficient portfolio, we implement a version with factor timing (Haddad, Kozak, and Santosh, 2020), and a version with volatility timing (Moreira and Muir, 2017). Finally, we use the characteristic-efficient factors of Daniel, Mota, Rottke, and Santos (2020) since Kozak and Nagel (2022) show that hedging the unpriced components of heuristic factor returns makes them more likely to span the stochastic discount factor. We also we perform several robustness tests by replacing the FF5 factors with the Hou, Xue, and Zhang (2015) q-factors and the PC-based model employed in Haddad, Kozak, and Santosh (2020) to construct the mean-variance portfolio.

We refer to the difference between cumulative (log) asset returns and the cumulative returns on the mean-variance efficient portfolio (built from one of the factor models described above) as to price deviations. As benchmark test assets, we use 90 portfolios from the long and short sides of 45 well-known and widely used characteristic-based strategies (Haddad, Kozak, and Santosh, 2020; Kelly, Kozak, and Giglio, 2020). Independently from how we construct the mean-variance portfolio, we show that price deviations forecast future anomaly returns with a negative sign, thus rejecting the restriction from the conditional beta-representation. Importantly, we document similar results even when using the classic 25 portfolios sorted on size and book-to-market or the large cross-section proposed by Chen and Zimmermann (2021) as test assets. The negative loading of future portfolio returns on the current price deviation implies that when asset prices are higher (lower) than the long-run price level implied by the factor model, we expect lower (higher) returns in the next period so that the deviations are corrected. Thus, it is natural to interpret the price deviations as the level of under- or over-pricing of a given asset relative to the price implied by the mean-variance portfolio.

The evidence in favor of anomalies' predictability is obtained by taking an out-of-sample perspective, i.e. by constructing price deviations in real time. The out-of-sample nature of our exercise is designed to combat overfitting and to detect demonstrable, ex ante mispricing. Also, importantly, our documented predictability already accounts for the possibility that the exposure of a given test asset to the mean-variance portfolio is time-varying. We do so in three ways: by using a classic fixed-length rolling window approach (e.g., Fama and French, 1997), by using short-window regressions with daily returns (e.g., Lewellen and Nagel, 2006), and by using the non-parametric method proposed by Ang and Kristensen (2012). The latter allows for tighter windows when there is more portfolio variation that can be picked up with greater precision. Despite these attempts, we continue to find evidence of sizable asset return predictability implied by the price deviations.

Similar to Dong, Li, Rapach, and Zhou (2021), the statistical model underlying our predictive framework assumes that price deviations eventually correct. However, the correction process can last for multiple periods. We study the timing of this correction process in our large cross-section of assets. Specifically, averaging across different factor models, we find that a value of the test asset above the target value implied by the mean-variance portfolio signals future negative returns over the next two to three years, at which point the price deviation is washed away. Interestingly, the long spell of time it takes for returns to revert toward their target value is in line with the evidence in Daniel, Klos, and Rottke (2022) who show that the beliefs of optimistic agents (who overreact to positive information) decay towards rational beliefs over a roughly 5-year period.

The mean-reverting behavior of price deviations toward the price implied by the mean-variance portfolio suggests that factor models provide a meaningful description of the financial system in the long run—a similar interpretation is discussed in Merton (1987)'s presidential address. Related, our documented portfolio-level return predictability can be interpreted in terms of speed of adjustment to the long-run price implied by the factor model. Our predictability disappears when the speed of adjustment is instantaneous while it becomes more apparent when the speed of adjustment to the long-run price reduces because of slow moving capital, slow adjustment to new information, or persistent behavioral pattern in expectations.

Importantly, the predictive content of our price deviations survives after controlling for alternative asset-specific predictors, including the test asset's book-to-market ratio, for the asset momentum or reversal effects captured by the 1- and 5-year past returns, respectively, and for measures of aggregate sentiment (Baker and Wurgler, 2006; Huang, Jiang, Tu, and Zhou, 2014). The result that our price deviations predict returns negatively and survive after controlling for the portfolio reversal based on long-term (5-years) past returns is striking. After all, our price deviations are obtained from the cumulative past returns *relative* to the cumulative mean-variance efficient ones. Thus, the fact that the price deviations series remains statistically significant after controlling for the (absolute) 5-years past returns, suggests that there is more information content in relative (to a given factor model) mispricing than in absolute mispricing as captured by the stand-alone past return series.

Although the out-of-sample R^2 from a forecasting model is a commonly used metric in the return predictability literature (see, e.g., Rapach and Zhou, 2022), Kelly, Malamud, and Zhou (2023) pointed out that it is an incomplete measure of the model economic value. Thus, we also implement a portfolio exercise to quantify the economic magnitude of the documented no-arbitrage rejections. Specifically, we form a zero-cost portfolio that buys anomalies with high one-year-ahead expected returns and sells anomalies with low one-year-ahead expected returns based on the signal provided by the anomaly-specific price deviations. Such a zero-cost investment strategy generates an out-of-sample annualized Sharpe ratio of 0.8 and 1.0 when the deviations are relative to the Fama-French five-factor model or to the Daniel, Mota, Rottke, and Santos (2020) hedged factors, respectively. Thus, the misspecification of the return dynamics in state-of-the-art models of the stochastic discount factors are quantitatively large.

We also show that the performance of our price deviations-based portfolio cannot be explained by other factor models, including those behavioral models that aim to capture temporary, long- and short-horizon deviations of prices from fundamental values (Daniel, Hirshleifer, and Sun, 2020), as well as models where factors are constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). Indeed, we find that regressing our zero-cost portfolio returns on the Daniel, Hirshleifer, and Sun (2020) behavioral factor model and the mispricing factor model of Bartram and Grinblatt

(2018) features large and statistically significant alphas. This evidence suggests that our investment strategy captures unexplained under/over reaction of asset price levels and, to capture such price dynamics, one needs additional mispricing factors outside those included in the candidate SDF model (which we use to infer the target price level). Consistent with this argument, we find that adding our price deviations-based strategy to the mean-variance portfolio leads to a significant reduction in risk-adjusted returns predictability, thus improving the factor model at hand.

Price deviations could be related to frictions that prevent rational traders from eliminating such deviations or to irrational behaviour of agents, or both. On the one hand, price divergences can be associated with noise trader risk (e.g., De Long, Shleifer, Summers, and Waldmann, 1990), holding costs (Tuckman and Vila, 1992), and idiosyncratic risk and transaction costs (e.g., Pontiff, 1996, 2006). Transitory price dislocations can also be due to the limited, and therefore slow-moving, capital of the currently available investors (e.g., Duffie, 2010) or slow adjustment of prices to new information (Amihud and Mendelson, 1987). Along this line, we show that a calibrated economy à la Amihud and Mendelson (1987) delivers a predictive coefficient on price deviations of similar magnitude to that found empirically. On the other hand, temporary price discrepancies that forecast return reversal (rather than continuation) can be the consequence of over-reaction to news. Behavioral models in which investors overreact to, e.g., news about firms' prospects have a long tradition; see, e.g., Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) and Gervais and Odean (2001). Interestingly, diagnostic expectation (e.g., Bordalo, Gennaioli, Porta, and Shleifer, 2019; Gennaioli and Shleifer, 2018) provides a modeling framework that yields overreaction to not just private but also public information, unlike model of investor overconfidence (Daniel, Hirshleifer, and Subrahmanyam, 1998) where decision makers exaggerate the precision of private information. Thus, inspired by the literature on diagnostic expectations, we conclude by linking our predictive framework to a model where the price deviation captures agents' over-reaction to news in prices that are subsequently corrected in return dynamics.

Related Literature. Our analysis builds upon, and relates to, the large empirical literature that studies temporary deviations of asset values from fundamentals. In an early contribution, Poterba and Summers (1988) find positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons which can be explained by persistent, but transitory, divergences between prices and fundamental values. Concurrently, Fama and French (1988) argue that the observed U-shaped pattern of the regression slope from forward h-period industry returns $r_{t,t+h}$ on past returns $r_{t-h,t}$ is consistent with the view that prices have a slowly decaying stationary component. Our finding that the deviations of a portfolio price from a given factor model forecast the portfolio returns is consistent with the permanent-transitory decomposition of prices proposed by Fama and French (1988).

Recently, Dong, Li, Rapach, and Zhou (2021) show that returns of the short- and (to a lesser extent) the long-leg of anomaly portfolios are positively related to the next period's market return. To explain this finding, these authors also exploit the permanent-transitory decomposition of an anomaly portfolio price. Differently from them, we show predictability at the individual anomaly level, rather than at the aggregate market level; furthermore, we link our predictive framework to a conditional test of a given factor model. Importantly, our evidence is that price deviations forecast reversal, not continuation, of returns. Thus our evidence complements that in Ehsani and Linnainmaa (2021) about positive auto-correlations in anomalies.

Our paper is related to a recent and rapidly growing literature that aims at explaining multi-period (cumulative) portfolio returns and portfolio price level.[1] The paper closest to ours is Chernov, Lochstoer, and Lundeby (2021). These authors propose to use multi-horizon returns to test over-identifying restrictions of a given factor model. Using their novel test, Chernov, Lochstoer, and Lundeby (2021) find that popular factor pricing models are unable to price their own factors at multiple return horizons even when one allows for state-of-the-art SDF sensitivities. We share a similar interest in (misspecification of) conditional dynamics. The conditional model misspecification documented in our paper is complementary to that analyzed by Chernov, Lochstoer, and Lundeby (2021). Whereas Chernov, Lochstoer, and Lundeby (2021) focus on the pricing of factors at multiple horizons, we instead test for mispecification in the risk-adjusted *short-run* dynamics of a test asset by exploiting information in *long-run* (cumulative) asset and factor returns.

Our work is also related to Baba-Yara, Boyer, and Davis (2022). We both focus on

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[1] See, e.g., Cohen, Polk, and Vuolteenaho (2009), Brennan and Wang (2010), Keloharju, Linnainmaa, and Nyberg (2019), Baba-Yara, Boons, and Tamoni (2020), Hendershott et al. (2020), Van Binsbergen and Opp (2019), Cho and Polk (2020), and Boons et al. (2021).

conditional models. In particular, Baba-Yara, Boyer, and Davis (2022) show that cutting-edge asset pricing models cannot explain the average returns of the implied mean-variance efficient (MVE) portfolios of other models. In this sense, Baba-Yara, Boyer, and Davis (2022) continue to use information external to a given model (but contained in other models) for their conditional asset pricing test. We instead rely only on information that is endogenous to a given model to study the (mispecification of) conditional dynamics of asset returns. Hence our paper speaks to the literature on factor timing (Cohen, Polk, and Vuolteenaho, 2003; Haddad, Kozak, and Santosh, 2020; Baba-Yara et al., 2020). In particular, we provide evidence that price deviations predict a vast array of portfolio returns. Also, we show that such predictability derives naturally from portfolio prices being anchored to factor prices.

Our paper also contribute to the literature that links the time-series and cross-sectional predictability. For example, Maio and Santa-Clara (2012) and Boons (2016) employ the I-CAPM to study the consistency between time-series and cross-sectional behavior of state variables and factors. Koijen, Lustig, and Van Nieuwerburgh (2017) propose a mode that prices the cross-section of equity while reproducing the time-series variation in bond returns. Lettau and Pelger (2020) discuss the tension between the time-series and cross-sectional objectives when designing a factor model. Our contribution is to show that cross-sectional models should incorporate the information in the limit multi-period returns (i.e. prices) in order to capture the time-series dynamics of portfolio returns.

Recently, Lopez-Lira and Roussanov (2022) show how to construct a portfolio that exploits individual stock return predictability while hedging all undiversifiable risk; they document that such portfolio delivers a Sharpe ratio above one. Kim, Korajczyk, and Neuhierl (2020) propose a procedure that gives characteristics maximal explanatory power for risk premiums before attributing any explanatory power to alphas (mispricing). Similar to these authors, our paper challenges the notion of a trade-off between systematic risk and expected returns. Whereas Lopez-Lira and Roussanov (2022) exploit a wide range of characteristics to forecast stock returns, we instead show how to construct a predictor that is endogenous to the factor model under scrutiny, and link this predictability to the conditional pricing ability of the model. We then show how to exploit this endogenous mispricing to form portfolios that hedge out the systematic risk associated with the MVE factor, in a spirit similar to the arbitrage portfolios of Kim, Korajczyk, and Neuhierl (2020).

Finally, despite the popularity of factor models in asset pricing (e.g., Ang, 2014), the literature on the relationship between the choice of factors and the investment horizon is less developed. Specifically, the factor-based approach to portfolio allocation and risk management has concentrated almost exclusively on modeling one-period returns, devoting less attention to the long-run relation between the performance of assets and factors.[2] In this paper, we propose a methodology that exploits long-horizon returns to test the short-run dynamic properties of asset pricing models.

2 Mean-variance returns, prices and predictability

Let R^e_{t+1} be the vector collecting the return on asset i in excess of the risk-free rate, R^{ei}_{t+1}. The conditional mean-variance efficient (CMVE) portfolio is given by[3]

$$R^{mv}_{t+1} = \left(k_t^{-1} V_t\left(R^e_{t+1}\right)^{-1} E_t\left[R^e_{t+1}\right]\right)^{\intercal} R^e_{t+1}, \tag{1}$$

where $V_t\left(R^e_{t+1}\right)$ and $E_t\left[R^e_{t+1}\right]$ are the conditional first and second moments of excess returns, and k_t is a time-varying scalar, known at time t, governing the leverage of the portfolio.

The no-arbitrage condition

$$E_t\left[R^e_{t+1}\right] = -\frac{Cov_t\left(M_{t+1}, R^e_{t+1}\right)}{E_t\left[M_{t+1}\right]}\ , \tag{2}$$

implies the conditional beta-pricing representation:[4]

$$E_t\left[R^e_{t+1}\right] = \beta_{i,t} E_t\left[R^{mv}_{t+1}\right] \tag{3}$$

For any return i included in the portfolio, the validity of Equation (3) requires that in a time

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[2] Hansen and Scheinkman (2009) and Backus, Chernov, and Zin (2014) have developed tools allowing researchers to characterize properties of equilibrium models at different horizons.

[3] See Hansen and Richard (1987); Ferson and Siegel (2001); Cochrane (2005); Chernov, Lochstoer, and Lundeby (2021). For completeness, we show the derivations of the CMVE portfolio in Appendix A.

[4] Consider the linear SDF $M_t = 1 - b_t\left(R^{mv}_{t+1} - E_t\left[R^{mv}_{t+1}\right]\right)$. Applying the no-arbitrage condition (2) to the minimum variance efficient portfolio we have: $E_t\left[M_{t+1}\right] = b_t \frac{Var_t\left[R^{mv}_{t+1}\right]}{E_t\left[R^{mv}_{t+1}\right]}$. By substituting this expression and $Cov_t\left(M_{t+1}, R^e_{t+1}\right) = -b_t Cov_t\left(R^{mv}_{t+1}, R^e_{t+1}\right)$ into (2), one obtains the desired expression.

series regression of the form:

$$R^{ei}_{t+1} = \beta_{i,t} R^{mv}_{t+1} + \varepsilon_{i,t+1} \tag{4}$$

the error terms should be unpredictable, i.e. $E_t\left[\varepsilon_{i,t+1}\right] = 0$ (see, e.g., Ferson and Harvey, 1991, 1999; Ferson and Korajczyk, 1995). Otherwise, one would buy (sell) the hedged portfolio $R^{ei}_{t+1} - \beta_{i,t} R^{mv}_{t+1}$ when the error is expected to be positive (negative), making a risk-adjusted profit and violating the fact that the SDF prices conditionally the given asset.

We propose to test for conditional misspecification of the SDF implied by (1) by generating a return predictor that is endogenous to the model (i.e., it depends solely on the candidate CMVE). We start by log-linearizing the Euler condition (2):[5]

$$E_t r^e_{i,t+1} + \frac{1}{2}\text{Var}_t r^e_{i,t+1} = \beta_{i,t} E_t r^{mv}_{t+1} \tag{5}$$

where $r^e_{i,t+1} = r_{i,t+1} - r_{f,t+1}$, and the conditional variance of the risky asset return on the left hand side of (5) is a Jensen's inequality correction that appears because we are working with logs.

Our test for conditional mispecification involves the coefficient δ_i in the following model-implied regression specification

$$r^e_{i,t+1} = c_{i,t} + \beta_{i,t} r^{mv}_{t+1} + \delta_i u_{i,t} + \epsilon_{i,t+1}\ . \tag{6}$$

where $c_{i,t}$ is a (possibly time-varying) intercept that captures the Jensen's effect. If the portfolio is CMVE and, hence, the associated SDF is correctly specified, one should have $\delta_i = 0$. One has, of course, many choices for $u_{i,t}$. We construct a predictor that is endogenous to the model as follows:

$$u_{i,t} = u_{i,t-1} + \underbrace{\left(r^e_{i,t} - c_{i,t-1} - \beta_{i,t-1} r^{mv}_t\right)}_{\varepsilon_{i,t}} \tag{7}$$

i.e., our predictor is the cumulative sum of risk-adjusted returns. To interpret $u_{i,t}$, it is convenient to define the log price of asset i as the cumulative log return: $\ln P_{i,t+1} = \ln P_{i,t} + r_{i,t+1}$. Similarly, we have $\ln P_{mv,t+1} = \ln P_{mv,t} + r_{mv,t+1}$ for the CMVE portfolio, and $\ln P_{rf,t+1} =$

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[5] This expression holds exactly if the SDF and the asset i returns have a joint conditional lognormal distribution.

$\ln P_{rf,t} + r_{f,t+1}$ for the risk-free asset. Note now that if $\beta_{i,t} \simeq \beta_{i,t-1}$, then

$$u_{i,t} = \ln P_{i,t} - \ln P_{rf,t} - \sum c_{i,t} - \beta_{i,t} \ln P_{mv,t} \ , \tag{8}$$

so that $u_{i,t}$ captures deviations of test asset prices from the price warranted by the CMVE portfolio (adjusted for a possible time-varying trend captured by $\sum c_{i,t}$).[6] The intuition behind our proposed predictor is that if there is persistent mispricing, it will show up in the price level (Shiller, 1981). Equation (8) suggests to compute the mispricing by comparing the portfolio price level to the value implied by the mean-variance portfolio, $\beta_{i,t} \ln P_{mv,t}$.

Although in our empirical analysis we work with time-varying exposures, the interpretation of $u_{i,t}$ as price deviations rest on the assumption $\beta_{i,t} \simeq \beta_{i,t-1}$, i.e. the portfolios' betas vary slowly and smoothly over time. This assumption is consistent with several economic models. E.g., Gomes, Kogan, and Zhang (2003) suggest that betas are a function of productivity shocks, which are often calibrated with an autocorrelation of 0.95 at the quarterly horizon. This translates into a monthly autocorrelation of conditional betas above 0.98. Similarly, in Santos and Veronesi (2006), stock betas change as the ratio of labor income to total consumption changes, which is also a highly persistent variable. Also, many previous empirical studies (see, e.g., Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001; Petkova and Zhang, 2005; Lewellen and Nagel, 2006; Ang and Chen, 2007; Pelger, 2020; Lopez-Lira and Roussanov, 2022) find that conditional betas are stable within short time window.[7]

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[6] Practically, by cumulating log excess returns on asset i we abtract from any source of long-run nominal comovement between the asset prices and the mean-variance efficient prices. To see this, consider for ease of exposition the CAPM model (the market is always included in the factor models studied in this paper) and constant betas. Under the null, we have: $(r_{i,t+1} - r_{f,t+1}) = \beta_i (r_{m,t+1} - r_{f,t+1}) + \varepsilon_{i,t+1}$. Compounding the left- and right-hand side yields:

$$\begin{aligned} \ln P_{i,t+1} - \ln P_{rf,t+1} &= \beta_i (\ln P_{m,t+1} - \ln P_{rf,t+1}) + u_{i,t+1}, \\ &\text{or, equivalently,} \\ \ln P_{i,t+1} &= \beta_i \ln P_{m,t+1} + (1-\beta_i) \ln P_{rf,t+1} + u_{i,t+1}. \end{aligned}$$

The term $(1-\beta_i) \ln P_{rf,t+1}$ effectively removes inflation-related trends that are common to the market factor and the asset prices.

[7] To account for the time-varying nature of factor betas in (period-by-period) neural networks, Fan, Ke, Liao, and Neuhierl (2023) propose local-PCA which also rely on conditional mispricing and risk premia varying slowly over time. Specifically, they use a 60-months window for estimation which they slide forward by one month, as we do.

Note that the test $\delta_i = 0$ in equation (6) is equivalent to testing the null:

$$u_{i,t} = u_{i,t-1} + \varepsilon_{i,t} \tag{H_0}$$

Indeed, in a correctly specified factor model there is an unpredictable error term $\varepsilon_{i,t}$. This error term leads to a random walk component in the price of the test asset, where prices are defined as cumulative returns. Hence, under the null of the model, the deviation of an asset price $\ln P_{i,t}$ from the price implied by the mean-variance portfolio (c.f., equation (8)) are permanent, and $u_{i,t}$ is a martingale. This implies that price deviations should not forecast risk-adjusted excess returns ($E_t\left[u_{i,t+1} - u_{i,t}\right] = E_t\left[\varepsilon_{i,t+1}\right] = 0$).

The alternative hypothesis, which we entertain in this paper, is that these price deviations are persistent instead. To be specific, we assume that the price deviations are mean reverting:

$$u_{i,t} = \rho_i u_{i,t-1} + \varepsilon_{i,t} \tag{H_1}$$

which implies $\delta_i = \rho_i - 1 < 0$ in Eq. (6).[8,9] In words, if asset prices are above the target value implied by the mean-variance portfolio, and if these price deviations are persistent but mean-reverting (i.e. $\rho_i < 1$), then future expected returns are lower (higher) on a risk-adjusted basis (i.e. after controlling for $\beta_{i,t} r^{mv}_{t+1}$).

Finally, note that under the null (H_0), $u_{i,t}$ is a martingale, i.e. $\rho_i = 1$ and $\delta_i = 0$ in Eq. (6). Thus one can view (H_1) as the unrestricted model, and (H_0) as the restriced model.

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[8] The result obtains by first differencing $u_{i,t} = (\ln P_{i,t} - \ln P_{rf,t}) - \sum c_{i,t} - \beta_{i,t} \ln P_{mv,t}$ (see Eq. (8)), using the autoregressive dynamics for $u_{i,t}$ under (H_1) and our definition of log prices as cumulative log returns, and using the assumption that betas are changing slowly over time, i.e. $\beta_{i,t+1} \simeq \beta_{i,t}$ which implies $\beta_{i,t+1} \ln P_{mv,t+1} - \beta_{i,t} \ln P_{mv,t} \simeq \beta_{i,t} r^{mv}_{t+1}$.

[9] In a contemporaneous paper, He and Zhou (2023) sort stocks into portfolios based on $\varepsilon_{i,t}$, i.e. on the difference between raw return and expected returns from a given factor model. Our approach is complementary. We study price deviations and their predictive ability for anomaly portfolios. Hence, we look at cumulative risk-adjusted returns $u_{i,t}$ and derive a novel prediction for the sign of its predictive coefficient δ_i in (6). He and Zhou (2023) instead study short-term reversal patterns which are potentially irrelevant for an analysis of the price level (albeit economically profitable, as they show).

2.1 Construction of Price Deviations

A large literature (e.g., Goyal and Welch, 2007, Rapach and Zhou, 2013, Martin and Nagel, 2020, Boudoukh, Israel, and Richardson, 2021) documents that out-of-sample tests provide the most rigorous and relevant evidence on stock return predictability. Therefore, to construct our predictor $u_{i,t}$ and to test the null $\delta_i = 0$ in (6), we take an out-of-sample perspective. The out-of-sample nature of our exercise is particularly important since we want to detect demonstrable, ex ante mispricing. Indeed, we will confirm that the price deviations captured by $u_{i,t}$ can be exploited in real time to predict asset returns.

First, following e.g., Fama and French (1997) and Ferson and Harvey (1999), we estimate the conditional betas using a regression over a 60-month rolling window:[10]

$$r^e_{\tau+1} = c_{i,t} + \beta_{i,t} r^{mv}_{\tau+1} + \varepsilon_{\tau+1}, \;\; \tau = t - 60 : t - 1. \tag{9}$$

We then construct the risk-adjusted return at time $t+1$ as:

$$\widehat{\varepsilon}_{i,t+1} = r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1}$$

where the beta and the constant are obtained from the rolling window regression (using information up to time t only, as denoted by the subscript). We repeat this same steps at time $t+2$ and construct $\widehat{\varepsilon}_{i,t+2}$ based on betas (and constant) from a rolling regressions over the period $t - 60 + 1$ to t. Our predictor is given by:

$$\widehat{u}_{i,t} = \sum_{\tau=0}^{t} \widehat{\varepsilon}_{i,\tau} \tag{10}$$

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[10] Using rolling windows to estimate conditional loadings gets around the problem of instrumenting time-varying factor loadings with the "right" state variables (e.g., Shanken, 1990; Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001. See also discussion in Lewellen and Nagel, 2006). As we use a rolling window of 60-months, our conditional betas estimates are not subject to overconditioning bias (Boguth, Carlson, Fisher, and Simutin, 2011). While the choice of the rolling window length is arbitrary, we document that our results are robust to a host of checks, including using daily returns, rolling windows of different length, and the optimal nonparametric technique developed in Ang and Kristensen (2012) to estimate time-varying betas. Additional unreported results show that results hold if one considers alternative window lengths of 12-months, 120-months, or an expanding window.

and, importantly, it is obtainable in real time. We then run the predictive regression

$$r^{e}_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1} = \delta_i \widehat{u}_{i,t} + \epsilon_{i,t+1} \ . \tag{11}$$

and test the null hypothesis $\delta_i = 0$ in Section 3.2. A rejection of the null, and in particular a negative δ_i, suggests that underpricing ($u_{i,t} < 0$) is followed by positive returns. We exploit this insight, and the fact that $u_{i,t}$ can be obtained in real time, to develop a trading strategy based on mispricing in Section 3.3. As we will see, the price deviations take time to be reabsorbed, which implies that our trading strategy does not require high-frequency rebalancing, reducing possible concerns about trading costs.

Note that our empirical analysis is using conditional betas since, as shown by Hansen and Richard (1987), assuming constant betas is not innocuous.[11] For example, with constant betas, price deviations could simply be a byproduct of time-varying loadings. In the Appendix, we repeat our analysis using progressively shorter time windows and high frequency (daily returns) (Appendices E.1 and E.2), or when we estimate time-varying betas using the kernel method proposed by Ang and Kristensen (2012) (Appendix E.3). The advantage of this method is that it allows the bandwidth of the kernel to vary across portfolios, i.e. to use tighter windows when there is more variation to be picked up with greater precision. Importantly, we will see that our findings continue to hold across these alternative specifications and different approaches.

The following steps summarize our approach to test for conditional misspecification:

1. Start from a factor model.
2. Construct the CMVE portfolio R^{mv}_{t+1} given in equation (1).
3. Using a rolling window (or kernel methods), estimate equation (9) to then construct real-time risk-adjusted returns $\widehat{\varepsilon}_{i,t+1}$ and price deviations $\widehat{u}_t$.
4. Run the predictive regression (11). An estimate of $\delta < 0$ leads to a rejection of the null (H_0) in favor of the alternative (H_1), thus implying that the model is misspecified.

Finally, if $\delta < 0$, we describe how to engage in anomaly timing in Section 3.3.

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[11] Gormsen and Jensen (2022) show that the alpha of major equity risk factors can partly be explained by time-varying market betas.

3 Main Results

3.1 Data

Our analysis focuses on characteristics-based factors. To pit models in a fair manner, we work with the MVE portfolio implied by the characteristics-based factors (this approach is also adopted by, e.g., Chernov, Lochstoer, and Lundeby, 2021 and Baba-Yara, Boyer, and Davis, 2022).[12] Specifically, we implement the mean-variance efficient portfolio using the following factor model representation:

$$R^{mv}_{t+1} = b_t^{\intercal} C_t R^e_{t+1} = b_t^{\intercal} \mathbf{f}_{t+1} \ , \tag{12}$$

where C_t is a $K \times N_t$ matrix of stock-level characteristics which define a set of K factors, $\mathbf{f}_{t+1} = C_t R^e_{t+1}$; and b_t is a $K \times 1$ timing vector that optimally combines these factors over time to get to the minimum variance portfolio (see, e.g., Haddad, Kozak, and Santosh, 2020; Moreira and Muir, 2017). Theoretically, the variation in the minimum variance portfolio weights must be driven by factor and volatility timing: $b_t \propto V_t\left(\mathbf{f}_{t+1}\right)^{-1} E_t\left[\mathbf{f}_{t+1}\right]$. We use the Fama and French (2015, FF5, henceforth) as factors, i.e. $\mathbf{f}'_t = [MKT_t \ SIZE_t \ HML_t \ RMW_t \ CMW_t]$ in equation (12), and entertain a version of FF5 with either factor return (factor-timing, henceforth) or volatility timing (vol-timing, henceforth). Mindful that standard factors are contaminated with unpriced components,[13] we also employ the hedging approach of Daniel, Mota, Rottke, and Santos (2020, DMRS) that aims at removing unpriced risks from the original factors. We call the residualized (with respect to the hedge portfolio returns) FF5 factors, FF5-DMRS.[14] This gives a total of four candidate SDFs. In addition, in Appendix F we repeat our main analysis when we use the Hou, Xue, and Zhang (2015) q-factors or statistical factors to construct the mean-variance efficient portfolio. In the latter case, we adopt principal components analysis (PCA) to extract factors from the 45 long-short portfolios.[15]

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[12] See Bessembinder, Burt, and Hrdlicka (2022) for a discussion on the time variation in the number of significant factors and its relation to time-varying economic complexity.

[13] For example, Gerakos and Linnainmaa (2017) find that the HML value factor is contaminated with unpriced components.

[14] We are grateful to Simon Rottke for sharing the up-to-date hedged FF5 factors.

[15] PCA is grounded in Ross (1976) seminal Arbitrage Pricing Theory (APT) and it is by far the most popular technique in finance to analyze latent factor models for returns with key empirical contributions dating back to Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986, 1988). Recently, Kelly,

Following, e.g., Kozak, Nagel, and Santosh (2020) and Haddad, Kozak, and Santosh (2020), we market-adjust the long-short strategies before computing the principal components. We then add back the aggregate market portfolio as a potentially important pricing factor. In other words, we study: $\mathbf{f}_t' = (R_{mkt,t}, PC_{1,t}, \dots PC_{n-1,t})$ and refer to this factor model as PCAn. Inspired by Haddad, Kozak, and Santosh (2020), we set $n = 6$.

Given our factors (e.g., the FF5), we estimate b such that the single-horizon monthly returns to the factors themselves are priced without error. For the volatility timing version, we follow Moreira and Muir (2017) and use $b_{i,t} = b_i V_t^{-1}(f_{i,t+1})$ which is computed using squared realized daily factor returns. For the factor timing version, we follow Haddad, Kozak, and Santosh (2020) and use $b_{i,t} = b_i E_t(f_{i,t+1})$ where the out-of-sample expectations for the factors are constructed using each factor's book-to-market ratio. In all cases, we estimate the constant of proportionality b_i for each factor i by matching the in-sample average returns to the timed factors in the model at hand, analogous to how we estimated the vector b in the baseline FF5 models.

We focus on U.S. data–NYSE, AMEX, and Nasdaq stocks from the Center for Research in Security Prices (CRSP) and Compustat data required for sorting – for the sample 1967–2019. In most of our analysis, we use monthly observations but we focus on 1-year holding-period return. A long holding period allows for a reaction of returns at time $t+1$ to the asset price deviations from the mean-variance portfolio prices at time t. So unless it is said otherwise, r_{t+1} will denote the one-year-ahead log excess returns. This choice is also in line with recent empirical studies on time-variation in anomaly returns (e.g., Lochstoer and Tetlock, 2020) and on the dynamics of equity portfolios (e.g., Kelly, Kozak, and Giglio, 2020). Interestingly, in our sample (see discussion of Figure 2) there is statistical evidence in favor of return predictability for one- up to twenty month ahead, i.e. $\hat{u}_{i,t}$ manifests forecasting ability for $r_{t+h/12}$ with $h = 1, \dots, 20$. Therefore, while we leave open the question of the exact timing of return reaction to price deviations, we repeat the relevant tests with monthly returns in Appendix D.2.

To investigate the validity of a given SDF, we consider as test assets a large cross-section

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Pruitt, and Su (2019) propose instrumental principal component analysis (IPCA) where the factor loadings are dynamic and can be instrumented with observable portfolio characteristics. Giglio and Xiu (2021) show that using ridge regression instead of PCA for reducing the dimensionality of the returns space yields similar results.

of anomaly portfolios based on single-sorts of 45 different characteristics. These test assets, or a subset of it, have been used by Kozak, Nagel, and Santosh (2020), Kelly, Kozak, and Giglio (2020), Haddad, Kozak, and Santosh (2020), and Lettau and Pelger (2020), among others.[16] In Appendix G, we provide robustness using an alternative, even larger, cross-section of portfolios from Chen and Zimmermann (2021).

3.2 Conditional Mispecification and Price Deviations

To test for the conditional validity of a given SDF, we run the following predictive regression:

$$\widetilde{r}_{i,t+1} = a + \delta \widehat{u}_{i,t} + \epsilon_{i,t+1} \tag{13}$$

where $\widetilde{r}_{i,t+1} = r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1}$ is the log excess return of test asset i at time $t+1$ net of the exposure to the log return on the mean-variance efficient portfolio r^{mv}_{t+1}; and $\widehat{u}_{i,t}$ measures the deviations of asset i prices from the mean-variance portfolio ones. The null is $H_0 : \delta = 0$ against $H_1 : \delta < 0$.[17,18]

We start by describing the properties of the price deviations $u_{i,t}$, i.e. our predictor in (13). In particular, Table 1 shows the half-life (Panel A) and the volatility of price deviations $u_{i,t}$ (Panel B) across our test assets $i = 1, \ldots, 90$. The price deviations are persistent but mean-reverting with an average half-life of about 2.5 years for all the models considered. Comparing the FF5 model to its timed or hedged versions, we observe very similar half-life distributions. Also, all models display price-deviations that are economically sizable, with a volatility of about 20% on average. In Panel C, we observe that price deviations from the

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[16] We thank Serhiy Kozak for making his data available at `https://sites.google.com/site/serhiykozak/data?authuser=0`. Appendix Table C.1 lists the categories and the portfolios included in each category.

[17] Note that the factor models considered perform well in pricing *unconditionally* the cross-section of the test assets. For example, the mean-variance efficient portfolio implied by the FF5 factor model and its hedged version (FF5-DMRS) reduces the mean absolute pricing errors on the 90 test assets from 5.21% per year in a risk-neutral benchmark economy to 2.77% and 2.64%, respectively, per year.

[18] Our test and the one proposed by Chernov, Lochstoer, and Lundeby (2021) are complementary. To see this, consider i.i.d. factor returns. In this case, the SDF prices the factors both conditionally and unconditionally and would pass the novel test of Chernov, Lochstoer, and Lundeby (2021) (which is solely based on factor dynamics). However, we show in Appendix B that this SDF could still admit test asset return predictability ($\delta < 0$), i.e. the SDF does not correctly price excess returns on test assets conditionally.

FF5 SDF have a high correlation of 0.8 or more with those obtained from its factor-timed or hedged versions. Finally, the price deviations display a strong factor structure (see Panel D), in particular for the version of FF5 with factor timing and for the hedged FF5-DMRS. Independently from the model considered, five PCs capture about 90% of the total variation in price deviations.

Table 2, column (1), shows the results from the pooled regression (13). Each panel refers to a different candidate SDFs, namely the FF5 model, its factor timed and volatility managed versions, and the FF5 residualized with respect to the DMRS hedge portfolio. Independently from the candidate SDF, we find a negative and statistically significant coefficient on the price deviations. The coefficient is economically large: for example, for the FF5-DMRS we find that a one standard deviation increase of (log) portfolio prices from the model-implied SDF value, implies an expected return that is lower by 4.6% over the next year (on average, across portfolios).[19] Also, note that the R^2 associated with the predictability induced by the price-deviations are about 10%, or larger, and thus comparable to the R^2 found in the aggregate market return predictability literature(e.g., Cochrane, 2008, 2011).

Figures 1 reports the asset specific $\widehat{\delta}_i$, along with its standard error, obtained from estimating equation (13) for each top decile portfolio.[20] The figure shows that $\widehat{\delta}_i$ is negative and significantly different from zero for all the portfolios and all the candidate SDFs considered. Hence, the evidence points to an ubiquitous rejection of the null in favor of price deviations that are persistent but mean reverting ($0 < \rho_i < 1$).

Next, we discuss the predictive ability of price deviations over alternative horizons. Recall that so far we have used annual returns in equation (13). Figure 2 shows the estimates of δ from a pooled regression when we forecast h-period ahead monthly returns, with $h = 1, ..., 60$ (i.e. returns are not compounded). For ease of exposition, we multiply the estimated coefficients by twelve so to make their magnitude comparable to the coefficient reported in Table 2 (which is based on annual returns). Across all models, there is statistical evidence in favor of return predictability for each of the future twenty months. Moreover, the magnitude of the coefficient is negative, similar across models, and decaying to zero only slowly as

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[19] From Table 1, we know that the average standard deviation is about 21% for FF5. Multiplying this value by the coefficient in Panel A of Table 2 we have: $-0.22 \times 0.21 = -4.6\%$. Equivalently, a pooled regression on standardized price deviations from FF5 yields a coefficient of -4.65%.

[20] Appendix Figure D.1 displays the analogous analysis for each bottom decile portfolio.

we increase the forecasting horizon. Comparing different SDFs, we observe that the price deviations from the FF5 and FF5-DMRS predict returns quite persistently, for up to forty months. On the other hand, the adoption of factor timing and, to a lesser extent, volatility management make the price deviations more transient as confirmed by a faster decay pattern in the coefficients, which become insignificant between 1.5 and 3 years. In sum, a value of the test asset above the target value implied by the mean-portfolio signals future negative returns over the next two to three years (in line with the average half-life reported in Table 1), at which point the price deviation is washed away. Interestingly, the long spell of time it takes for returns to revert toward their target value is in line with the evidence in Daniel, Klos, and Rottke (2022) who show that the beliefs of optimistic agents (who overreact to positive information) decay towards rational beliefs over a roughly 5-year period.

Recall that our price deviations signal is obtained in real time, using only information up-to-time t. Thus, we now evaluate its predictive ability for each portfolio using the out-of-sample (OOS) R^2 metric proposed by Campbell and Thompson (2008). Table 3 shows the results for each test asset. Each panel refers again to a given SDF model. On average (across models), we document positive OOS R^2 for more than 70% of anomaly portfolios. Most importantly, we find significant out-of-sample R^2 for relevant characteristics-sorted portfolios such as value, duration (Weber, 2018; Gormsen and Lazarus, 2020), and investment (Cooper, Gulen, and Schill, 2008). Although the out-of-sample R^2 from a forecasting model is a commonly used metric in the return predictability literature (see, e.g., Rapach and Zhou, 2022), Kelly, Malamud, and Zhou (2023) pointed out that it is an incomplete measure of the model economic value. Thus, in Section 3.3 we implement a portfolio exercise to quantify the economic profits of a market timer that exploits the price deviations implied by a given SDF model. Before doing so, in the next subsection we make sure that the predictive power of the price deviations is not subsumed by well known predictors.

3.2.1 The information content of price deviations

Our predictor $u_{i,t}$ is endogenous to the model: it accounts for the conditioning information (characteristics and possible timing variables) used in the construction of the SDF, and it allows to test conditional aspects of the model, namely the dynamics of future returns. However, one may wonder how $u_{i,t}$ relates to other portfolio return predictors.

To address this question, we run the following pooled regression:

$$\widetilde{r}_{i,t+1} = a + \delta \widehat{u}_{i,t} + \gamma \mathrm{X}_{i,t} + \epsilon_{i,t} \tag{14}$$

where $X_{i,t}$ is an alternative candidate predictor for the anomaly portfolio i. Columns (2) to (5) of Table 2 show the estimates when we control for (the portfolio) long-term reversal, past one-year returns, the book-to-market ratios, and aggregate sentiment as measured by the Baker and Wurgler (2006) investor sentiment index. Each panel refers to a specific SDF. We focus on results from the pooled regression only for ease of exposition, but all our conclusions hold when we run asset-specific individual regressions.

In column (2) we consider the reversal signal based on past 5-year returns (skipping the most recent year) as an additional anomaly portfolio predictor. After all, our price deviations are obtained from the cumulative past returns relative to the cumulative mean-variance efficient ones. We see that the series of past returns *relative to* the mean-variance portfolio remains statistically significant after controlling for the (absolute) 5-years past return series. Moreover, the loading $\hat{\delta}$ is always negative and of similar magnitude to the value reported in column (1). This result suggests that there is more information content in relative (to a given factor model) mispricing than in absolute mispricing as captured by the stand-alone past return series.

In column (3), we report results for regression (14) when we include the portfolio's performance over the prior year from month $t-12$ to $t-1$ along with the price deviations. Ehsani and Linnainmaa (2021) document that most factors are positively autocorrelated, and propose a factor that bets on the continuation in factor returns. Contrary to their work, our framework focuses on price deviations that forecast reversal, not continuation, of returns. It is then not surprising to see that our price deviations (capturing reversal) continue to be statistically significant after controlling for the portfolio momentum (capturing continuation).

Column (4) reports results for regression (14) when the control variable $X_{i,t}$ is the portfolio' book-to-market ratio. Indeed, valuation ratios are often used in return forecasting regressions (e.g., Cochrane, 2005; Campbell, 2017) as they represent a natural predictor according to the Campbell-Shiller (1988) log-linear present value model. Even after controlling for the book-to-market ratio, the coefficient on the price deviations is statistically significant, and negative: e.g., for the FF5 model, a one standard deviation increase of log portfolio prices from the model-implied MVE portfolio, implies a return that is lower by 4.6% over the next year (c.f., computation in footnote 19).

Finally, Shen, Yu, and Zhao (2017) document a negative predictive relation between the returns to portfolios sorted on macro-related risk factors and investor sentiment proxied by Baker and Wurgler (2006) index. Related, Avramov, Chordia, Jostova, and Philipov (2019) show that mispricing occurs across financial distressed firms during periods of high market sentiment because in these times both retail and institutional investors are overly optimistic about the likelihood and consequences of financial distress. The sluggish investors' response to correct overpricing leads to a wide range of anomalies in the cross-section of stocks and bonds. Column (5) of Table 2 displays the results from a predictive regression that controls for sentiment. Once again, we find that the predictive content of the price deviations is not driven away by aggregate sentiment. This result continue to hold true when we use the improved aggregate sentiment of Huang, Jiang, Tu, and Zhou (2014).

Overall, our evidence suggests that price deviations convey information about the time-series dynamics of risk-adjusted returns, $r^e_{i,t+1} - \beta_{i,t} r^{mv}_{t+1}$, for a wide range of portfolios. The predictive informative content of these price deviations is not subsumed by valuation ratios, momentum or reversal in individual factors, or aggregate sentiment.

3.2.2 Robustness

We provide several robustness checks for the evidence that anomaly returns can be predicted by the deviation of a portfolio price from the mean-variance target.

First, we verify the robustness of our results to alternative ways of computing time-varying betas. Specifically, Appendix Figures E.1, E.2, and E.3 report the asset specific $\widehat{\delta}_i$, along with its standard error, when we estimate betas using, respectively: (1) a shorter 2-year window; (2) a 1-year window but employing daily observations of returns (in the spirit of, e.g., Lewellen and Nagel, 2006, and following the estimation approach described in Welch, 2022); and (3) the nonparametric method proposed by Ang and Kristensen (2012).[21]

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[21] We report the results for the FF5 mean-variance portfolio but identical conclusions hold for the other

In all the cases, we observe $\widehat{\delta}_i$ that are negative and significantly different from zero for all the portfolios.

Second, we evaluate the effect of using other well-established factor models in the construction of the mean-variance portfolios. Appendix Table F.1 shows the results from the pooled predictive regressions (13) and (14) when we employ deviations of asset prices from the mean-variance portfolios implied by the HXZ factors (Panel A) or by their volatility managed version (Panel B), and for the mean-variance portfolio constructed using statistical (PCA-based) factors (Panel C). For all these candidate factor models, we find a negative and significant loading on price deviations even after controlling for well-known predictors. In Appendix Table F.2, we show that this predictability translates in out-of-sample R^2s that are positive for 86.7% and 68.9% of the portfolios when we use the HXZ and PCA-based factor models, respectively.

Finally, we evaluate the FF5 factor model using as test assets the 25 FF book-to-market and size sorted portfolios, i.e., a small cross-section of assets built using the very same characteristics that feature in the FF factor model.[22] Figure G.1 shows that even for this cross-section we find evidence that price deviations forecast anomaly portfolio returns negatively, providing a conditional rejection of the FF5 model.

Overall, the evidence points to an ubiquitous rejection of the null in favor of mean reverting price deviations ($0 < \rho_i < 1$) that convey information about future anomaly returns.

3.3 Return Dynamics, Mispricing, and Trading Strategy

In Section 3.2 we provided evidence against the null that price deviations follow a random walk and that risk-adjusted returns are unpredictable. In this section we evaluate the economic magnitude of this predictability by studying the performance of a strategy that – according to our signal – longs underpriced portfolios and short those that are overpriced.

We proceed as follows. First, recall that $u_{i,t}$ measures the deviation of the portfolio price i from the mean-variance target price, and $\widehat{u}_{i,t}$ is obtained in real-time using only

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factor models.

[22] The mean-variance efficient portfolio implied by the FF5 factor model reduces the mean absolute pricing errors on these 25 test assets from 6.87% per year in a risk-neutral benchmark economy to 2.03% per year.

information up to time t (c.f., Section 2.1). Second, in our framework $\delta_i u_{i,t}$ is a proxy for anomaly expected returns. We estimate δ_i using an expanding window with a burn-in sample the first 20 years of observations (1967–1987). This provides us with an out-of-sample estimate of expected returns $\widehat{\delta}_{i,t}\widehat{u}_{i,t}$ for portfolio i. We go long portfolios with prices below the model-implied target ($u_{i,t} < 0$), and consequent positive expected returns given the ubiquitous negative δ_i (c.f., Section 3.2), and go short those with prices higher than what the mean-variance portfolio would suggest ($u_{i,t} > 0$) and negative expected returns.[23] This timing strategy that increases exposure to assets after they have fallen and decreased exposure to stocks after they have risen in price emerges naturally, e.g., in a world where noise traders' misperception of returns follows a mean-reverting process (c.f. Section IV in De Long, Shleifer, Summers, and Waldmann, 1990).

We focus on a large cross-section of anomaly portfolios based on single-sorts of 45 different characteristics (see Kozak, Nagel, and Santosh, 2020), for a total of 90 portfolios. We sort these 90 anomaly portfolios once per year (in December) according to the portfolio-specific expected return, $\widehat{\delta}_{i,t}\widehat{u}_{i,t}$ and hold the position for one year, at which point we repeat the sorting procedure. Rather than choosing an arbitrary quantile, we implement rank-based strategy that invest in all portfolios.[24]

Figure 3 shows the performance of the long and short sides of our mispricing strategy, along with the aggregate market returns. The top left panel refer to the results obtained when we compute price deviations relative to the FF5 mean-variance portfolio. The next two panels refer to the results for the factor and volatility timed version of the FF5. The bottom right panel refers to DMRS hedged version of FF5. As expected, the long leg which contains underpriced test assets outperforms the market, whereas the short leg with overpriced portfolios underperforms. A strategy that goes long underpriced test assets and short overpriced assets generates an annualized average excess return of 5.2% and 6.6% for the FF5 model and its version that hedges unpriced risks. The associated annualized Sharpe Ratios are 0.80 and 1, respectively. A version of the mean-variance portfolio that times factors' returns obtains even stronger performance with annualized average excess return of

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[23] Estimation of equation (9) introduces measurement noise in the computation of $u_{i,t}$. As discussed in Burt and Hrdlicka (2021), this noise can lead to lower predictability. Thus, the reported performance of the anomaly timing strategy represents a lower bound.

[24] Furthermore, using ranks of the signal as portfolio weights helps mitigate the influence of outliers. Specifically, the weight on portfolio i at time t is: $w_{i,t} \propto \text{rank}\left(\delta_i u_{i,t}\right) - \sum_i \text{rank}\left(\delta_i u_{i,t}\right)/N$.

6.8% and a Sharpe ratio of 1.05. The performance of a strategy based on deviations from a volatility-timed mean-variance portfolio attains an average return of 4% and a Sharpe ratio of 0.73, instead.

Of course, it is important to ascertain that our mispricing zero-cost portfolio displays alphas relative to factor models, in particular those behavioral models aiming at capturing temporary, long- and short-horizon deviations of prices from fundamental values (Daniel, Hirshleifer, and Sun, 2020), as well as models where factors are constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). Table 4 shows the alpha of our zero-cost strategy based on model-implied price deviations. Several observations stand out. First, the average return of our strategy is not captured by standard characteristics-based or behavioral factors. In particular, our price deviations convey different information from that captured by the Daniel, Hirshleifer, and Sun (2020) behavioral factors and from the mispricing factor of Bartram and Grinblatt (2018): In correspondence of these two models (see rightmost two columns of Table 4), we observe that the constant remains large and statistically, for every panel (i.e. independently of the factors used to construct the mean-variance portfolio).

Second, we observe that, for three our of the four candidate SDFs (the exception being the volatility-managed FF5), the lowest alpha obtains in correspondence of the mispricing factor model of Stambaugh and Yuan (2016). Nevertheless, focusing on the FF5-DMRS in Panel (d), the alpha remains large and significant at 4.08% per year. Third, we find that our strategy has a positive and (often) statistically significant loading (not reported) on the Mgmt and Perf factors of Stambaugh and Yuan (2016), on the Daniel, Hirshleifer, and Sun (2020) behavioral factor FIN which is designed to capture long-horizon mispricing, as well as on the mispricing factor of Bartram and Grinblatt (2018). This suggests that indeed our strategy captures under/over reaction of asset price levels and, to describe accurately such price dynamics, one needs additional mispricing factors outside those included in the candidate SDF model (which, in turn, dictates the target price level). Having said that, none of the mispricing factor proposed in the literature subsumes the performance of our strategy.

3.3.1 Robustness

We provide a robustness tests for the performance of our mispricing portfolio along four main dimensions.

First, we investigate the effect of a more frequent rebalancing on our strategy. In particular, we repeat our analysis when we rebalance our portfolio at monthly frequency, rather than annual. In this case, an investment strategy using price deviations from the FF5 mean-variance portfolio achieve an annualized return and Sharpe ratio of 4.8% and 0.69, respectively. Table D.1 quantifies the performance of our strategy. We continue to see statistically significant alphas even after controlling for the Daniel, Hirshleifer, and Sun (2020) behavioral and Stambaugh and Yuan (2016) mispricing models, and independently of the candidate mean-variance portfolios. Since increasing the rebalancing frequency leaves largely unaffected the magnitude of alphas, while it would likely increase transtaction costs, our recommendation is to rebalance the mispricing strategy once per year.

Second, we investigate the effect of using alternative estimates for the time-varying betas. Specifically, Appendix Table E.1 shows the performance of our strategy when the price deviations $u_{i,t}$ are computed using a shorter length for the rolling window of 2-years (i.e., 24-months), as in, e.g., Hasler and Martineau (2023). In general, we see that our strategy continues to deliver statistically significant alphas. In Table E.2, we also verify that our results continue to hold when we use an even shorter window of 1-year together with daily returns. Table E.3, instead, shows the performance of our strategy when the time-varying exposures are computed with the non-parametric approach of Ang and Kristensen (2012). Note that this approach adjusts the length of the window (over which to compute betas) based on how much variation there is in portfolio betas. For example, the growth portfolio does not exhibit much variation in beta so the window estimation procedure picks a long bandwidth, corresponding to (a windows of about) 60 months. In contrast, we find significant time variation in beta for the value portfolio and the procedure picks a relatively tighter windows that allow this variation to be picked up with greater precision. Despite this more challenging set-up, we confirm the presence of statistically significant alphas for our strategy that longs portfolio with negative price deviations, and shorts portfolio with positive price deviations even after controlling for those behavioral and mispricing models proposed in the literature. In fact, quite surprisingly, we find that the shorter 2-year windows leaves the economic value of our mispricing strategies almost unaffected, whereas the use of non-parametric betas leads to larger gains relative to the benchmark case of betas estimated with a 5-year window.

Third, we show in Table F.3 the performance of our strategy when the price deviations are computed relative to either an SDF that employs the q-factors of Hou, Xue, and Zhang (2015) (Panel (a)) or a volatility-timed version of the same q-model (Panel (b)), or five principal components plus the market extracted from the underlying test assets. The annualized return and Sharpe ratio of the strategy which uses deviations of prices from the q-factors are 4.1% and 0.65. When we instead use principal components as (statistical) factors, we obtain an average return of 6.3% and a Sharpe ratio of 0.86. The table shows that the alpha from these strategies remains significant even after controlling for benchmark models featuring behavioral or mispricing factors.

Fourth, DeMiguel, Garlappi, and Uppal (2009) show that measurement error in the mean-variance weights has a nontrivial impact on the the portfolio performance. Thus, a concern is that our results are due to a mismeasurement of the factor weights in the SDF. To address this concern, we replicate our main results using the unconstrained FF5 model, i.e., using the five-Fama and French (2015) factors instead of the implied mean-variance portfolio. Table D.2 reports results for our strategy in this case. This strategy generates a Sharpe ratio of 1.15 with an alpha that is statistically significant even after controlling for benchmark models. Thus, measurement errors in SDF is not a concern for a real-time investor who uses price deviations to time anomaly portfolios.

So far we have used a cross-section of 45 characteristics, and the underlying 90 portfolios from the long and short sides of these strategies. In our last robustness check we instead employ an even larger cross-section of anomaly portfolios obtained from Chen and Zimmermann (2021). A rank-weighted investment strategy using this large cross-section generates an annualized performance of 7.2% with an associated Sharpe ratio of 1.13, an increase of more than 40% with respect the our benchmark case with 90 test assets. This is perhaps not surprising given that a larger cross-section features more possibilities to find highly over- and under-priced portfolios. Table G.2 reports results for our strategy when we use this larger universe of tests assets. The table shows large abnormal returns of a strategy that buys underpriced anomaly portfolios and sells overpriced portfolios (independently of the factors used to construct the mean-variance portfolio). Importantly, in each panel we see a statistically significant alpha, even after controlling for the Stambaugh and Yuan (2016) or the Bartram and Grinblatt (2018) mispricing factors, and for the Daniel, Hirshleifer, and Sun (2020) behavioral model. This evidence suggests that increasing the universe of test assets leads to even larger economic gains for an investor who uses price deviations to predict anomaly returns.

Overall, our analysis suggests that the deviations of a portfolio price from its long-term level implied by the mean-variance contain timely information to predict anomaly returns out-of-sample. Our conclusions is robust to alternative factors used to construct the SDF (HXZ or FF5-factor models, and their timed version), to the use of non-parametric procedure for the computation of the time-varying exposures, to the universe of test assets used, and to alternative ways to construct the strategy.

3.4 Improving Factor Models Using Price Deviations

A unique feature of our approach is that conditional mispecification tests are based on a predictive framework that uses only information about test assets and the factor model one wants to test. We explore the idea that price deviations—our endogenous portfolio-specific predictors—convey aggregate information potentially useful to improve factor models, in a similar spirit to Stambaugh and Yuan (2016). As shown in the previous Section 3.3, the price deviations-based portfolios implied by different mean-variance portfolios are unspanned by existing factor models. Thus, we use these portfolios together with their respective mean-variance portfolios to re-examine the documented risk-adjusted return predictability.

Specifically, for every candidate stochastic discount factor, we compute a new set of price deviations—$\tilde{u}_t$—in a similar way as in (10) but including both the mean-variance and the price deviations-based portfolios. We then use $\tilde{u}_t$ to re-test equation (13). Table 5 show the results from the pooled predictive regression (14) that use price deviations computed as described above. Comparing the result with those in Column (1) of Table 2, it is evident that the magnitude of the δ estimates is significantly reduced. Furthermore, for FF5 and DMRS, there is no evidence of risk-adjusted returns predictability associated with the new price deviations. Thus, for these cases, we cannot reject the null (H_0).

Finally, we also consider a more general approach to construct a mispricing portfolio. Specifically, we exploit the fact that the price deviations are an effective signals for the time-series dynamics of anomaly returns (c.f., Section 3.2), and use them as instruments in the IPCA framework developed by Kelly, Pruitt, and Su (2019). Using price deviations as an observable anomaly characteristic in IPCA enables us to construct a price deviations-based latent factor for a given factor model. Then, we consider this latent factor together with the mean-variance portfolio to compute price deviations and again test equation (13). Appendix Table D.3 shows the results. Similarly to Table 5, we find no evidence of predictability for both FF5 and DMRS. Furthermore, when using this improved mispricing portfolio, we cannot reject the null (H_0) also for FF5 with factor timing.

Overall, we find that portfolio-specific components unrelated to the mean-variance portfolio convey important information about return dynamics and can be used to improve a given factor model. This finding is complementary to the recent evidence documented by Dello Preite, Uppal, Zaffaroni, and Zviadadze (2023) in an unconditional setting.

4 Price Deviations and Predictability: Discussion

4.1 A Statistical Interpretation

In their seminal contribution, Fama and French (1988) argue that the (log of) stock price, $\ln P_{i,t}$, is composed of two parts: a permanent component $q_{i,t}$, modeled as a random walk with drift, and a temporary component $u_{i,t}$, modeled as a stationary AR(1) process,

$$\begin{aligned} \ln P_{i,t} &= q_{i,t} + u_{i,t} \\ q_{i,t} &= q_{i,t-1} + \alpha_i + \eta_{i,t} \\ u_{i,t} &= \rho_i u_{i,t-1} + v_{i,t} \end{aligned} \tag{15}$$

where $\eta_{i,t}$ and $v_{i,t}$ are independent processes with zero mean and constant variance and $|\rho_i| < 1$. Fama and French (1988) argue that the slowly mean reverting temporary component induces predictability in returns.

It is easy to map our alternative hypothesis (H_1) in the Fama and French (1988) framework: just assume that the permanent component for the (log of) stock price is $q_{i,t} = \beta_i \ln P_{mv,t}$; i.e. the permanent component is common across assets, but the loadings are asset specific. Thus, our analysis uncovers the return predictability induced by the deviation of asset prices from their (common) permanent trend captured by the mean-variance portfolio.

4.2 An Economic Interpretation

In the introductory session of our paper we stated that the return predictablity from price deviations could originate from either slow adjustment of prices to new information or the presence of Diagnostic Expectations. In this section we develop these two alternative interpretations in the light of our empirical results.

Consider first the case of slow adjustment of prices to new information as considered in the model proposed by Amihud and Mendelson (1987). Let $\ln P_t$ be the observed log asset price, and $\ln V_{t+1}$ its intrinsic value. Prices adjust slowly towards their intrinsic value; specifically, $\ln P_t$ evolves according to the following dynamics:

$$\ln P_{t+1} = \ln P_t + k(\ln V_{t+1} - \ln P_t) \tag{16}$$

where k is a parameter controlling the adjustment of prices towards the asset intrinsic value. If the adjustment parameter satisfies $0 < k < 1$, then the observed asset price $\ln P_t$ adjusts slowly to the fundamental price $\ln V_t$:

$$\ln P_{t+1} = k \ln V_{t+1} + (1 - k) \ln P_t \tag{17}$$

In our language, $\ln V_t$ is the price of the CMVE portfolio and the difference $\ln V_{t+1} - \ln V_t = r^V_{t+1}$ is the CMVE portfolio log return. For $0 < k < 1$, Eq. (17) describes the dynamics of a security that manifests temporary deviations from its intrinsic value.

We calibrate r^V_t to the CMVE portfolio return constructed using the Fama and French (2015) five-factor model over the period 1967–2019. Specifically, r^V_t is normally distributed with an annualized mean of 1.23% and an annualized volatility of 1.12%. The price vector is constructed as $\ln V_{t+1} = \ln V_t + r^V_{t+1}$. We then simulate a sample of 636 observations of $\ln P_{t+1}$ using equation (17). Using simulated prices, we construct returns. Then, we run regressions (13) and store the estimated δ. We repeat the simulation 10'000 times.

Figure H.1 reports the distribution of δ for three different calibrations of the adjustment parameter. For $k = 0.5$ (top panel), the simple partial-adjustment model features a significant and negative coefficient on price deviations. The average δ is about -0.25, which is comparable to the mean value across the 90 anomaly portfolios reported in Table 2 Panel A. As the adjustment parameter gets closer to one (i.e. full price adjustment to information), price deviation loadings get closer to zero (c.f., bottom panel with $k = .95$). Indeed, the extreme case of an economy without slow adjustments (i.e. $k = 1$), features a δ centered exactly at zero.

Our empirical evidence could also be interpreted through the lens of an expectation formation mechanism based on representativeness heuristic where temporary discrepancies are a consequence of over-reaction to news (e.g., Bordalo, Gennaioli, Porta, and Shleifer, 2019). Diagnostic Expectations have been proposed to model transitory deviations from Rational Expectations for stationary univariate processes. Agents with Diagnostic Expectations extrapolate into the future current news about the generic univariate process of interest, x_t:

$$E_t^{\theta}\left[x_{t+1}\right] = E_t\left[x_{t+1}\right] + \theta\left[E_t\left[x_{t+1}\right] - E_{t-1}\left[x_{t+1}\right]\right], \tag{18}$$

where the parameter θ controls the size of the deviations from rational expectations. Expectations are diagnostic when $\theta > 0$, and are rational when $\theta = 0$. Diagnostic Expectations converge to Rational Expectations in the long-run but in the short-run current news in x_t (e.g., excess returns) are extrapolated into the future. Our framework in Section 2 is related to this belief formation mechanism albeit with important differences. First, our framework is bi-variate and, thus, it requires a description of the test asset and mean-variance portfolio dynamics. Second, our framework requires non-stationary processes for the prices (of the individual asset and of the mean-variance portfolio), consistent with the Fama and French (1988) decomposition discussed in Section 4.1. In particular, time-t expectations of next period asset returns depend on their expectations conditional upon the returns on the CMVE portfolio and on the deviations of prices from their expectations conditional upon the price of the CMVE portfolio at time t:

$$E_t\left[r_{i,t+1}\right] = E_t\left[r_{i,t+1}|r_t^{mv}\right] + \delta_i\left(\ln P_{i,t} - E\left[\ln P_{i,t}|\ln P_{mv,t}\right]\right), \tag{19}$$

where negative values for δ_i imply that returns increase (decrease) when prices are below (above) their conditional expectations at time t. In other words, while prices are non stationary, the deviations of prices $\ln P_{i,t}$ from their projection on the minimum-variance portfolio prices ($E\left[\ln P_{i,t} \mid \ln P_{mv,t}\right]$) are temporary (i.e. stationary); in this sense, the price deviations play a similar role as the news term in the DE framework (18).

5 Conclusion

Standard asset pricing theory establishes that risk-adjusted returns should be unpredictable. Instead, this paper documents that deviations of portfolio prices from the value implied by leading factor models predict future risk-adjusted portfolio returns with a negative sign.

This predictability is endogenous to the model, i.e. it does not need any conditioning variables beyond those used in the construction of a heuristic mean-variance efficient portfolio. We also show how such a predictability can be used to test the conditional validity of any given SDF. Furthermore, the anomaly return predictability we document generates large economic gains. In particular, a real-time strategy that exploits mean-reverting price deviations generates Sharpe ratios between 0.7 and 1.2. Finally, we show that our price deviations-based portfolios are unspanned by existing factor models and convey aggregate information potentially useful to improve factor models.

From a data generating process point of view, we show that our results are consistent with a permanent-transitory decomposition of prices (Fama and French, 1988). From an economic perspective, our empirical evidence can be rationalized within an economy featuring slow adjustment of prices or where agents overweight "representative" events in reacting to news. Our results have implications for the practical implementation of asset allocation and risk management based on the parsimonious factor representation of large cross-sections of assets.

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Table 1: Price Deviations: Summary Statistics

This table reports descriptive statistics for price deviations computed using different heuristic mean-variance efficient portfolios. Price deviations $\hat{u}$ are computed as in equation (10). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020); see Appendix Table C.1 for a description of the anomalies. To compute different heuristic mean-variance efficient portfolios, we employ Fama and French (2015, FF5), its factor-timing and volatility-managed versions, and its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Panel A reports the half-life (in months) distribution of price deviations for each factor model. Panel B reports the standard deviation (in percentage) distribution of price deviations for each factor model. Panel C reports the average correlation across price deviations for each factor model. Panel D reports the proportion of variance explained by the first five principal components (PC). Monthly observations. The sample period is 1967 to 2019.

Panel A: Half-Life

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	30.1	14.4	23.6	27.4	36.1	62.7
FF5 with factor-timing	34.3	17.0	24.4	32.7	40.4	76.7
Volatility-managed FF5	30.4	16.6	23.6	28.5	34.3	86.3
FF5-DMRS	30.3	14.1	25.3	28.5	34.3	53.0

Panel B: Standard Deviation

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	21.0	13.2	17.8	20.5	23.6	33.2
FF5 with factor-timing	26.3	18.6	23.3	25.9	28.6	34.6
Volatility-managed FF5	23.6	10.7	20.0	22.9	27.1	39.7
FF5-DMRS	25.2	15.1	21.6	25.2	27.8	40.4

Panel C: Correlation Matrix

	FF5	factor-timing	vol-timing	FF5-DMRS
FF5	1	0.797	0.593	0.841
FF5 with factor-timing		1	0.272	0.851
Volatility-managed FF5			1	0.553
FF5-DMRS				1

Panel D: Principal Component Analysis

Model	PC1	PC2	PC3	PC4	PC5	cumulative
FF5	0.542	0.169	0.111	0.037	0.028	0.888
FF5 with factor-timing	0.756	0.065	0.055	0.030	0.020	0.926
Volatility-managed FF5	0.669	0.134	0.055	0.029	0.020	0.908
FF5-DMRS	0.736	0.076	0.061	0.032	0.017	0.922

Table 2: Predicting Anomaly Returns with Price Deviations

This table reports pooled estimates for δ_i from predictive regression (14). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)
δ	-0.222^{***}	-0.183^{***}	-0.229^{***}	-0.217^{***}	-0.223^{***}
	(0.029)	(0.028)	(0.033)	(0.028)	(0.028)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.097	0.131	0.098	0.102	0.128

Panel B: FF5 with factor-timing

	(1)	(2)	(3)	(4)	(5)
δ	-0.218^{***}	-0.173^{***}	-0.202^{***}	-0.202^{***}	-0.197^{***}
	(0.031)	(0.033)	(0.029)	(0.027)	(0.028)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.112	0.143	0.116	0.119	0.131

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)
δ	-0.273^{***}	-0.258^{***}	-0.279^{***}	-0.285^{***}	-0.307^{***}
	(0.059)	(0.053)	(0.056)	(0.058)	(0.052)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.125	0.219	0.125	0.151	0.203

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)
δ	-0.265^{***}	-0.188^{***}	-0.267^{***}	-0.252^{***}	-0.245^{***}
	(0.040)	(0.042)	(0.041)	(0.039)	(0.037)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.118	0.173	0.118	0.125	0.169

Table 3: Out-of-Sample Predictability

This table reports the out-of-sample R^2 (R^2_{OOS}) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return at time $t+1$ and price deviations $\hat{u}$ are computed as in equation (10). Test assets are the long legs for the 45 anomalies constructed in Kozak, Nagel, and Santosh (2020). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. The R^2_{OOS} is computed as in Campbell and Thompson (2008); *p*-values for R^2_{OOS} are computed as in Clark and West (2007). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. Monthly observations of annual returns.

Panel A: FF5

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	17.02***	indmom	2.87***	price	−19.42
age	5.21***	indmomrev	6.55***	prof	−17.56
aturnover	−7.48	indrrev	12.00***	roaa	−12.00
betaarb	−11.68	indrrevlv	−0.56	roea	−11.01
cfp	0.49***	inv	16.10***	season	8.94***
ciss	−1.32	invcap	10.65***	sgrowth	18.28***
divg	13.49***	ivol	−14.82	shvol	−10.73
divp	1.38***	lev	−6.82	size	7.48***
dur	7.35***	lrrev	12.02***	sp	18.31***
ep	9.61***	mom	5.88***	strev	9.75***
exchsw	−3.44	mom12	7.16***	valmom	11.3***
fscore	2.03***	momrev	12.60***	valmomprof	7.82***
gmargins	−14.57	nissa	−5.53	valprof	20.24***
growth	11.78***	nissm	1.69***	value	0.97***
igrowth	8.70***	noa	−18.42	valuem	−1.6

Panel B: FF5 with factor-timing

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	5.93***	indmom	−9.04	price	−15.97
age	10.68***	indmomrev	−10.2	prof	−13.76
aturnover	−6.23	indrrev	4.17***	roaa	−21.12
betaarb	−10.55	indrrevlv	−15.69	roea	−18.27
cfp	15.24***	inv	3.14***	season	−10.99
ciss	3.75***	invcap	3.46***	sgrowth	7.85***
divg	−8.55	ivol	−5.43	shvol	−0.53
divp	−1.52	lev	−2.02	size	8.17***
dur	11.2***	lrrev	−3.49	sp	12.91***
ep	−5.49	mom	1.31***	strev	3.94***
exchsw	−9.81	mom12	−5.81	valmom	−1.35
fscore	−5.61	momrev	−3.79	valmomprof	−8.12
gmargins	−22.75	nissa	−16.46	valprof	14.95***
growth	9.10***	nissm	−4.38	value	16.51***
igrowth	−2.34	noa	−17.56	valuem	−3.04

Panel C: Volatility-managed FF5

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	4.49***	indmom	12.74***	price	9.1***
age	9.28***	indmomrev	3.37***	prof	16.11***
aturnover	−5.07	indrrev	2.89***	roaa	12.5***
betaarb	12.36***	indrrevlv	−5.41	roea	12.19***
cfp	−0.24	inv	8.79***	season	7.74***
ciss	15.23***	invcap	16.45***	sgrowth	14.11***
divg	15.27***	ivol	10.58***	shvol	6.58***
divp	−0.01	lev	−4.73	size	9.28***
dur	−10.07	lrrev	7.85***	sp	2.31***
ep	2.30***	mom	18.06***	strev	5.25***
exchsw	15.9***	mom12	15.05***	valmom	1.76***
fscore	18.9***	momrev	15.39***	valmomprof	10.15***
gmargins	8.65***	nissa	6.19***	valprof	−2.7
growth	11.62***	nissm	8.27***	value	−1.43
igrowth	13.84***	noa	6.37***	valuem	5.78***

Panel D: FF5-DMRS

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	11.55***	indmom	1.22***	price	−13.99
age	6.76***	indmomrev	−1.87	prof	−17.42
aturnover	−5.7	indrrev	18.94***	roaa	−19.36
betaarb	4.69***	indrrevlv	−0.56	roea	−16.73
cfp	17.1***	inv	15.95***	season	1.2***
ciss	0.9***	invcap	14.27***	sgrowth	12.52***
divg	8.49***	ivol	−12.15	shvol	−5.88
divp	5.21***	lev	7.29***	size	9.28***
dur	12.10***	lrrev	6.71***	sp	17.18***
ep	13.08***	mom	8.61***	strev	18.87***
exchsw	−2.54	mom12	10.07***	valmom	10.29***
fscore	0.84***	momrev	6.59***	valmomprof	−2.63
gmargins	−16.49	nissa	−10.15	valprof	8.23***
growth	14.67***	nissm	−0.43	value	21.74***
igrowth	13.17***	noa	−19.09	valuem	13.4***

Table 4: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. We control for the following factor models: Carhart (1997) (C4), Fama and French (2018) (FF6), Hou, Xue, and Zhang (2015) (q), Stambaugh and Yuan (2016) (SY4), Daniel, Hirshleifer, and Sun (2020) (DHS3), Bartram and Grinblatt (2018) (BG3). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.39***	0.29***	0.28***	0.23***	0.33***	0.51***
	(0.07)	(0.07)	(0.09)	(0.07)	(0.10)	(0.11)
Adjusted R^2	0.62	0.67	0.49	0.55	0.41	0.26

Panel B: FF5 with factor-timing

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.50***	0.42***	0.43***	0.33***	0.39***	0.64***
	(0.09)	(0.08)	(0.09)	(0.07)	(0.09)	(0.12)
Adjusted R^2	0.54	0.57	0.38	0.46	0.34	0.16

Panel C: Volatility-managed FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.32***	0.27***	0.20**	0.25***	0.28***	0.34***
	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)
Adjusted R^2	0.40	0.43	0.28	0.32	0.31	0.20

Panel D: FF5-DMRS

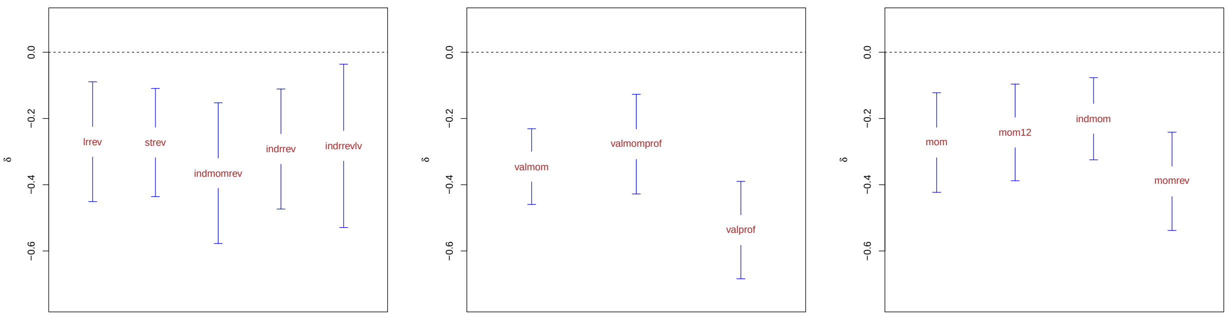
	C4	FF6	q	SY4	DHS3	BG3
Constant	0.51***	0.41***	0.40***	0.34***	0.42***	0.58***
	(0.11)	(0.10)	(0.10)	(0.08)	(0.11)	(0.13)
Adjusted R^2	0.49	0.53	0.37	0.46	0.32	0.18

Table 5: Improving Factor Models Using Price Deviations

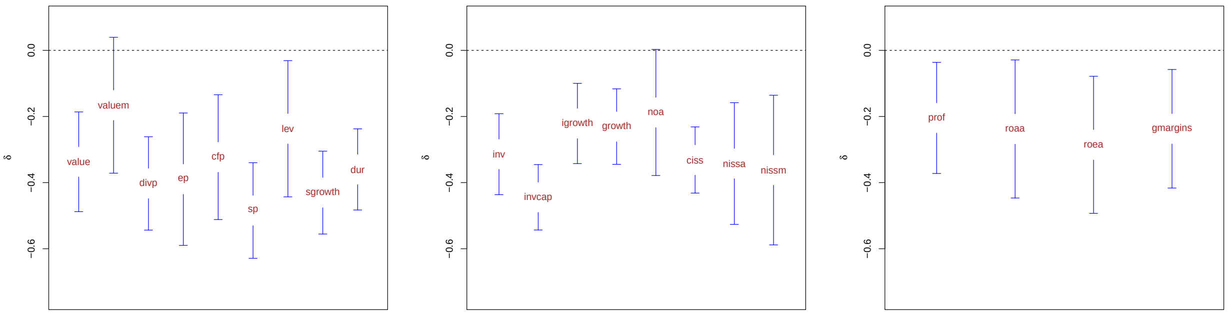
This table reports pooled estimates for δ_i from predictive regression (14). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\tilde{u}$ are computed as described in Section 3.4. We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Column (1) reports results for the Fama and French (2015, FF5) factor model, Columns (2) and (3) report results for its factor-timing and volatility-managed versions, and Column (4) reports results for its characteristics-efficient version computed in Daniel et al. (2020). Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. Constant estimates are not tabulated ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

	(1)	(2)	(3)	(4)
δ	-0.025	-0.031^{**}	-0.078^{***}	-0.012
	(0.018)	(0.015)	(0.029)	(0.013)
Observations	28,170	28,170	28,170	28,170
Adjusted R^2	0.006	0.010	0.026	0.002

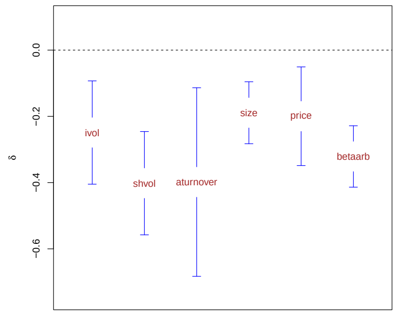
Panel A: FF5



(a) reversal **(b)** value interaction **(c)** momentum



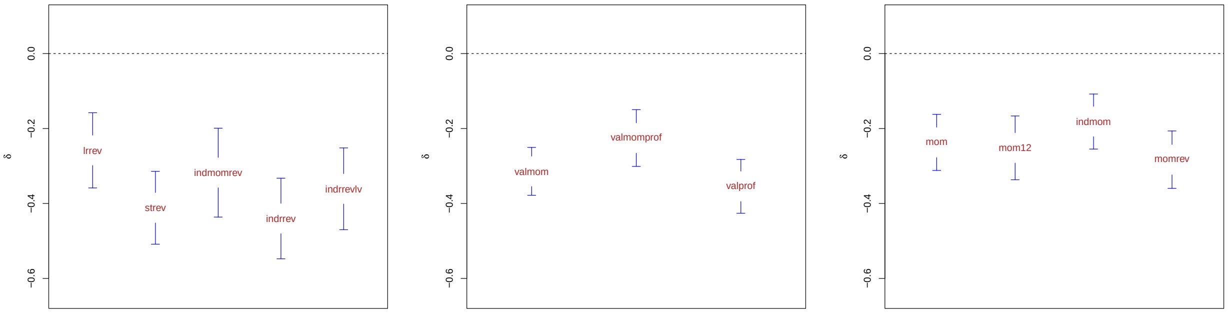
(d) value **(e)** investment **(f)** profitability



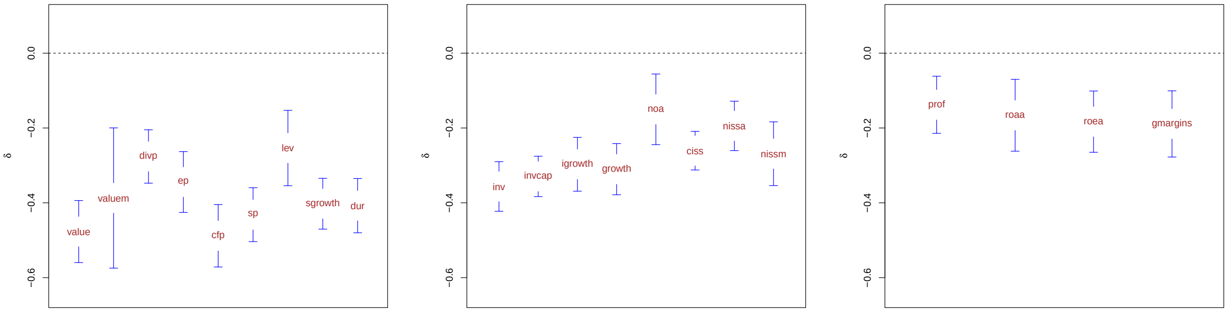
(g) trading frictions

Figure 1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

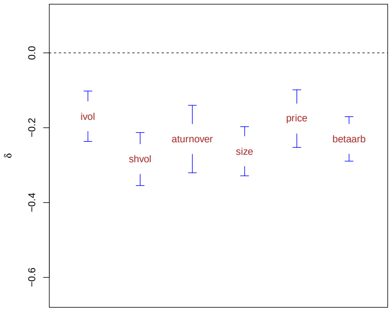
Panel B: FF5 with factor-timing



(a) reversal **(b)** value interaction **(c)** momentum

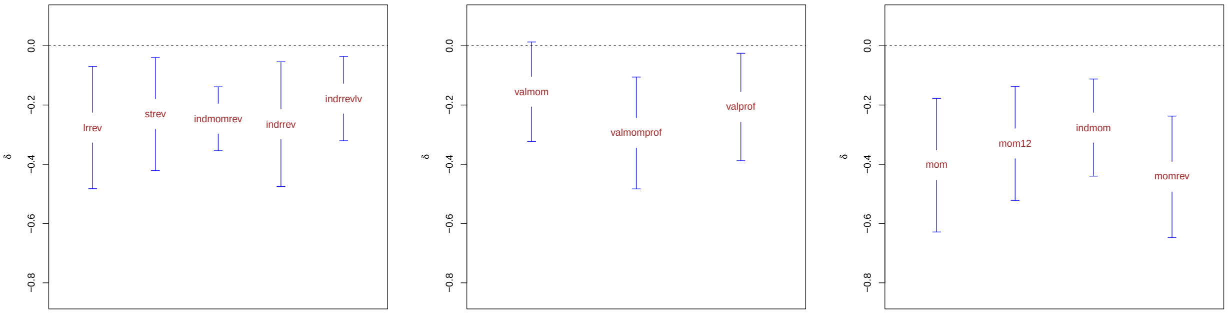


(d) value **(e)** investment **(f)** profitability

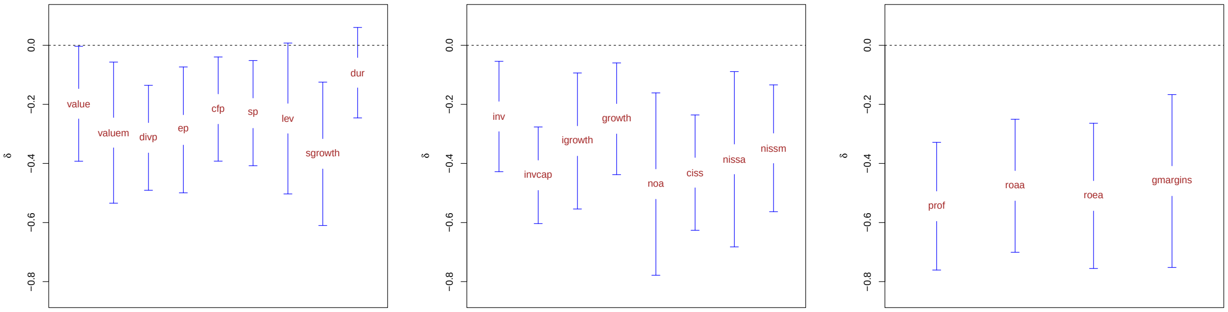


(g) trading frictions

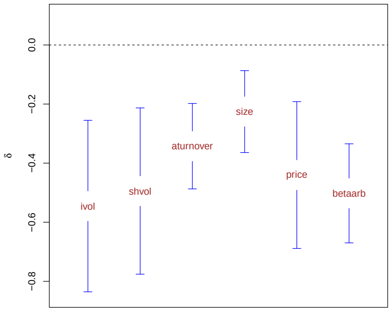
Panel C: Volatility-Managed FF5



(a) reversal **(b)** value interaction **(c)** momentum

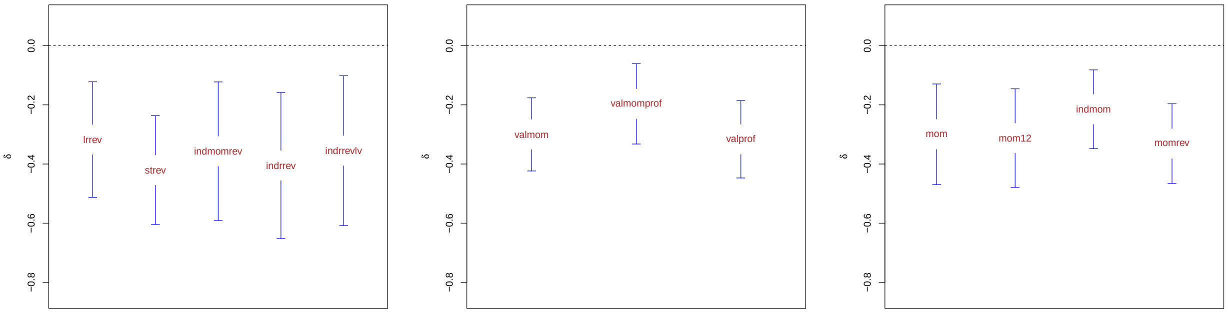


(d) value **(e)** investment **(f)** profitability

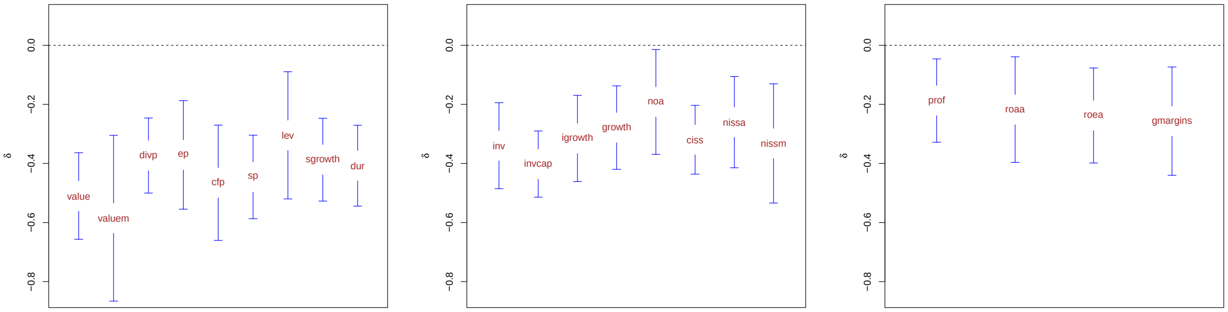


(g) trading frictions

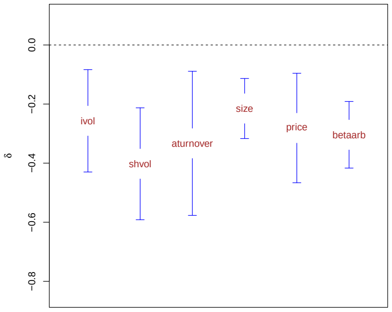
Panel D: FF5-DMRS



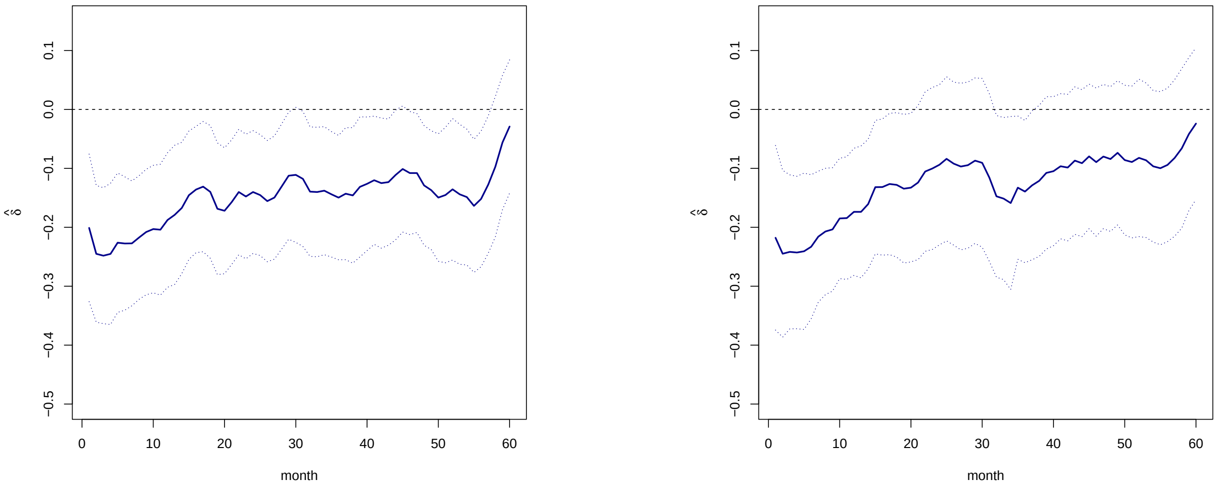
(a) reversal **(b)** value interaction **(c)** momentum



(d) value **(e)** investment **(f)** profitability

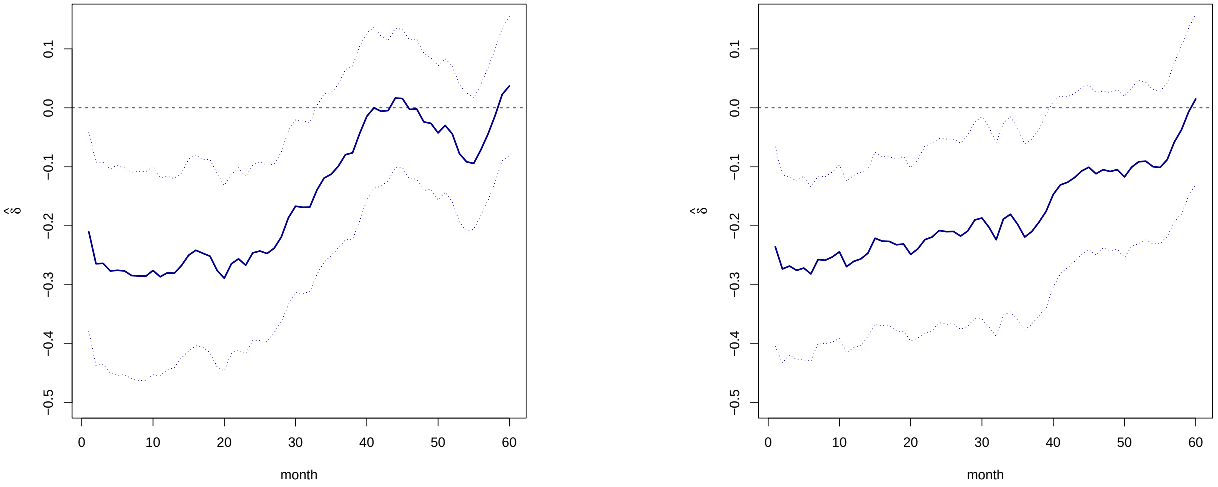


(g) trading frictions



(a) FF5

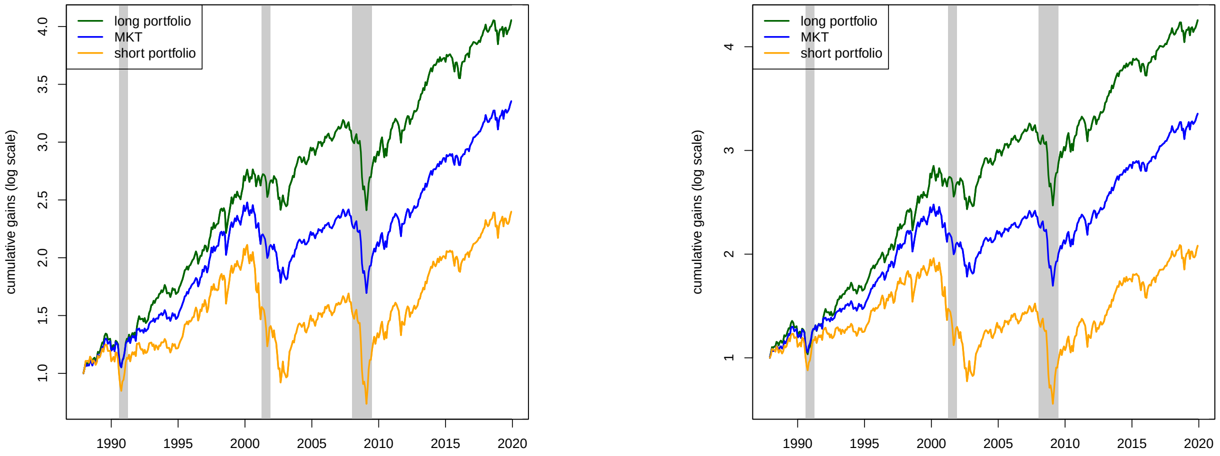
(b) FF5 with factor-timing



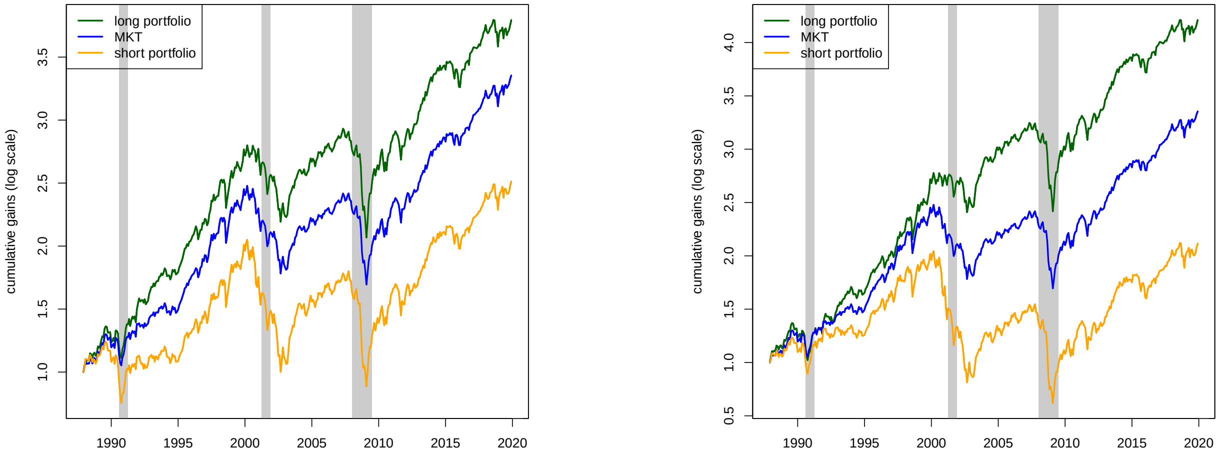
(c) Volatility-managed FF5

(d) FF5-DMRS

Figure 2: Price Deviations and Horizon of Predictability. This figure shows pooled regression estimates of δ_i for equation (13) for h-period ahead monthly returns ($h = 1, \ldots, 60$). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Standard errors are computed as in Newey and West (1987) with automatic bandwidth selection procedure as described in Newey and West (1994). Non-overlapping monthly observations. The sample period is 1967 to 2019.



(a) FF5 **(b)** FF5 with factor-timing



(c) Volatility-Managed FF5 **(d)** FF5-DMRS

Figure 3: Anomaly Portfolios Rotation using Real-Time Price Deviations. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Long (short) portfolio corresponds to the cumulative gains of a dynamic strategy that goes long on the 45 portfolios associated with the highest (lowest) expected return implied by the portfolio-specific price deviation. MKT is the performance of a static buy-and-hold strategy on the market portfolio in excess of the risk-free rate. Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Shaded areas are NBER recessions. Monthly observations.

Appendix

A Mean-variance efficient portfolio and the SDF

Define $\Sigma_t = V_t\left[R^e_{t+1}\right]$ and $\mu_t = E_t\left(R^e_{t+1}\right)$ and consider the following portfolio:

$$R^C_{t+1} = w_t^\intercal R^e_{t+1} \tag{A.1}$$

$$w_t = k_t^{-1}\Sigma_t^{-1}\mu_t \tag{A.2}$$

Next, we show that the SDF

$$\boxed{M^C_{t+1} = 1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)}$$

prices all assets conditionally:

$$\begin{aligned}
E_t\left[M^C_{t+1}R^e_{t+1}\right] &= E_t\left[1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^e_{t+1}\right]\\
&= E_t\left[R^e_{t+1}\right] - k_t E_t\left[\left(w_t^\intercal R^e_{t+1} - w_t^\intercal E_t\left[R^e_{t+1}\right]\right)R^e_{t+1}\right]\\
&= E_t\left[R^e_{t+1}\right] - k_t w_t^\intercal E_t\left[\left(R^e_{t+1} - E_t\left[R^e_{t+1}\right]\right)R^e_{t+1}\right]\\
&= E_t\left[R^e_{t+1}\right] - k_t k_t^{-1}\mu_t^\intercal\Sigma_t^{-1}\Sigma_t\\
&= 0
\end{aligned} \tag{A.3}$$

The parameter k_t is found by pricing the portfolio R^C_{t+1} itself:

$$\begin{aligned}
E_t\left[M^C_{t+1}R^C_{t+1}\right] &= E_t\left[1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^C_{t+1}\right]\\
&= E_t\left[R^C_{t+1}\right] - k_t E_t\left[\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^C_{t+1}\right]\\
&= E_t\left[R^C_{t+1}\right] - V_t\left[R^C_{t+1}\right]k_t\\
&= 0 \Leftrightarrow k_t = \left(V_t\left[R^C_{t+1}\right]\right)^{-1}E_t\left[R^C_{t+1}\right]
\end{aligned}$$

B Price deviations when factor returns are i.i.d.

This example is inspired by Section 2.4 in Chernov, Lochstoer, and Lundeby (2021). Suppose that the true model is given by:

$$M_{t+1} = 1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right) \ , \quad \mathbf{b} = V\left(\mathbf{f}_{t+1}\right)^{-1} E\left[\mathbf{f}_{t+1}\right]$$

where the factors $\mathbf{f}_{t+1}$ are excess returns to traded portfolios.
Suppose also that the factor returns are i.i.d. Thus, the model prices the factors both conditionally and unconditionally.

Despite the factors being i.i.d., our predictive model (6) implies that test assets' returns are not, since their dynamics feature the (persistent) $u_{i,t}$ term:

$$\begin{aligned} r^e_{i,t+1} &= \beta_i' \mathbf{f}_{t+1} + \underbrace{\Delta u_{i,t+1}}_{\delta_i u_{i,t} + \varepsilon_{i,t+1}} \ . \\ u_{i,t} &= (\ln P_{i,t} - \ln P_{f,t}) - \beta_i \ln P_{f,t}, \\ u_{i,t} &= \rho_i u_{i,t-1} + \varepsilon_{i,t} \end{aligned}$$

where $\delta_i = 1 - \rho_i$ and for simplicity we have omitted the constant.

Note that the SDF prices r^i_{t+1} *unconditionally*:

$$\begin{aligned} E\left[M_{t+1} r^e_{i,t+1}\right] &= E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\left(\beta_i' \mathbf{f}_{t+1} + \Delta u_{i,t+1}\right)\right] \\ &= (\beta_i' \underbrace{E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\mathbf{f}_{t+1}\right]}_{=0 \text{ using the definition of } \mathbf{b}} + E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\Delta u_{i,t+1}\right] \end{aligned}$$

where the last term is zero given our assumption of factors being independent over time and the price deviations $u_{i,t}$ being zero mean. However, the SDF does not prices $r^e_{i,t+1}$

conditionally:

$$\begin{aligned}
E_t\left[M_{t+1} r^e_{i,t+1}\right] &= E_t\left[\left(1 - \mathbf{b}^\intercal\left(\mathbf{f}_{t+1} - E[f_{t+1}]\right)\right)\left(\beta_i'\mathbf{f}_{t+1} + \Delta u_{i,t+1}\right)\right] \\
&= \beta_i' \underbrace{E_t\left[\left(1 - \mathbf{b}^\intercal\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\mathbf{f}_{t+1}\right]}_{=0 \text{ using the definition of } b \text{ and factors being iid}} \\
&+ \underbrace{E_t\left[\left(\mathbf{b}^\intercal\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\Delta u_{i,t+1}\right]}_{=0 \text{ since factors are i.i.d and property of } u_{i,t}} + E_t\left[\Delta u_{i,t+1}\right] \\
&= \delta_i u_{i,t}
\end{aligned}$$

where in the last step we exploit the AR(1) dynamics for $u_{i,t}$.
Furthermore, we have that

$$\text{Cov}\left(u_{i,t-1}, u_{i,t}\right) \neq 0$$

In words, through our predictive system we document that test assets feature persistent pricing errors.[25]

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[25] In the notation of Chernov, Lochstoer, and Lundeby (2021), $\delta_i u_{i,t}$ is the conditional pricing error at horizon $h = 1$.

C Test Assets

Table C.1: Categories

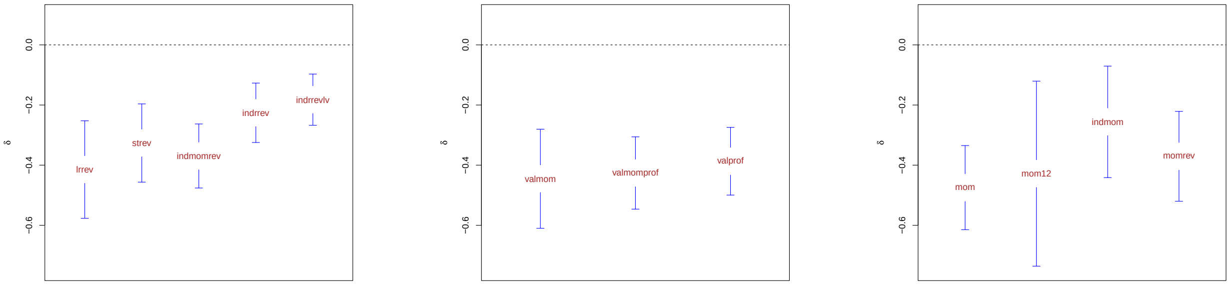
We group anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) following Lettau and Pelger (2020). This table lists the categories and the portfolios that we include in each category. Differently from Lettau and Pelger (2020), we allocate some of the portfolios in the category "others" across the other categories. In total, we consider 8 categories and 45 anomaly portfolios. Anomalies are defined in Kelly, Kozak, and Giglio (2020), Haddad, Kozak, and Santosh (2020), and Kozak, Nagel, and Santosh (2020).

Category	Anomaly Portfolios
reversal	indmomrev, indrrev, indrrevlv, lrrev, strev
value interaction	valmom, valmomprof, valprof
momentum	indmom, mom, mom12, momrev
value	cfp, divp, dur, ep, lev, sgrowth, sp, value, valuem
investment	ciss, inv, invcap, igrowth, growth, nissa, nissm, noa
profitability	gmargins, prof, roaa, roea
trading frictions	aturnover, betaarb, ivol, price, shvol, size
others	accruals, age, divg, exchsw, fscore, season

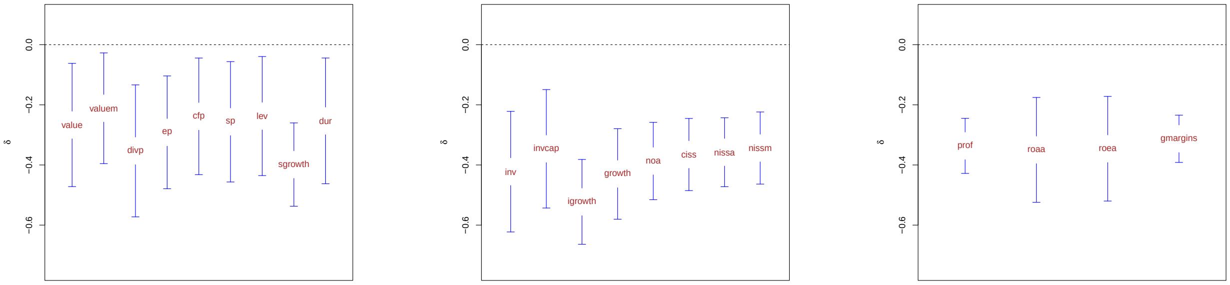
Notes: lrrev is long-term reversal calculated as in De Bondt and Thaler (1985). strev is short-term reversal calculated as in Jegadeesh (1990). indmomrev is industry momentum-reversal reversal calculated as in Moskowitz and Grinblatt (1999). indrrev is industry relative reversal calculated as in Da, Liu, and Schaumburg (2014). indrrevlv is industry relative reversal low volatility calculated as in Da, Liu, and Schaumburg (2014). valmom is value-momentum calculated as in Novy-Marx (2013). valmomprof is value-momentum-profitability calculated as in Novy-Marx (2013). valprof is value-profitability calculated as in Novy-Marx (2013). mom is 6-months momentum calculated as in Jegadeesh and Titman (1993). mom12 is 12-months momentum calculated as in Jegadeesh and Titman (1993). indmom is long-term reversal calculated as in Moskowitz and Grinblatt (1999). momrev is momentum-reversal calculated as in Jegadeesh and Titman (1993). value is annual value calculated as in Fama and French (1993). valuem is monthly value calculated as in Asness and Frazzini (2013). divp is dividend yield calculated as in Naranjo, Nimalendran, and Ryngaert (1998). ep is earnings/price calculated as in Basu (1977). cfp is cash-flow/market value of equity calculated as in Lakonishok, Shleifer, and Vishny (1994). sp is sales-to-price calculated as in Barbee Jr, Mukherji, and Raines (1996). lev is leverage calculated as in Bhandari (1988). sgrowth is sales growth calculated as in Lakonishok, Shleifer, and Vishny (1994). inv is investment calculated as in Chen, Novy-Marx, and Zhang (2011). invcap is investment-to-capital calculated as in Xing (2008). igrowth is investment growth calculated as in Xing (2008). growth is asset growth calculated as in Cooper, Gulen, and Schill (2008). noa is net operating asset calculated as in Hirshleifer et al. (2004). ciss is composite issuance calculated as in Daniel and Titman (2006). prof is ross profitability calculated as in Novy-Marx (2013). roaa is annual return on assets calculated as in Chen, Novy-Marx, and Zhang (2011). roea is annual return on equity calculated as in Haugen, Baker et al. (1996). gmargins is gross margins calculated as in Novy-Marx (2013). ivol is idiosyncratic volatility calculated as in Ang et al. (2006). shvol is share volume calculated as in Datar, Naik, and Radcliffe (1998). aturnover is asset turnover calculated as in Soliman (2008). size is size calculated as in Fama and French (1993).

D Fama and French (2015): Further Results

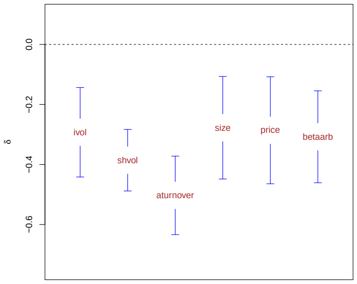
D.1 Bottom Deciles



(a) reversal **(b)** value interaction **(c)** momentum



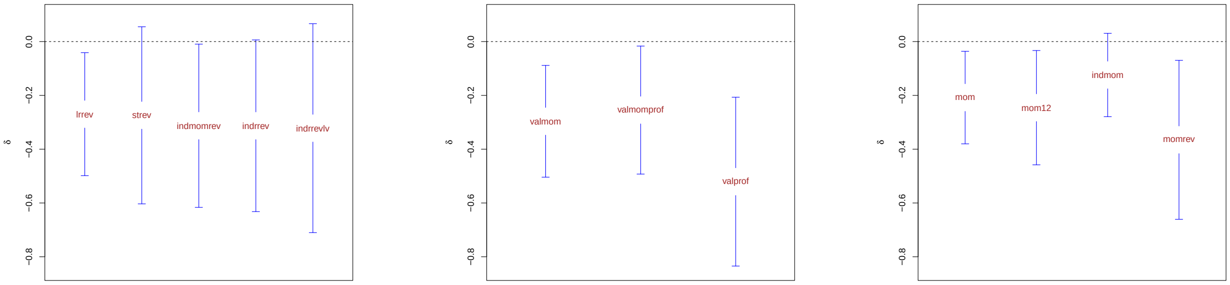
(d) value **(e)** investment **(f)** profitability



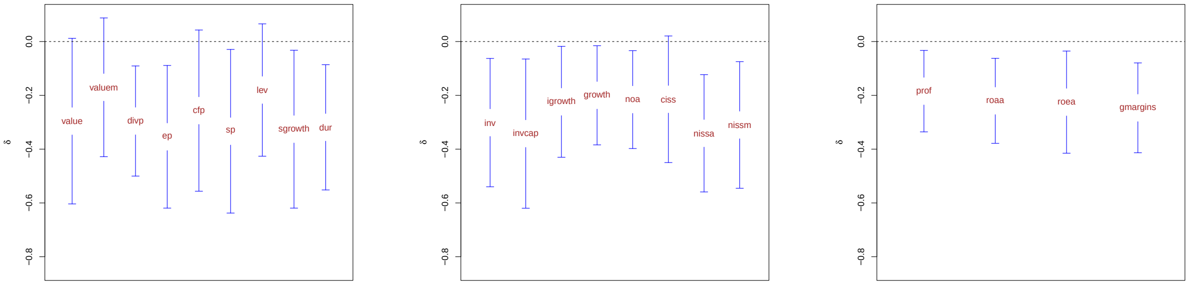
(g) trading frictions

Figure D.1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

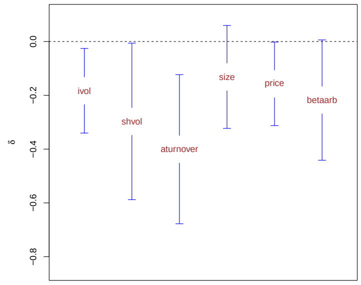
D.2 Monthly (Non-Overlapping) Observations



(a) reversal **(b)** value interaction **(c)** momentum



(d) value **(e)** investment **(f)** profitability



(g) trading frictions

Figure D.2: Anomaly Portfolios and Price Deviations – Top Deciles. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table D.1: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. The price deviations are relative to the mean-variance efficient portfolio implied by the Fama and French (2015) factor model. Once per month, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.35***	0.25***	0.25**	0.19**	0.32***	0.49***
	(0.09)	(0.09)	(0.10)	(0.08)	(0.12)	(0.14)
MKT	−0.13***	−0.08***	−0.11***	−0.06**	−0.10***	−0.22***
	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.05)
SMB	0.13***	0.17***		0.24***		0.10**
	(0.04)	(0.04)		(0.05)		(0.04)
HML	0.43***	0.27***				
	(0.06)	(0.04)				
Mom	0.18***	0.16***				
	(0.02)	(0.03)				
RMW		0.13*				
		(0.08)				
CMA		0.23***				
		(0.05)				
ME			0.21***			
			(0.05)			
IA			0.45***			
			(0.10)			
ROE			0.22***			
			(0.06)			
Mgmt				0.45***		
				(0.09)		
Perf				0.14***		
				(0.05)		
PEAD					0.11	
					(0.10)	
FIN					0.19*	
					(0.10)	
BG						0.10***
						(0.04)
Observations	384	384	384	384	384	384
Adjusted R^2	0.58	0.62	0.46	0.49	0.28	0.23

D.3 Unconstrained FF5

Table D.2: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. The price deviations are relative to the Fama and French (2015) factor model. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.45^{***}	0.34^{***}	0.34^{***}	0.30^{***}	0.35^{***}	0.53^{***}
	(0.06)	(0.06)	(0.06)	(0.05)	(0.06)	(0.09)
MKT	-0.09^{***}	-0.04^{*}	-0.07^{***}	-0.03	-0.05^{**}	-0.15^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)
SMB	0.00	0.05^{**}		0.08^{***}		-0.03
	(0.02)	(0.02)		(0.03)		(0.03)
HML	0.30^{***}	0.16^{***}				
	(0.04)	(0.02)				
Mom	0.10^{***}	0.09^{***}				
	(0.02)	(0.01)				
RMW		0.17^{***}				
		(0.03)				
CMA		0.23^{***}				
		(0.03)				
ME			0.06^{**}			
			(0.03)			
IA			0.40^{***}			
			(0.05)			
ROE			0.16^{***}			
			(0.03)			
Mgmt				0.35^{***}		
				(0.05)		
Perf				0.09^{***}		
				(0.03)		
PEAD					0.08^{***}	
					(0.03)	
FIN					0.21^{***}	
					(0.04)	
BG						0.08^{***}
						(0.02)
Observations	384	384	384	384	384	384
Adjusted R^2	0.59	0.68	0.55	0.58	0.49	0.25

D.4 Improving Factor Models: IPCA

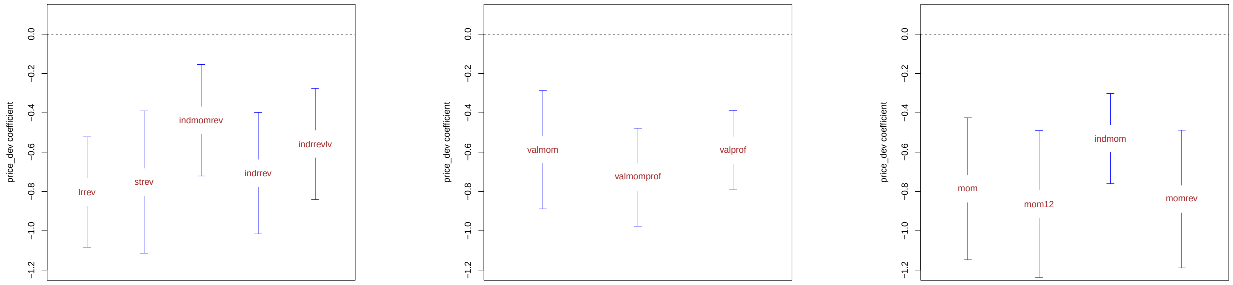
Table D.3: Improving Factor Models Using Price Deviations

This table reports pooled estimates for δ_i from predictive regression (14). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations are computed as described in Section 3.4. We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Column (1) reports results for the Fama and French (2015, FF5) factor model, Columns (2) and (3) report results for its factor-timing and volatility-managed versions, and Column (4) reports results for its characteristics-efficient version computed in Daniel et al. (2020). Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. Constant estimates are not tabulated ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

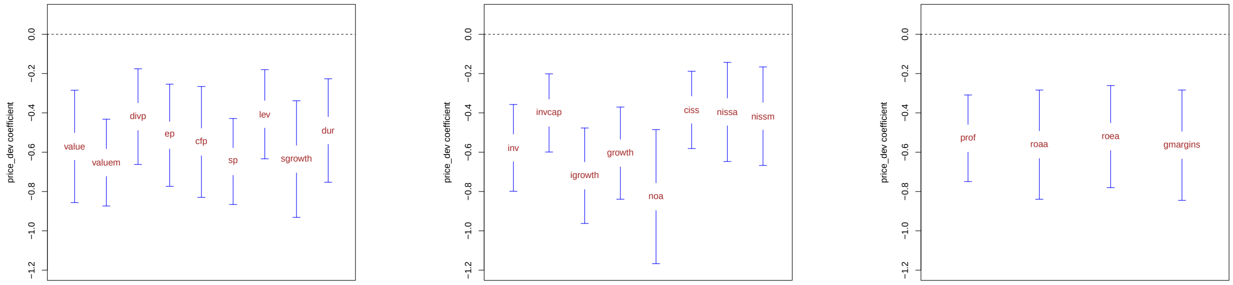
	(1)	(2)	(3)	(4)
δ	-0.014	0.007	-0.131^{***}	-0.026
	(0.016)	(0.027)	(0.036)	(0.018)
Observations	45,360	45,270	45,360	45,360
Adjusted R^2	0.001	0.000	0.061	0.007

E Alternative Conditional Betas: Robustness

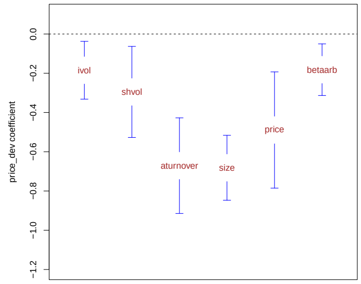
E.1 24-Months Estimation Window



(a) reversal **(b)** value interaction **(c)** momentum



(d) value **(e)** investment **(f)** profitability



(g) trading frictions

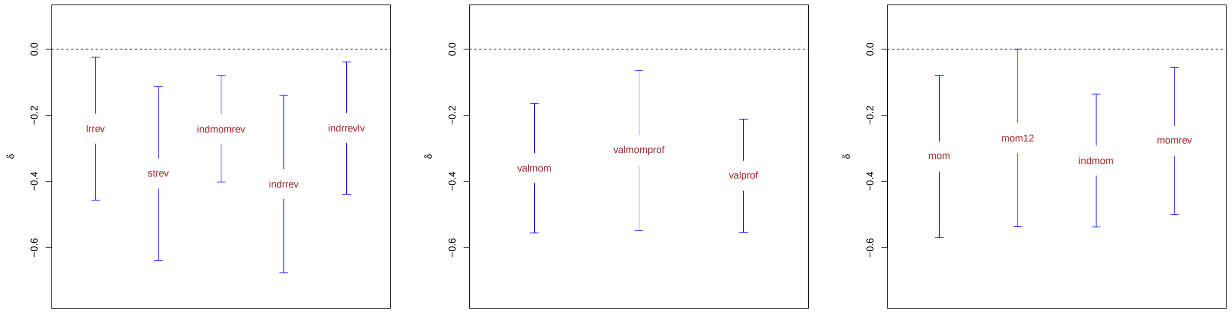
Figure E.1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10) using a 2-year rolling window to calculate time-varying parameters. We report results for price deviations computed using the Fama and French (2015) factor model to compute the mean-variance efficient portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table E.1: Long-Short Anomaly Portfolio Alphas

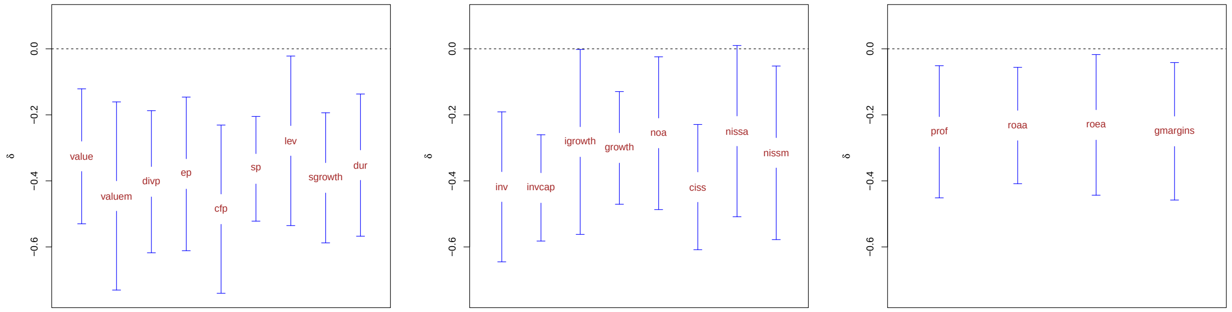
This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10) using a 2-year rolling window to calculate time-varying parameters. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.37^{***}	0.27^{***}	0.23^{***}	0.22^{***}	0.33^{***}	0.39^{***}
	(0.08)	(0.06)	(0.07)	(0.07)	(0.09)	(0.11)
MKT	-0.07^{***}	-0.03^{*}	-0.03	-0.01	0.00	-0.11^{**}
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.05)
SMB	0.01	0.09^{***}		0.09		-0.05
	(0.03)	(0.03)		(0.06)		(0.05)
HML	0.43^{***}	0.34^{***}				
	(0.06)	(0.04)				
Mom	0.03	0.01				
	(0.03)	(0.02)				
RMW		0.22^{***}				
		(0.04)				
CMA		0.08				
		(0.05)				
ME			0.06			
			(0.05)			
IA			0.47^{***}			
			(0.10)			
ROE			0.13^{*}			
			(0.07)			
Mgmt				0.43^{***}		
				(0.06)		
Perf				-0.01		
				(0.04)		
PEAD					-0.12^{***}	
					(0.05)	
FIN					0.26^{***}	
					(0.05)	
BG						0.19^{***}
						(0.04)
Observations	384	384	384	384	384	384
Adjusted R^2	0.60	0.68	0.39	0.51	0.46	0.26

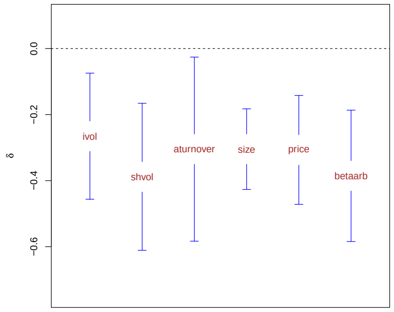
E.2 Using Daily Returns for Calculating Betas



(a) reversal **(b)** value interaction **(c)** momentum



(d) value **(e)** investment **(f)** profitability



(g) trading frictions

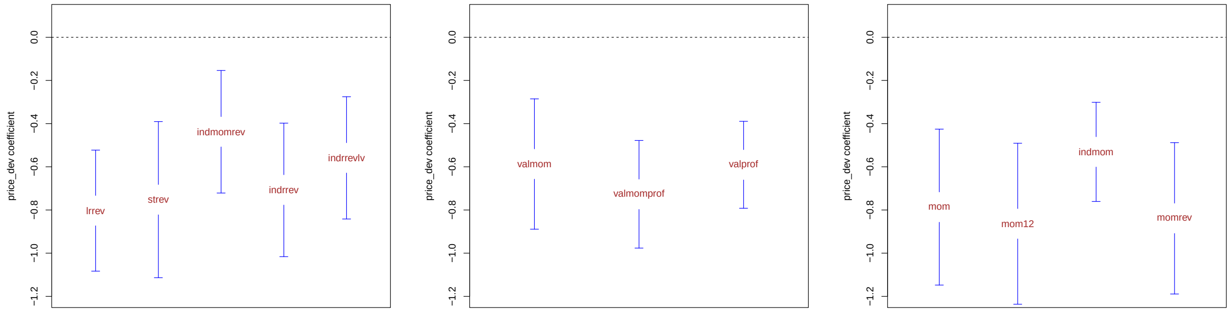
Figure E.2: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10) using daily returns to calculate time-varying betas over an estimation window of one year. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table E.2: Long-Short Anomaly Portfolio Alphas

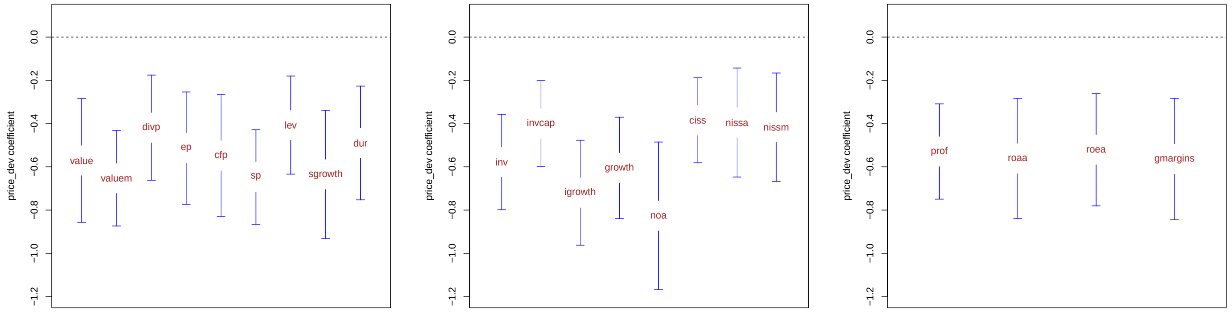
This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10) using daily returns to calculate time-varying betas over an estimation window of one year. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.53^{***}	0.43^{***}	0.40^{***}	0.34^{***}	0.42^{***}	0.52^{***}
	(0.13)	(0.11)	(0.11)	(0.08)	(0.12)	(0.13)
MKT	−0.04	0.00	0.01	0.04^{*}	0.04	−0.07
	(0.03)	(0.02)	(0.04)	(0.03)	(0.04)	(0.06)
SMB	−0.05	0.00		0.04		-0.13^{**}
	(0.04)	(0.05)		(0.07)		(0.05)
HML	0.43^{***}	0.32^{***}				
	(0.09)	(0.05)				
Mom	−0.01	−0.02				
	(0.05)	(0.04)				
RMW		0.17^{**}				
		(0.08)				
CMA		0.18^{*}				
		(0.10)				
ME			−0.02			
			(0.06)			
IA			0.56^{***}			
			(0.13)			
ROE			0.05			
			(0.08)			
Mgmt				0.49^{***}		
				(0.08)		
Perf				−0.02		
				(0.06)		
PEAD					−0.05	
					(0.08)	
FIN					0.29^{***}	
					(0.07)	
BG						0.23^{***}
						(0.05)
Observations	384	384	384	384	384	384
Adjusted R^2	0.44	0.47	0.30	0.43	0.30	0.22

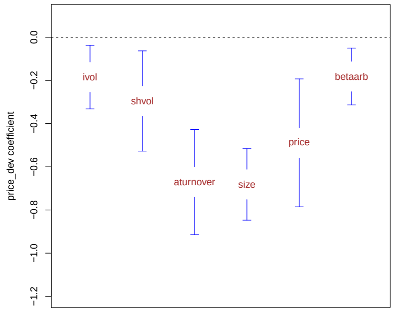
E.3 Nonparametric Conditional Betas (Ang and Kristensen, 2012)



(a) reversal **(b)** value interaction **(c)** momentum



(d) value **(e)** investment **(f)** profitability



(g) trading frictions

Figure E.3: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 45 top anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10) using the methodology proposed by Ang and Kristensen (2012) to calculate time-varying parameters. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table E.3: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10) using the methodology proposed by Ang and Kristensen (2012) to calculate time-varying parameters. We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.61^{***}	0.57^{***}	0.58^{***}	0.51^{***}	0.50^{***}	0.73^{***}
	(0.09)	(0.09)	(0.11)	(0.09)	(0.10)	(0.11)
MKT	-0.14^{***}	-0.12^{***}	-0.14^{***}	-0.09^{***}	-0.10^{***}	-0.20^{***}
	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)
SMB	0.02	0.05		0.09^{**}		0.01
	(0.04)	(0.04)		(0.04)		(0.05)
HML	0.24^{***}	0.19^{***}				
	(0.04)	(0.05)				
Mom	0.14^{***}	0.14^{***}				
	(0.02)	(0.02)				
RMW		0.08				
		(0.05)				
CMA		0.07				
		(0.05)				
ME			0.07			
			(0.04)			
IA			0.24^{***}			
			(0.07)			
ROE			0.14^{***}			
			(0.05)			
Mgmt				0.28^{***}		
				(0.05)		
Perf				0.11^{***}		
				(0.03)		
PEAD					0.17^{***}	
					(0.06)	
FIN					0.18^{***}	
					(0.06)	
BG						0.03
						(0.03)
Observations	384	384	384	384	384	384
Adjusted R^2	0.43	0.44	0.31	0.37	0.36	0.22

F Alternative Factor Models: Robustness

F.1 Predictive regressions

Table F.1: Pooled Regressions for Alternative Factor Models

This table reports pooled estimates for δ_i from predictive regression (14). Test assets are the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (10). We report results for different heuristic mean-variance efficient portfolios. Panel A reports results for the Hou, Xue, and Zhang (2015, HXZ) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in Haddad, Kozak, and Santosh (2020). Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: HXZ

	(1)	(2)	(3)	(4)	(5)
δ	-0.347^{***}	-0.300^{***}	-0.352^{***}	-0.332^{***}	-0.340^{***}
	(0.049)	(0.052)	(0.046)	(0.046)	(0.046)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.159	0.185	0.159	0.167	0.183

Panel B: Volatility-managed HXZ

	(1)	(2)	(3)	(4)	(5)
δ	-0.377^{***}	-0.316^{***}	-0.405^{***}	-0.371^{***}	-0.365^{***}
	(0.065)	(0.066)	(0.063)	(0.063)	(0.059)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.176	0.226	0.181	0.198	0.227

Panel C: PCA

	(1)	(2)	(3)	(4)	(5)
δ	-0.261^{***}	-0.207^{***}	-0.268^{***}	-0.247^{***}	-0.242^{***}
	(0.044)	(0.050)	(0.045)	(0.043)	(0.041)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment
Adjusted R^2	0.115	0.144	0.115	0.124	0.152

F.2 Out-Of-Sample R^2

Table F.2: Out-of-Sample Predictability for Alternative Factor Models

This table reports the out-of-sample R^2 (R^2_{OOS}) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return and price deviations $\hat{u}$ are computed as in equation (10). Test assets are the long legs for the 45 anomalies constructed in Kozak, Nagel, and Santosh (2020). See Appendix Table C.1 for a description of the anomalies. We report results for different heuristic mean-variance efficient portfolios. Panel A reports results for the Hou, Xue, and Zhang (2015, HXZ) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in Kelly, Kozak, and Giglio (2020). The R^2_{OOS} is computed as in Campbell and Thompson (2008); *p*-values for R^2_{OOS} are computed as in Clark and West (2007). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. Monthly observations of annual returns.

Panel A: HXZ

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	14.71^{***}	indmom	11.64^{***}	price	-2.61
age	10.88^{***}	indmomrev	1.96^{***}	prof	3.17^{***}
aturnover	10.12^{***}	indrrev	18.07^{***}	roaa	-3.22
betaarb	12.37^{***}	indrrevlv	-7.58	roea	-3.4
cfp	16.86^{***}	inv	27.41^{***}	season	5.49^{***}
ciss	18.04^{***}	invcap	15.59^{***}	sgrowth	29.52^{***}
divg	18.59^{***}	ivol	2.59^{***}	shvol	15.42^{***}
divp	21.39^{***}	lev	4.26^{***}	size	12.32^{***}
dur	3.01^{***}	lrrev	20.48^{***}	sp	32.9^{***}
ep	21.87^{***}	mom	11.89^{***}	strev	17.54^{***}
exchsw	9.17^{***}	mom12	17.12^{***}	valmom	11.31^{***}
fscore	9.67^{***}	momrev	19.18^{***}	valmomprof	10.83^{***}
gmargins	-3.67	nissa	6.63^{***}	valprof	32.43^{***}
growth	24.42^{***}	nissm	16.87^{***}	value	24.63^{***}
igrowth	20.3^{***}	noa	-1.29	valuem	6.74^{***}

Panel B: Volatility-managed HXZ

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	-1.78	indmom	18.46***	price	0.75***
age	8.06***	indmomrev	-0.6	prof	15.01***
aturnover	17.93***	indrrev	6.16***	roaa	4.33***
betaarb	3.15***	indrrevlv	-19.48	roea	6.09***
cfp	23.58***	inv	19.8***	season	3.03***
ciss	18.64***	invcap	3.96***	sgrowth	25.54***
divg	19.11***	ivol	-2.24	shvol	-0.75
divp	19.83***	lev	11.16***	size	19.7***
dur	6.17***	lrrev	23.44***	sp	29.72***
ep	15.09***	mom	22.99***	strev	15.72***
exchsw	12.82***	mom12	21.83***	valmom	5.54***
fscore	14.66***	momrev	30.72***	valmomprof	17***
gmargins	-1.85	nissa	9.77***	valprof	23.06***
growth	22.26***	nissm	14.98***	value	21.95***
igrowth	25.88***	noa	3.58***	valuem	25.64***

Panel C: PCA

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	9.93***	indmom	6.71***	price	-6.26
age	12.02***	indmomrev	1.43***	prof	-13.45
aturnover	-19.47	indrrev	7.60***	roaa	-11.37
betaarb	-1.27	indrrevlv	1.48***	roea	-17.9
cfp	18.92***	inv	17.85***	season	-4.2
ciss	4.77***	invcap	5.43***	sgrowth	19.30***
divg	5.46***	ivol	-6.66	shvol	3.42***
divp	12.95***	lev	-3.95	size	10.27***
dur	8.8***	lrrev	9.97***	sp	19.42***
ep	14.96***	mom	10.16***	strev	7.91***
exchsw	-3.09	mom12	10.55***	valmom	18.42***
fscore	-3.68	momrev	9.05***	valmomprof	16.55***
gmargins	-11.36	nissa	3.24***	valprof	23.20***
growth	16.50***	nissm	4.97***	value	18.72***
igrowth	18.41***	noa	-8.28	valuem	-8.96

F.3 Anomaly Rotation using Price Deviations

Table F.3: Long-Short Anomaly Portfolio Alphas for Alternative Factor Models

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10). We report results for different heuristic mean-variance efficient portfolios. Panel A reports results for the Hou, Xue, and Zhang (2015, HXZ) factor model, Panel B reports results for its volatility timed version, and Panel C reports results for the principal component model employed in Kelly, Kozak, and Giglio (2020). We control for the following factor models: Carhart (1997) (C4), Fama and French (2018) (FF6), Hou, Xue, and Zhang (2015) (q), Stambaugh and Yuan (2016) (SY4), Daniel, Hirshleifer, and Sun (2020) (DHS3), Bartram and Grinblatt (2018) (BG3). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: HXZ

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.36***	0.29***	0.25***	0.24***	0.32***	0.29***
	(0.09)	(0.08)	(0.09)	(0.07)	(0.11)	(0.11)
Adjusted R^2	0.54	0.57	0.25	0.46	0.31	0.28

Panel B: Volatility-managed HXZ

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.29***	0.25***	0.19**	0.24***	0.27***	0.21*
	(0.09)	(0.07)	(0.09)	(0.08)	(0.10)	(0.11)
Adjusted R^2	0.41	0.41	0.14	0.27	0.19	0.17

Panel C: PC6

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.47***	0.33***	0.33***	0.25***	0.38***	0.57***
	(0.10)	(0.11)	(0.10)	(0.09)	(0.11)	(0.15)
Adjusted R^2	0.54	0.61	0.43	0.53	0.37	0.19

G Alternative Test Assets: Robustness

G.1 25 Fama-French Portfolios Sorted on Size and Book-to-Market

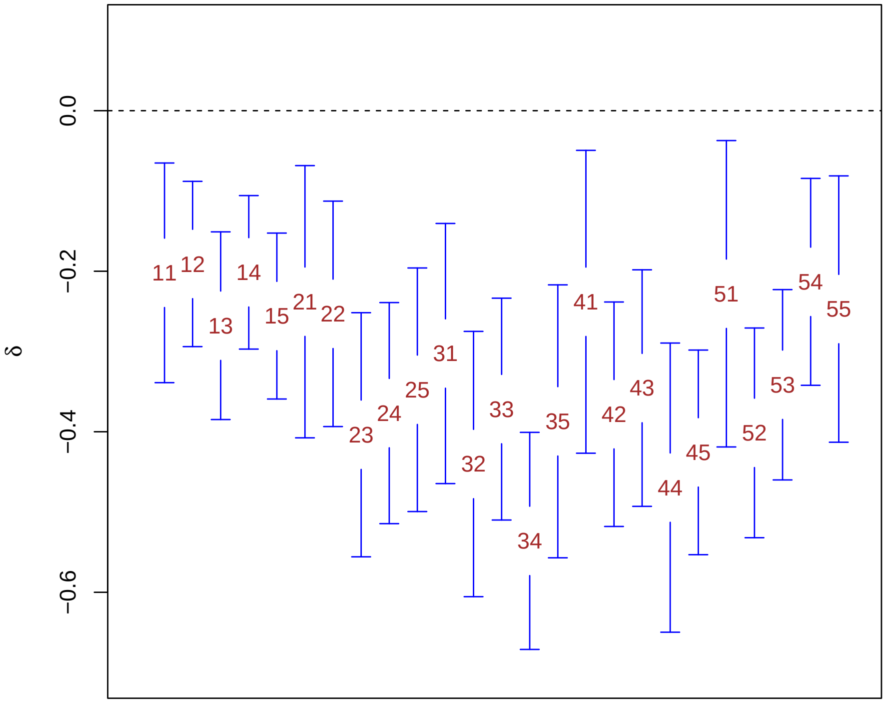


Figure G.1: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (13) with respective confidence intervals at 5% level of significance. Test assets are the 25 Fama-French portfolios sorted on size and B/M. Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using the Fama and French (2015, FF5) factor model. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

Table G.1: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 90 top and bottom anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) using a zero-cost rank-based strategy. Price deviations $\hat{u}$ are computed as in equation (10). We report results for price deviations computed using the Fama and French (2015) factor model to calculate the mean-variance efficient portfolio. Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.33^{***}	0.23^{**}	0.17	0.25^{***}	0.26^{**}	0.36^{**}
	(0.09)	(0.10)	(0.14)	(0.10)	(0.12)	(0.15)
MKT	-0.08^{***}	-0.03	-0.02	-0.05	0.01	-0.14^{***}
	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)
SMB	0.06	0.09		0.12		-0.05
	(0.06)	(0.07)		(0.09)		(0.09)
HML	0.63^{***}	0.50^{***}				
	(0.05)	(0.05)				
Mom	0.04	0.03				
	(0.03)	(0.04)				
RMW		0.14^{***}				
		(0.05)				
CMA		0.19^{***}				
		(0.07)				
ME			0.09			
			(0.06)			
IA			0.74^{***}			
			(0.09)			
ROE			0.10			
			(0.07)			
Mgmt				0.51^{***}		
				(0.05)		
Perf				-0.12^{**}		
				(0.05)		
PEAD					-0.08	
					(0.07)	
FIN					0.33^{***}	
					(0.05)	
BG						0.28^{***}
						(0.04)
Observations	384	384	384	384	384	384
Adjusted R^2	0.59	0.61	0.39	0.42	0.32	0.24

G.2 Evidence from Chen and Zimmermann Open-Source Library

Table G.2: Long-Short Anomaly Portfolio Alphas

This table reports factor exposures and alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. Once per year, we sort the 252 top and bottom anomaly portfolios constructed in Chen and Zimmermann (2021) using a zero-cost rank-based strategy. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. We control for: Carhart (1997) (C4), Fama and French (2018) (FF6), Hou, Xue, and Zhang (2015) (q), Stambaugh and Yuan (2016) (SY4), Daniel, Hirshleifer, and Sun (2020) (DHS3), Bartram and Grinblatt (2018) (BG3). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.58***	0.44***	0.45***	0.39***	0.45***	0.70***
	(0.10)	(0.08)	(0.08)	(0.07)	(0.08)	(0.11)
Adjusted R^2	0.62	0.70	0.58	0.65	0.55	0.32

Panel B: FF5 with factor-timing

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.66***	0.55***	0.57***	0.48***	0.50***	0.79***
	(0.10)	(0.09)	(0.09)	(0.07)	(0.08)	(0.10)
Adjusted R^2	0.60	0.66	0.51	0.60	0.51	0.29

Panel C: Volatility-managed FF5

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.48***	0.41***	0.35***	0.37***	0.35***	0.52***
	(0.09)	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)
Adjusted R^2	0.48	0.52	0.41	0.45	0.44	0.30

Panel D: FF5-DMRS

	C4	FF6	q	SY4	DHS3	BG3
Constant	0.58***	0.45***	0.43***	0.37***	0.42***	0.68***
	(0.09)	(0.08)	(0.08)	(0.07)	(0.09)	(0.12)
Adjusted R^2	0.63	0.70	0.55	0.67	0.55	0.30

H A Model of Slow Adjustment to Information

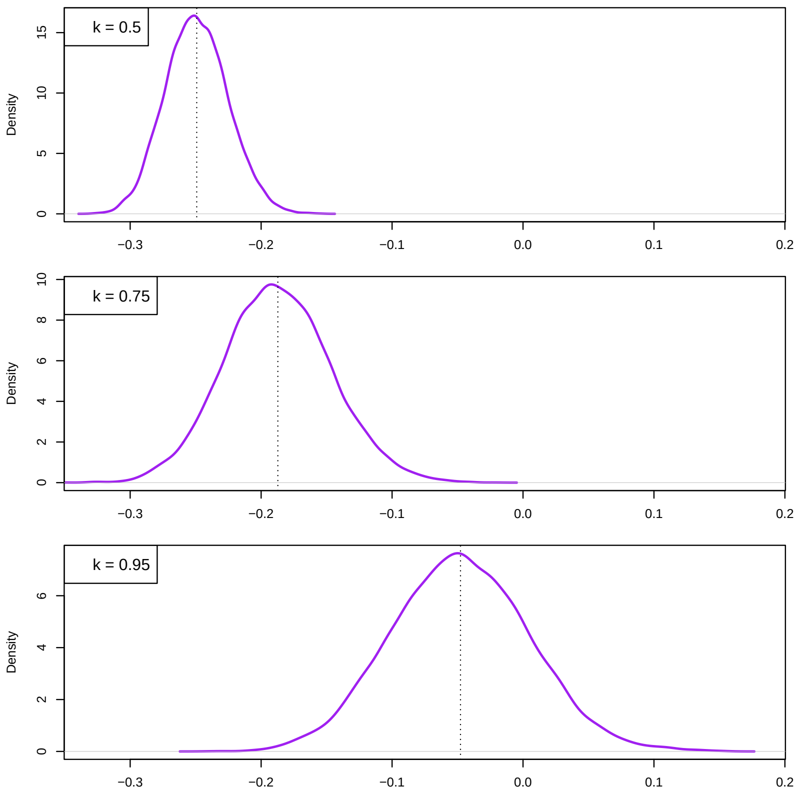


Figure H.1: Predictability Using Price Deviations and Slow Adjustment to Information. This figure shows ex-post densities for δ coefficients in specification (13) for different calibrations of the adjustment parameter k in equation (17). We calibrate r_t^V to the CMVE portfolio log return over the period 1967–2019, with an annualized (percentage) mean of 1.23% and an annualized volatility of 1.12%. Prices are constructed as $\ln V_{t+1} = \ln V_t + r_{t+1}^V$. We then simulate 10000 times a sample of 636 observations of $\ln P_{t+1}$ using equation (17). The case $k = 1$ is full price adjustment to information.