Misallocation and Asset Prices

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Abstract

We develop an endogenous growth model with heterogeneous firms facing financial frictions, in which misallocation emerges explicitly as a crucial state variable. The model demonstrates that macroeconomic shocks that affect misallocation can generate persistent effects on aggregate growth. In equilibrium, the slow-moving misallocation endogenously generates long-run uncertainty about economic growth by distorting innovation decisions. When agents have recursive preferences, the misallocation-driven low-frequency growth fluctuations lead to significant welfare losses and risk premia in capital markets. Using an empirical misallocation measure motivated by the model, we provide evidence that misallocation captures low-frequency fluctuations in both aggregate growth and stock returns.

Keywords: Distribution of firms, Financial frictions, Misallocation, Endogenous growth, Welfare cost of business cycles.
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1 Introduction

Misallocation plays an important role in helping us understand economic growth, both during economic transitions (e.g., Buera and Shin, 2013, 2017) and in the long-run steady states (e.g., Acemoglu et al., 2018; Peters, 2020). Various firm-level dispersion measures indicate that the allocation efficiency of capital seems to exhibit cyclical properties (e.g., Eisfeldt and Rampini, 2006; Bloom, 2009; Kehrig, 2015; Bloom et al., 2018). The link between misallocation and growth prospects can potentially shed light on the fundamental forces that drive low-frequency growth fluctuations, a mechanism that quantitatively rationalizes many asset pricing moments (e.g., Bansal and Yaron, 2004; Hansen, Heaton and Li, 2008) and justifies the large welfare cost of business cycles (e.g., Barlevy, 2004).

This paper develops an analytically tractable general-equilibrium model with endogenous stochastic growth to quantitatively investigate the connection between misallocation and the systematic risk that shapes asset prices in capital markets. As the key insight of this paper, our model demonstrates that the slow-moving misallocation drives endogenous low-frequency growth fluctuations as a primitive source of systematic risk. When agents have recursive preferences, the misallocation-driven low-frequency growth fluctuations can have first-order asset pricing implications in capital markets and lead to substantial welfare losses.

Our model builds on the framework of Moll (2014) in which there is endogenous misallocation across firms producing final goods due to financial frictions. We introduce two additional ingredients. First, in addition to the final goods sector, we introduce intermediate goods and R&D sectors. By expanding the variety of intermediate goods used in production, R&D generates technological advances and thus endogenous growth (Romer, 1990). Second, there are aggregate shocks that drive slow-moving misallocation.

The novel combination of these two ingredients allows the model to generate endogenous low-frequency growth fluctuations. The mechanism is as follows. A higher misallocation in production capital reduces the aggregate demand for intermediate goods and thus the profits of innovation. This reduces the R&D sector's incentive to innovate, leading to a lower equilibrium growth rate.[1] As misallocation is endogenously persistent and slow moving, aggregate shocks that affect misallocation could in turn drive

[1]Our theoretical mechanism that links misallocation and growth is similar to that in the model of Peters (2020), where firms' innovation rates are negatively affected by misallocation (in production labor) through the aggregate demand for their outputs. This mechanism is different from that of Acemoglu et al. (2018), who emphasize the role of misallocation in R&D inputs (skilled labor), rather than production inputs, in determining equilibrium growth.

low-frequency growth fluctuations.

Our model economy has three sectors. The innovation sector uses final goods and existing stock of knowledge to produce new knowledge. The intermediate goods sector uses the blueprints from the innovation sector together with final goods to produce differentiated intermediate goods. The final goods sector uses capital, labor, and intermediate goods to produce final goods. There exists a representative agent that owns firms in all sectors, a continuum of heterogeneous firms in the final goods sector and homogeneous firms in intermediate goods and innovation sectors.

Firms in the final goods sector are different in productivity and capital. Because of agency conflicts, firms face an equity market constraint for payout and issuance and a collateral constraint for debt. The collateral constraint generates capital misallocation among firms. A higher misallocation results in a lower productivity in the final goods sector, which reduces the aggregate demand for intermediate goods. This, in turn, motivates innovators to invent new intermediate goods less intensively, leading to a lower growth rate.

Firms endogenously choose their capacity utilization intensity. A higher intensity allows firms to produce more outputs at the cost of bearing a higher depreciation rate of capital. The only aggregate shock is the capital depreciation shock, as in Storesletten, Telmer and Yaron (2007), Gourio (2012), and Brunnermeier and Sannikov (2017), etc. In equilibrium, because firms with higher productivity use their capital more intensively, they are also more exposed to aggregate capital depreciation shocks than firms with lower productivity. Thus, the aggregate shocks generate fluctuations in the economy's misallocation, which in turn result in fluctuations in the economy's growth rate.

Our model is a general equilibrium model with heterogeneous firms and aggregate uncertainty. The standard way to solve this type of models is to do numerical approximations based on a few moments that summarize the cross-sectional distribution of firms (e.g., Krusell and Smith, 1998). Instead of following this route, we directly propose a parametric approximation for the distribution of log productivity and log capital using a bivariate normal distribution. We justify its validity using the Berry-Esseen bound (Tikhomirov, 1980; Bentkus, Gotze and Tikhomoirov, 1997) under certain conditions. By characterizing the distribution of firms using a few moments, our parametric approximation is in spirit similar to standard methods of numerical approximations. We discuss the connections in Online Appendix 2.2. To show robustness, we also compare our results with the results solved by numerical approximations (see Online Appendix 2).

The proposed parametric approximation provides two major benefits. First, it allows us to derive a closed-form expression for the misallocation of capital in the final goods sector, which emerges as a crucial and explicit endogenous state variable that summarizes the cross-sectional distribution of firms. Specifically, misallocation in our model can be characterized by the covariance between log marginal revenue product of capital (MRPK) and log capital, normalized by the variance of log MRPK. This covariance-type measure of misallocation is intuitive, and similar measures have been implemented in empirical studies to gauge capital allocation efficiency (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013). Second, it makes the model highly tractable and transparent. The evolution of the model economy can be analytically characterized by the evolution of two endogenous state variables, misalloation and the knowledge stock-capital ratio. This enables us to elucidate the relationship between the dynamics of misallocation and the dynamics of aggregate growth.

To illustrate the key theoretical mechanism, we first focus on the balanced growth in the absence of aggregate shocks. We show that a one-time shock that increases misallocation can have an overly persistent effect on economic growth. Specifically, due to financial frictions, the reallocation of capital across firms takes time. Thus, the one-time shock not only increases contemporaneous misallocation, but also future misallocation. In other words, misallocation is slow moving and persistent. Because misallocation determines economic growth through R&D incentive, the persistently high misallocation further generates a persistently low economic growth. Taken together, a one-time (temporary) shock brings a persistent effect on economic growth. We further show that the persistence of misallocation and growth is closely related to the persistence of firms' idiosyncratic productivity. This complements the key insight of Moll (2014) that when firms' idiosyncratic productivity becomes more persistent, it takes a longer time for the economy to reach the steady state. For us, the persistence of idiosyncratic productivity determines the persistence of aggregate growth through its effect on the persistence of endogenous misallocation.

Building on this mechanism, we show that in the full model with aggregate shocks, misallocation is slow moving, thereby generating low-frequency growth fluctuations. Quantitatively, the yearly autocorrelation of misallocation is 0.73 and that of consumption growth is 0.46, both of which are similar to what we measure in the data. Thus, our model demonstrates a novel channel through which business cycle fluctuations are endogenously associated with low-frequency growth fluctuations. This provides a misallocation-based explanation for the observed low-frequency covariation in the time series of consumption growth and output growth (e.g., Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Müller and Watson, 2008, 2018).

We further show that the model is able to rationalize several important asset pricing moments and imply large welfare costs of business cycles. Specifically, our model implies a high Sharpe ratio of 0.39 and a smooth risk-free rate, as in the data. The representative agent would experience a welfare gain of about 10% if consumption fluctuations are eliminated. The large quantitative effects generated by misallocation fluctuations hinge on two properties of the model, the low-frequency growth fluctuations driven by the slow-moving misallocation and the recursive preference of the representative agent. We show that if misallocation does not affect economic growth or if it is not sufficiently slow moving to generate low-frequency growth fluctuations, both of the quantified Sharpe ratio and welfare gain would be very small. Moreover, if the representative agent has a standard (non-recursive) preference with constant relative risk aversion (CRRA), these two values would also be very small. Intuitively, the recursive preference ensures that the representative agent's marginal utility today is affected not only by news about contemptuous consumption growth, but also, crucially, by news about future consumption growth. Thus, innovations in future consumption growth can generate valuation effects through the pricing kernel. In addition, because consumption growth is persistent, a temporary innovation in consumption growth would generate a persistent effect in future, which significantly amplifies the impacts of future consumption growth on the marginal utility today.

While our main contribution is theoretical, we also empirically test the main predictions of our model. Motivated by our theory, we construct a misallocation measure based on the covariance between log MRPK and log capital using the U.S. Compustat data. We show that the misallocation measure is persistent, with a yearly autocorrelation of 0.75. Moreover, the value of our empirical measure of misallocation increases during economic downturns. We further show that an increase in misallocation predicts declines in R&D intensity and lower growth of aggregate consumption and output over long horizons.

Although our model does not focus on cross-sectional asset pricing implications, we provide evidence in the cross section to support the main theoretical mechanism. First, we show that, as a macroeconomic factor, the empirical misallocation measure has significant cross-sectional asset pricing implications. A two-factor model with market and misallocation factors prices size, book-to-market, momentum, and bond portfolios with a mean absolute pricing error (MAPE) of 1.46, which is even lower than that implied by the Fama-French three-factor model. Importantly, future accumulated consumption growth, as a proxy for the low-frequency component of consumption growth that is shown to contain important asset pricing information, would have little importance in explaining asset returns once our misallocation measure is taken into account as a factor. Second, we find that the cash flows of value firms load more negatively on misallocation than the cash flows of growth firms. This is consistent with the robust evidence found in the asset pricing literature that the cash flows of value firms load more positively on accumulated consumption growth than those of growth firms (Bansal, Dittmar and Lundblad, 2005; Parker and Julliard, 2005; Hansen, Heaton and Li, 2008; Santos and Veronesi, 2010), providing further support for our model's key mechanism that fluctuations in misallocation drive the low-frequency growth fluctuations.

Finally, we provide direct evidence to support the model's mechanism that misallocation drives long-run growth through its impact on R&D. We consider the policy shock from the American Jobs Creation Act (AJCA) passed in 2004, which presumably relaxes the financial constraints of firms with pre-tax income from abroad. By exploiting industries' differential exposure to this policy shock in a difference-in-differences (DID) setting, we find that AJCA results in significantly lower industry-level misallocation and higher R&D expenditure in treated industries. Moreover, the impact of AJCA on industry-level R&D expenditure becomes statistically insignificant after controlling for industry-level misallocation.

Related Literature. Our paper is related to several strands of the literature. First, we contribute to the asset pricing literature. Various theoretical studies provide micro foundations to justify low-frequency growth fluctuations (e.g., Ai, 2010; Kaltenbrunner and Lochstoer, 2010; Garleanu, Panageas and Yu, 2012; Croce, 2014; Kung and Schmid, 2015; Collin-Dufresne, Johannes and Lochstoer, 2016; Ai, Li and Yang, 2020; Gârleanu and Panageas, 2020; Croce, Nguyen and Raymond, 2021). Our paper is mostly related to Kung and Schmid (2015) who show that R&D endogenously drives a small, persistent component in productivity, which generates long-run uncertainty about economic growth. Our model's main departure from their model is that the aggregate TFP is determined by cross-sectional misallocation whereas it is exogenous in their model. This difference allows our theory to rationalize low-frequency growth fluctuations through the equilibrium interactions between endogenous slow-moving misallocation and R&D incentives, a mechanism supported by the data.

Second, our paper contributes to the large and growing literature that emphasizes the role of misallocation in economic development (e.g., Banerjee and Duflo, 2005; Foster, Haltiwanger and Syverson, 2008; Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Jones, 2011, 2013; Bartelsman, Haltiwanger and Scarpetta, 2013; Asker, Collard-Wexler and Loecker, 2014*a*; König et al., 2022; Edmond, Midrigan and Xu, 2023; Glode and Ordonez, 2023). Our paper is particularly related to the literature on financial frictions and misallocation. The bulk of this literature focuses on the long-run TFP and welfare losses of misallocation in the deterministic stationary equilibrium (e.g., Amaral and Quintin, 2010; Greenwood, Sanchez and Wang, 2010, 2013; Caselli and Gennaioli, 2013; Midrigan and Xu, 2014; Buera, Kaboski and Shin, 2015) whereas a few papers also analyze transitional dynamics (e.g., Jeong and Townsend, 2007; Buera and Shin, 2013; Moll, 2014; Gopinath et al., 2017).[2] Our paper develops a stochastic growth model in which the slow-moving misallocation endogenously drives low-frequency growth cycles. On the technical side, our model extends the tractable framework of Moll (2014) with intermediate inputs, R&D, and aggregate shocks to generate endogenous stochastic growth. Our paper provides the following insights to the literature. First, the misallocation in the final goods sector can affect economic growth because it determines the profits of R&D through the aggregate demand. This mechanism is similar to Peters (2020), but for us, misallocation arises from financial frictions rather than markup dispersions. Second, when agents have recursive preferences, the misallocation caused by financial frictions can generate large risk premia and welfare losses through endogenous low-frequency growth fluctuations. Third, the persistence of firm-level idiosyncratic productivity plays an important role in generating the slow-moving misallocation, which in turn, generates low-frequency growth fluctuations.[3] Our results complement the key insight of Moll (2014), who shows that as idiosyncratic productivity becomes more persistent, the transition speed from a distorted initial state to the steady state is slower, resulting in potentially larger welfare losses during transitions.

[2] Several papers measure the importance of financing costs in generating misallocation. For example, Gilchrist, Sim and Zakrajsek (2013) find a limited role of the costs of debt based on a sample that consists of about 500 firms issuing corporate bonds, most of which are large firms. David, Schmid and Zeke (2022) find a more important role played by the costs of equity in generating misallocation. Whited and Zhao (2021) find significant variation in the costs of debt and equity across U.S. firms.

[3] By connecting the persistence in idiosyncratic productivity with the persistence in aggregate consumption growth, our model implies that low-frequency growth fluctuations can be identified using granular firm-level data, which potentially helps address the issues of weak identification in the asset pricing literature (Chen, Dou and Kogan, 2022; Cheng, Dou and Liao, 2022).

Relative to the macro-development literature, much less work focuses on the role of misallocation in the finance literature.[4] Our paper is most related to David, Schmid and Zeke (2022), who analyze the implications of macroeconomic risk for misallocation. Our paper complements their work by analyzing the implications of misallocation for macroeconomic risk. Specifically, our model demonstrates that the economy's misallocation itself can be a macroeconomic risk factor as it enters the investors' pricing kernel by affecting consumption growth, whereas their model takes an exogenous pricing kernel to analyze its impacts on the economy's misallocation.

Finally, our paper is related to the literature on the welfare cost of business cycles (e.g., Lucas, 1987). Barlevy (2004) shows that business cycles can generate large welfare losses by affecting the growth rate of consumption through capital investment. Alvarez and Jermann (2004) estimate a large welfare gain from eliminating consumption uncertainty using asset prices. Through the slow-moving misallocation, our model rationalizes low-frequency growth fluctuations, implying a large welfare cost of business cycles when the representative agent has recursive preferences.

2 Model

There are three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. There is a representative agent that owns firms in all sectors, a continuum of heterogeneous firms in the final goods sector and homogeneous firms in the intermediate goods and R&D sectors.

2.1 Final Goods Sector

In the final goods sector, there is a continuum of firms of measure one, indexed by $i \in \mathcal{I} \equiv [0, 1]$ and operated by managers. Firms are different from each other in their idiosyncratic productivity $z_{i,t}$ and capital $a_{i,t}$. At each point in time t, the distribution of firms is characterized by the joint probability density function (PDF), $\varphi_t(a, z)$.

The firm produces output at intensity $y_{i,t}$ over $[t, t + \mathrm{d}t)$ using a production technology

[4] E.g., Eisfeldt and Rampini (2006, 2008*b*), Rampini and Viswanathan (2010), Opp, Parlour and Walden (2014), Fuchs, Green and Papanikolaou (2016), Ehouarne, Kuehn and Schreindorfer (2017), van Binsbergen and Opp (2019), Ai, Li and Yang (2020), Ai et al. (2020), and Lanteri and Rampini (2021).

with constant returns to scale (CRS):

$$y_{i,t} = \left[(z_{i,t}u_{i,t}k_{i,t})^{\alpha}\ell_{i,t}^{1-\alpha}\right]^{1-\varepsilon} x_{i,t}^{\varepsilon}, \quad \text{with } \alpha,\ \varepsilon \in (0,1), \tag{1}$$

where labor $\ell_{i,t}$ is hired in a competitive labor market at the equilibrium wage w_t. The variable $k_{i,t} = a_{i,t} + \widehat{a}_{i,t}$ is the capital installed in production, which includes the firm's own capital $a_{i,t}$ and the leased capital $\widehat{a}_{i,t}$ borrowed from a competitive rental market at the equilibrium risk-free rate $r_{f,t}$.[5] The final goods are the numeraire.

The firm's output increases with its idiosyncratic productivity $z_{i,t}$ and endogenous choice of capacity utilization intensity $u_{i,t} \in [0,1]$. Utilizing capital at intensity $u_{i,t}$ leads to an amount of $u_{i,t}k_{i,t}\mathrm{d}\Delta_t$ depreciation over $[t, t + \mathrm{d}t)$, where $\mathrm{d}\Delta_t$ captures the stochastic depreciation rate,

$$\mathrm{d}\Delta_t = \delta_k \mathrm{d}t + \sigma_k \mathrm{d}W_t. \tag{2}$$

The standard Brownian motion W_t captures the aggregate capital depreciation shock (e.g., Storesletten, Telmer and Yaron, 2007; Albuquerue and Wang, 2008; Gourio, 2012). The parameters $\delta_k, \sigma_k > 0$ capture the constant and stochastic components of capital depreciation. As we show in Lemma 1 below, in equilibrium, more productive firms utilize capital more intensively by optimally choosing larger $u_{i,t}$. As a result, more productive firms are more exposed to the aggregate depreciation shock $\mathrm{d}W_t$ than less productive firms.[6]

We assume that the firm's own capital stock evolves according to

$$\mathrm{d}a_{i,t} = -\delta_a a_{i,t}\mathrm{d}t + \sigma_{a,t}a_{i,t}\mathrm{d}W_t + \mathrm{d}I_{i,t}, \tag{3}$$

where $\delta_a > 0$ is the constant depreciation rate, and $\sigma_{a,t}a_{i,t}\mathrm{d}W_t$ captures improvements in capital efficiency. We assume that a single aggregate shock $\mathrm{d}W_t$ enters both equations (2) and (3) mainly for tractability. This implies that improvement in the efficiency of new capital is associated with depreciation of existing capital, capturing the displacement effect of new capital (e.g., Gârleanu, Kogan and Panageas, 2012; Kogan et al., 2017; Kogan, Papanikolaou and Stoffman, 2020).[7] The variable $\mathrm{d}I_{i,t}$ in equation (3) is the amount of

[5] The capital leasing market is relevant for firms' production and financial decisions (e.g., Eisfeldt and Rampini, 2008*a*; Rampini and Viswanathan, 2013; Li and Tsou, 2021).

[6] The only role of utilization intensity $u_{i,t}$ is to endogenously generate differential exposure to the aggregate shocks in the cross section of firms, so that $\mathrm{d}W_t$ will have an impact on the economy's misallocation. The same results can be obtained if we exogenously specify the aggregate risk exposure across firms.

[7] The modeling of capital efficiency shocks has been widely adopted in the literature, e.g., Sundaresan

final goods that is converted to capital over $[t, t + \mathrm{d}t)$. Similar to Pástor and Veronesi (2012), we assume that profits are reinvested, so that the firm's investment rate $\mathrm{d}I_{i,t}$ is equal to its profit after paying operation expenses, interests, and dividends (see equation (19) below). As we show in Section 3.2, the aggregate shock $\mathrm{d}W_t$ generates time variation in misallocation of production capital in the final-goods sector, enabling us to study the asset pricing implications of misallocation.[8]

The composite $x_{i,t}$ in equation (1) consists of differentiated intermediate goods, given by the constant elasticity of substitution (CES) aggregation:

$$x_{i,t} = \left(\int_0^{N_t} x_{i,j,t}^{\nu} \mathrm{d}j \right)^{\frac{1}{\nu}}, \tag{4}$$

where $x_{i,j,t}$ is the quantity of intermediate goods $j \in [0, N_t]$. The elasticity of substitution among differentiated intermediate goods is $1/(1-\nu) > 0$. The economy's stock of knowledge (i.e., the variety of differentiated intermediate goods created based on existing blueprints) at t is N_t. Technological advances through the expansion of N_t drive endogenous growth, as in Romer (1987, 1990) and Jones (1995).

The firm's idiosyncratic productivity $z_{i,t}$ evolves according to

$$\mathrm{d}\ln(z_{i,t}) = -\theta \ln(z_{i,t})\mathrm{d}t + \sigma\sqrt{\theta}\mathrm{d}W_{i,t}, \tag{5}$$

where the standard Brownian motion $W_{i,t}$ captures idiosyncratic shocks to firm i's productivity. The specification of the idiosyncratic process $z_{i,t}$ is similar to that of Moll (2014). The parameter θ determines the persistence of idiosyncratic productivity $z_{i,t}$. A higher θ makes $z_{i,t}$ less persistent, implying that firms face higher uncertainty in their future idiosyncratic productivity. Importantly, a change in θ does not affect the dispersion in idiosyncratic productivity across firms, because θ scales both the drift term and the diffusion term in equation (5).

(1984), Cox, Ingersoll and Ross (1985), Kogan (2001, 2004), Gourio (2012), Di Tella (2017), and Dou (2017).

[8] Other aggregate shocks can also generate time variation in misallocation. For example, the aggregate shocks to firms' financial constraints considered by Jermann and Quadrini (2012). However, Jermann and Quadrini (2012) show that financial shocks cannot generate persistent macroeconomic effects unless the shocks themselves are calibrated to be persistent. The aggregate shock $\mathrm{d}W_t$ in our model directly affects firms' capital. Because capital accumulation is a gradual process, even with our i.i.d. shocks, the macroeconomic effects can be persistent.

2.2 Intermediate Goods Sector

There is a continuum of intermediate goods producers, indexed by $j \in [0, N_t]$. Each producer j has monopoly power in setting prices, facing a downward sloping demand for its output. It buys final goods and transform them to intermediate inputs, based on the blueprints created by firms in the R&D sector. We assume that one unit of final goods can be transformed into one unit of intermediate goods, meaning that the marginal cost of producing intermediate goods is unity. Denote by $p_{j,t}$ and p_t the prices of the intermediate good j and the composite of intermediate goods, respectively. The producer of intermediate good j solves

$$\max_{p_{j,t}} \pi_{j,t} = p_{j,t} e_{j,t} - e_{j,t}, \tag{6}$$

subject to the demand curve:

$$e_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{\frac{1}{\nu - 1}} X_t, \tag{7}$$

where $X_t \equiv \int_{i \in \mathcal{I}} x_{i,t} \mathrm{d}i$ is the aggregate demand for the composite of intermediate goods.

The value of a blueprint, denoted by $v_{j,t}$, is the value of owning the exclusive rights to produce intermediate good j, which is given by the Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \Lambda_t \left(\pi_{j,t} - \delta_b v_{j,t} \right) \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t v_{j,t}) \right], \tag{8}$$

where Λ_t is the SDF, and δ_b is the hazard rate at which an existing blueprint becomes obsolete. Because of symmetry and homogeneity, all blueprints have identical values, $v_{j,t} \equiv v_t$, and all intermediate good producers make identical flow profits, $\pi_{j,t} \equiv \pi_t$.

2.3 R&D Sector

Intermediate goods producers are competitive and do not make profits in equilibrium. They buy blueprints from innovators at the price v_t. That is, innovators have full bargaining power and seize all the surplus v_t. Thus, v_t is the value of creating the blueprint, which shapes the incentive to innovate.

Blueprints are created by conducting R&D using final goods. The stock of knowledge N_t evolves as follows:

$$\mathrm{d}N_t = \vartheta_t S_t \mathrm{d}t - \delta_b N_t \mathrm{d}t, \tag{9}$$

where δ_b is the patent obsolescence rate, S_t is the aggregate R&D expenditure, and ϑ_t captures the productivity of innovations, which is taken as exogenously given by individual innovators. In equilibrium, the free-entry condition implies that the marginal return of R&D is equal to its marginal cost:

$$v_t \vartheta_t = 1. \tag{10}$$

Following Comin and Gertler (2006) and Kung and Schmid (2015), we specify

$$\vartheta_t = \chi \left(\frac{N_t}{S_t} \right)^h, \tag{11}$$

where $h \in (0, 1)$. Equation (11) implies that there are positive spillovers of the aggregate stock of knowledge (the term N_t^h) as in Romer (1990) and Jones (1995), and that aggregate R&D investment has decreasing marginal returns (the term S_t^{-h}), capturing the congestion effect in developing new blueprints.

2.4 Agents

There is a continuum of agents, which include workers and managers. As in Dou (2017), only managers decide firms' investments and operations. The managers can be executives, directors, and entrepreneurs; more broadly, they can also be the controlling shareholders who are fully entrenched and have complete control over firms' investment and payout policies (e.g., Albuquerue and Wang, 2008). Each manager manages a firm in the final goods sector subject to agency problems. Workers lend funds to firms and hold equity claims on all firms. We assume that a full set of Arrow-Debreu securities is available to agents, so that idiosyncratic consumption risks can be fully insured and there exists a representative agent.

Preferences. The representative agent has stochastic differential utility as in Duffie and Epstein (1992):

$$U_0 = \mathbb{E}_0 \left[\int_0^\infty f(C_t, U_t) \mathrm{d}t \right], \tag{12}$$

where

$$f(C_t, U_t) = \left(\frac{1-\gamma}{1-\psi^{-1}} \right) U_t \left[\left(\frac{C_t}{[(1-\gamma)U_t]^{1/(1-\gamma)}} \right)^{1-\psi^{-1}} - \delta \right]. \tag{13}$$

This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The felicity function f is an aggregator over the current consumption rate C_t of final goods and future utility level U_t. The coefficient δ is the subjective discount rate, the parameter ψ is the elasticity of intertemporal substitution (EIS), and the parameter γ captures risk aversion.

The representative agent maximizes utility (12) subject to the following budget constraint:

$$\mathrm{d}B_t = \left(w_t L_t + r_{f,t} B_t + D_t - C_t\right) \mathrm{d}t, \tag{14}$$

where $w_t L_t$ is the wage income intensity, D_t is the dividend intensity of all firms, and B_t is the amount of bonds held by the representative agent at t. The aggregate labor supply is inelastic and normalized to be $L_t \equiv 1$.

The representative agent's SDF is

$$\Lambda_t = \exp\left(\int_0^t f_U(C_s, U_s) ds\right) \delta^{\frac{1-\gamma}{1-\psi^{-1}}} R_t^{\frac{\gamma-\psi^{-1}}{1-\psi^{-1}}} C_t^{-\gamma}, \tag{15}$$

where R_t is the consumption-wealth ratio of the representative agent.

Limited Enforcement. An equity market constraint for payout/issuance and a credit market collateral constraint for borrowing endogenously arise from limited enforcement problems of equity and debt contracts.

The manager extracts pecuniary rents $\tau a_{i,t} \mathrm{d}t$ over $[t, t + \mathrm{d}t)$ when running the firm i.[9] These rents represent the cash compensation above the manager's wage (e.g., Myers, 2000; Lambrecht and Myers, 2008, 2012). Shareholders have the option to intervene and take control of the firm by replacing the manager. Intervention is costly because it requires collective action (e.g., Myers, 2000) and can damage the firm's talent-dependent customer capital (e.g., Dou et al., 2020*b*). In particular, we assume that a fraction τ/ρ of capital $a_{i,t}$ is lost upon intervention with $\tau < \rho$, after which shareholders will become the new manager of the firm. In equilibrium, the manager will pay dividend up to the point where shareholders would have no incentive to intervene, implying a payout intensity

[9]Managers can extract rents because corporate governance is imperfect. In practice, it is difficult to verify the cash flows generated by firms' assets, even though cash flows are observable and shareholders' property rights to firm assets are protected. For example, it is difficult to distinguish and verify rents and business expenses. The rents here do not include nonpecuniary private benefits, such as prestige from empire building (Eisfeldt and Rampini, 2008*b*).

policy $d_{i,t} = \rho a_{i,t}$ over $[t, t + \mathrm{d}t)$.[10]

Moreover, the installed capital for production is $k_{i,t} = a_{i,t} + \widehat{a}_{i,t}$, and the manager can divert a fraction $1/\lambda$ of leased capital $\widehat{a}_{i,t}$ with $\lambda \geq 1$. As a punishment, the firm would lose his own capital $a_{i,t}$. In equilibrium, the manager is able to borrow up to the point where the manager has no incentive to divert leased capital, implying a collateral constraint $\widehat{a}_{i,t} \leq \lambda a_{i,t}$. The same form of collateral constraints is widely adopted in the literature (e.g., Jermann and Quadrini, 2012; Buera and Shin, 2013; Moll, 2014).

The financial frictions are summarized in the following proposition.

Proposition 1. *Because of the agency problem with limited enforcement, the firm's payout/issuance policy is subject to the following equity market constraint:*

$$d_{i,t} = \rho a_{i,t}; \tag{16}$$

moreover, the firm's leased capital is subject to the following collateral constraint:

$$-a_{i,t} \leq \widehat{a}_{i,t} \leq \lambda a_{i,t}. \tag{17}$$

Several points are worth further discussions. First, other agency problems can give rise to above equity market and collateral constraints, e.g., Gertler and Kiyotaki (2010); Gertler and Karadi (2011). Second, the equity market constraint is widely studied in the corporate finance literature (e.g., Myers, 2000; Lambrecht and Myers, 2008, 2012). It essentially implies that shareholders cannot freely move funds in and out of firms. Third, our analytically tractable formulation of capital market imperfections captures the fact that external funds available to a firm are limited and costly. Fourth, one specific interpretation of inter-firm borrowing and lending is the existence of a competitive rental market in which firms can rent capital from each other (e.g., Jorgenson, 1963; Hall and Jorgenson, 1969; Buera and Shin, 2013; Rampini and Viswanathan, 2013; Moll, 2014).

Managers' Problem. The manager of firm i makes leasing ($\widehat{a}_{i,s}$) and production ($u_{i,s}, \ell_{i,s}, x_{i,j,s}$) decisions for all $s \geq t$ to maximize the present value $J_{i,t}$ of his own rents as in Lambrecht

[10] Technically, because the firm's dividend intensity is a constant fraction of its capital, the model has linear solutions and tractable aggregation. This property follows the model of Moll (2014) in which the entrepreneur has log utility and CRS technology, and as a result, his consumption intensity is a constant fraction of his wealth.

and Myers (2008, 2012),

$$J_{i,t} = \max_{\widehat{a}_{i,s}, u_{i,s}, \ell_{i,s}, x_{i,j,s}} \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau a_{i,s} \mathrm{d}s \right], \tag{18}$$

subject to the equity market constraint (16), the collateral constraint (17), and the intertemporal budget constraint (3) with $\mathrm{d}I_{i,t}$ given by

$$\mathrm{d}I_{i,t} = y_{i,t}\mathrm{d}t - \int_0^{N_t} p_{j,t} x_{i,j,t} \mathrm{d}j\mathrm{d}t - w_t \ell_{i,t}\mathrm{d}t - u_{i,t} k_{i,t} \mathrm{d}\Delta_t - r_{f,t} \widehat{a}_{i,t}\mathrm{d}t - d_{i,t}\mathrm{d}t, \tag{19}$$

where the SDF Λ_t evolves according to

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = -r_{f,t}\mathrm{d}t - \eta_t \mathrm{d}W_t. \tag{20}$$

The variable η_t is the endogenous market price of risk. Because the technology, budget constraint, and collateral constraint are all linear in $a_{i,t}$, the value $J_{i,t}$ is also linear in $a_{i,t}$ with the following form:

$$J_{i,t} \equiv J_t(a_{i,t}, z_{i,t}) = \xi_t(z_{i,t}) a_{i,t}, \tag{21}$$

where $\xi_{i,t} \equiv \xi_t(z_{i,t})$ captures the marginal value of capital to the manager, which depends on the firm's idiosyncratic productivity $z_{i,t}$ and the aggregate state of the economy. The variable $\xi_{i,t}$ evolves as follows:

$$\frac{\mathrm{d}\xi_{i,t}}{\xi_{i,t}} = \mu_{\xi,t}(z_{i,t})\mathrm{d}t + \sigma_{\xi,t}(z_{i,t})\mathrm{d}W_t + \sigma_{w,t}(z_{i,t})\mathrm{d}W_{i,t}, \tag{22}$$

where $\mu_{\xi,t}(z_{i,t})$, $\sigma_{\xi,t}(z_{i,t})$, and $\sigma_{w,t}(z_{i,t})$ are endogenously determined in equilibrium.

Exploiting the homogeneity of $J_{i,t}$ in capital $a_{i,t}$, we obtain the manager's optimal decisions, summarized in Lemma 1.

Lemma 1. *Factor demands and profits are linear in capital, and there is a productivity cutoff* $\underline{z}_t$

for being active:

$$u_t(z) = \begin{cases} 1, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases}, \qquad k_t(a,z) = \begin{cases} (1+\lambda)a, & z \geq \underline{z}_t \\ 0 & z < \underline{z}_t \end{cases} \tag{23}$$

$$\ell_t(a,z) = \left[\frac{(1-\alpha)(1-\varepsilon)}{w_t}\right]^{\frac{1}{\alpha}} \left(\frac{\varepsilon}{p_t}\right)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} z u_t(z) k_t(a,z), \tag{24}$$

$$x_{j,t}(a,z) = \left(\frac{p_{j,t}}{p_t}\right)^{\frac{1}{\nu-1}} x_t(a,z), \;\; \textit{for all} \;\; j \in [0, N_t], \tag{25}$$

where p_t is the price index and $x_t(a,z)$ is the demand for the composite of intermediate goods,

$$p_t = \left(\int_0^{N_t} p_{j,t}^{\frac{\nu}{\nu-1}} dj\right)^{\frac{\nu-1}{\nu}}, \tag{26}$$

$$x_t(a,z) = \left(\frac{\varepsilon}{p_t}\right)^{\frac{1-(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)}} \left[\frac{(1-\alpha)(1-\varepsilon)}{w_t}\right]^{\frac{1-\alpha}{\alpha}} z u_t(z) k_t(a,z). \tag{27}$$

The productivity cutoff $\underline{z}_t$ is determined by:

$$\underline{z}_t \kappa_t = r_{f,t} + \delta_k + \sigma_k(\sigma_{\xi,t}(\underline{z}_t) - \eta_t). \tag{28}$$

where κ_t is

$$\kappa_t = \alpha(1-\varepsilon)\left(\frac{\varepsilon}{p_t}\right)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} \left[\frac{(1-\alpha)(1-\varepsilon)}{w_t}\right]^{\frac{1-\alpha}{\alpha}}. \tag{29}$$

At any point in time t, only firms with productivity above $\underline{z}_t$ produce, and these firms rent the maximal amount $\widehat{a}_{i,t} = \lambda a_{i,t}$ allowed by the collateral constraint. Equations (25) to (27) are standard results of CES aggregation. The productivity cutoff $\underline{z}_t$ is determined by equation (28), where the marginal return, $\underline{z}_t\kappa_t$, is equal to the marginal cost of leased capital, $r_{f,t} + \delta_k + \sigma_k\left(\sigma_{\xi,t}(\underline{z}_t) - \eta_t\right)$, which includes the locally deterministic user cost of capital and the term $\sigma_k\left(\sigma_{\xi,t}(\underline{z}_t) - \eta_t\right)$, reflecting the firm's exposure to aggregate risk.

Using Lemma 1, equation (19) can be simplified as[11]

$$\mathrm{d}I_{i,t} = (1+\lambda)\left(\kappa_t z_{i,t}\mathrm{d}t - \mathrm{d}\Delta_t - r_{f,t}\mathrm{d}t\right) a_{i,t} \mathbb{1}_{z_{i,t} \geq \underline{z}_t} + (r_{f,t} - \rho) a_{i,t}\mathrm{d}t. \tag{30}$$

[11] As in Moll (2014), the drift term in capital accumulation is proportional to the firm's capital $a_{i,t}$. This is a direct consequence of the constant payout ratio (16) and the constant-returns-to-scale production technology (1) for a fixed N_t.

2.5 Equilibrium and Aggregation

The dividend intensity D_t is given by

$$D_t = \rho A_t + \int_{j=0}^{N_t} \pi_{j,t} \mathrm{d}j - S_t, \tag{31}$$

where A_t is the aggregate capital held by firms in the final goods sector, given by

$$A_t = \int_0^\infty \int_0^\infty a\varphi_t(a,z)\mathrm{d}a\mathrm{d}z. \tag{32}$$

In equation (31), the first term ρA_t captures the dividend of the final goods sector. The second term $\int_{j=0}^{N_t} \pi_{j,t}\mathrm{d}j$ captures the profits from the intermediate goods sector and the third term S_t captures the expenditure on R&D. The aggregate capital K_t in the economy is

$$K_t = \int_0^\infty \int_0^\infty k_t(a,z)\varphi_t(a,z)\mathrm{d}a\mathrm{d}z. \tag{33}$$

Definition 2.1 (Competitive Equilibrium)**.** *At any point in time t, the competitive equilibrium of the economy consists of prices w_t, $r_{f,t}$, and $\{p_{j,t}\}_{j=0}^{N_t}$, and corresponding quantities, such that*

(i) firms in the final goods sector maximize (18) by choosing $\widehat{a}_{i,t}$, $u_{i,t}$, $\ell_{i,t}$, and $x_{i,j,t}$, subject to (16), (17), and (19), given equilibrium prices;

(ii) intermediate goods producers maximize (6) by choosing $p_{j,t}$ for $j \in [0, N_t]$;

(iii) the equilibrium R&D expensiture S_t is determined by equation (10);

(iv) the SDF Λ_t is given by equation (15) and the risk-free rate $r_{f,t}$ is determined by

$$r_{f,t} = -\frac{1}{dt}\mathbb{E}_t\left[\frac{d\Lambda_t}{\Lambda_t}\right]; \tag{34}$$

(v) the labor market-clearing condition determines w_t:

$$L_t = \int_{\underline{z}_t}^\infty \int_0^\infty \ell_t(a,z)\varphi_t(a,z)dadz; \tag{35}$$

(vi) the leased capital market-clearing condition determines the representative agent's bond holdings B_t:

$$B_t = \int_0^\infty \int_0^\infty \widehat{a}_t(a,z)\varphi_t(a,z)dadz. \tag{36}$$

The aggregate capital is the sum of capital in the final goods sector and bonds

$$K_t = A_t + B_t. \tag{37}$$

(vii) the resource constraint is satisfied because of Walras's law (see Online Appendix 3.1).

Because managers' problem is linear in capital $a_{i,t}$ (see equation (21)), it is not necessary to track the marginal distribution of capital conditional on each productivity type z.[12] We thus follow Moll (2014) and introduce the capital share $\omega_t(z)$ to fully characterize the distribution of firms in the final goods sector:

$$\omega_t(z) \equiv \frac{1}{A_t} \int_0^\infty a\varphi_t(a,z)\mathrm{d}a. \tag{38}$$

Intuitively, the capital share $\omega_t(z)$ plays the role of a density, and it captures the share of firms' capital held by each productivity type z. We define the analogue of the corresponding cumulative distribution function (CDF) as

$$\Omega_t(z) \equiv \int_0^z \omega_t(z')\mathrm{d}z'. \tag{39}$$

To ensure well-behaved equilibrium growth, as in standard growth models, we need output Y_t to be homogenous of degree one in the accumulating factors N_t and K_t, i.e., $\frac{(1-\nu)\varepsilon}{\nu(1-\varepsilon)} + \alpha = 1$, as in Kung and Schmid (2015). For the rest of the paper, we assume this parameter restriction.

Proposition 2. *At any point in time $t \geq 0$, given the capital share $\omega_t(z)$, the equilibrium aggregate output is*

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha}, \tag{40}$$

where Z_t is the economy's TFP, given by

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} H_t N_t^{1-\alpha} \ \ with \ \ H_t = \left[\frac{\int_{\underline{z}_t}^\infty z\omega_t(z)dz}{1-\Omega_t(\underline{z}_t)}\right]^\alpha. \tag{41}$$

The variable H_t captures the endogenous productivity of the final goods sector. The equilibrium

[12] In fact, similar to the model of Moll (2014), the marginal distribution of capital is not stationary due to the constant-returns-to-scale production technology.

K_t/A_t ratio is determined by the productivity cutoff $\underline{z}_t$ in equation (28):

$$K_t/A_t = (1+\lambda)\left[1-\Omega_t(\underline{z}_t)\right]. \tag{42}$$

Factor prices are

$$p_{j,t} = 1/\nu, \ \textit{for all} \ \ j \in [0, N_t], \quad \textit{and} \quad p_t = N_t^{\frac{\nu-1}{\nu}}/\nu, \tag{43}$$

$$w_t = (1-\alpha)(1-\varepsilon)Y_t/L_t, \tag{44}$$

where κ_t in equation (29) is simplified to $\kappa_t = \alpha(1-\varepsilon)H_t^{-\frac{1}{\alpha}}Y_t/K_t$. The aggregate profits of the intermediate goods sector and R&D expenditure are

$$N_t\pi_t = (1-\nu)\varepsilon Y_t, \tag{45}$$

$$S_t = (\chi v_t)^{\frac{1}{h}} N_t. \tag{46}$$

Equation (40) shows that the economy's aggregate TFP is $(\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} H_t N_t^{1-\alpha}$, which depends on the knowledge stock N_t and the productivity H_t of the final goods sector. A higher H_t increases the demand for intermediate goods through equation (7), and thus the incentive to innovate. This, in turn, increases growth rate of N_t and hence the growth rate of aggregate TFP. In equation (41), H_t is the average firm-level productivity z weighted by the capital share $\omega_t(z)$.[13] The equilibrium productivity cutoff $\underline{z}_t$ is determined directly by the CDF of capital share (see equation (42)) due to the bang-bang solution in equation (23). The value of H_t is higher when more productive firms are associated with more capital, which reflects a more efficient capital allocation across firms.

Equation (43) is a direct consequence of homogeneous intermediate goods producers facing the a constant elasticity of substitution, $1/(1-\nu)$. Equation (44) implies that the equilibrium wage is competitive, given by the labor share, $(1-\alpha)(1-\varepsilon)$, in the production function times the aggregate per-capita output, Y_t/L_t. In the intermediate goods sector, equation (45) implies that the aggregate profit flow, $N_t\pi_t$, equals the share of intermediate goods in aggregate output, εY_t, multiplied by the profitability of

[13] The formula (41) is related to the industry-level TFP formula derived by Hsieh and Klenow (2009). The key difference is that in our model, firms in the final goods sector produce homogeneous goods. But firms in the model of Hsieh and Klenow (2009) produce differentiated goods. In Online Appendix 3.3, we show that by driving the elasticity of substitution among goods to infinity and wedges to zero, the industry-level TFP formula of Hsieh and Klenow (2009) coincides with our productivity H_t in equation (41).

intermediate-goods producers, as captured by the inverse of the elasticity of substitution, $1-\nu$, among differentiated intermediate goods. In equation (46), innovators' R&D expenditure increases with the value of blueprints v_t with an elasticity of $1/h$.

3 Model Solution and Mechanism

In this section, we present the model solutions to highlight the key mechanism of the model. In Section 3.1, we propose an parametric approximation of the model using parametric functions to represent the distribution of firms. Under this approximation, we derive an endogenous state variable that intuitively captures the misallocation of the model economy. In Sections 3.2 and 3.3, we characterize the evolution of the economy in the presence of aggregate shocks and the balanced growth path in the absence of aggregate shocks, respectively. Finally, in Section 3.4, we focus on the balanced growth path to illustrate the key theoretical mechanism: a one-time shock has a persistent effect on misallocation, which in turn generates a persistent effect on aggregate growth.

3.1 Parametric Approximation: Misallocation as A State Variable

The capital share $\omega_t(z)$ is crucial in determining the final goods sector's productivity H_t in equation (41), whose value reflects the misallocation of capital. As in the model of Moll (2014) and many other general-equilibrium models with heterogeneous agents, the capital share is an infinite-dimensional object that evolves endogenously.

We propose a parametric approximation of $\omega_t(z)$ for three purposes: First, it yields a simple endogenous state variable that intuitively captures the misallocation of capital in the final goods sector; second, it allows us to elucidate the relationship between the dynamics of misallocation and the dynamics of aggregate growth in a transparent way, shedding light on the key theoretical mechanism that generates low-frequency growth fluctuations; and third, it allows us to analytically characterize the evolution of the model economy, making the computation of model dynamics highly tractable.[14]

Our idea of using tractable parametric approximations to deliver key model mechanisms is in spirit similar to several important works in the finance literature. For example,

[14]To evaluate the accuracy of our parametric approximation, we show in Online Appendix 2 that, under our baseline calibration, the approximation yields solutions sufficiently close to the solutions of numerical methods that directly track the evolution of $\omega_t(z)$ using higher-order moments in both the balanced growth path and the stochastic steady state.

Campbell and Shiller (1988*b*) propose log-linear present value approximations to decompose the impact of discount-rate news and cash-flow news on stock valuations. Gabaix (2007, 2012) develops the class of "linearity-generating" processes to achieve analytical convenience when revisiting a set of macro-finance puzzles. In Online Appendix 2.2, we provide detailed discussions for the relationship between our parametric approximation method and the numerical approximation methods developed in the literature (e.g., Krusell and Smith, 1998; Winberry, 2018).[15]

Specifically, at any point in time $t \geq 0$, we approximate the distribution of log capital $\widetilde{a}_{i,t} = \ln a_{i,t}$ and log productivity $\widetilde{z}_{i,t} = \ln z_{i,t}$ across firms in the final goods sector using a bivariate normal distribution. This approximation is intuitive because according to equation (5), $\ln z_{i,t}$ across firms follows a normal distribution, $\widetilde{z}_{i,t} \sim N(0, \sigma^2/2)$, in the stationary equilibrium. Moreover, using the Berry-Esseen bound, we can heuristically show that $\widetilde{a}_{i,t}$ across firms approximately follows a normal distribution (see Online Appendix 2.1). This joint-normality assumption allows us to derive a closed-form representation for the capital share $\omega_t(z)$ as follows.

Proposition 3. *For any $t \geq 0$, the capital share $\omega_t(z)$ can be approximated by the PDF of a log-normal distribution,*

$$\omega_t(z) = \frac{1}{z\sigma\sqrt{\pi}} \exp\left[-\frac{(\ln z + \sigma^2 M_t/2)^2}{\sigma^2}\right], \tag{47}$$

where $M_t \equiv -Cov(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t}) = -2Cov(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\sigma^2$.

Proposition 3 implies that under our approximation, the endogenous state variable $M_t \equiv -\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$ is a sufficient statistic that characterizes $\omega_t(z)$. We further characterize the economy's TFP Z_t in closed form.

Proposition 4. *Under our approximation, the TFP Z_t is*

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} N_t^{1-\alpha} \left[(1+\lambda)\frac{A_t}{K_t} \exp\left(-\frac{\sigma^2}{2}M_t + \frac{\sigma^2}{4}\right) \Phi\left(\Phi^{-1}\left(\frac{1}{1+\lambda}\frac{K_t}{A_t}\right) + \frac{\sigma}{\sqrt{2}}\right)\right]^{\alpha}, \tag{48}$$

[15]Briefly, our parametric approximation method is similar to these numerical approximation methods in terms of using a few moments to summarize the infinite-dimensional cross-sectional distribution of agents or firms. However, the difference is that our method directly imposes the parametric functional form to characterize the distribution at any point in time, which allows us to derive closed-form equations for the evolution of these moments. By contrast, in the numerical approximation methods, these moments are obtained by numerically fitting the distribution using the parametric functional form and as a result, the evolution of these moments is not characterized in closed form.

where $\Phi(\cdot)$ represents the CDF of a standard normal variable. The productivity cutoff $\underline{z}_t$ is

$$\underline{z}_t = \exp\left[-\frac{\sigma^2}{2}M_t - \Phi^{-1}\left(\frac{1}{1+\lambda}\frac{K_t}{A_t}\right)\frac{\sigma}{\sqrt{2}}\right]. \tag{49}$$

Equation (48) shows that the economy's TFP Z_t strictly decreases with the state variable M_t. Thus, the state variable M_t reflects the degree of misallocation in our model economy. In fact, M_t captures the distribution of MRPK. To elaborate, substituting out labor and intermediate inputs in firms' technology using Lemma 1, we obtain

$$y_{i,t} = q_{i,t}k_{i,t}, \tag{50}$$

where $q_{i,t} = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}}\left[(1-\alpha)(1-\varepsilon)/w_t\right]^{\frac{1-\alpha}{\alpha}} z_{i,t}$. Because final goods are the numeraire, $q_{i,t}$ measures firm i's MRPK at t. Define $\widetilde{q}_{i,t} = \ln q_{i,t}$. We obtain a theoretically motivated measure for misallocation:

$$M_t \equiv -\frac{\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})}{\mathrm{var}(\widetilde{z}_{i,t})} = -\frac{\mathrm{Cov}(\widetilde{q}_{i,t}, \widetilde{a}_{i,t})}{\mathrm{var}(\widetilde{q}_{i,t})}. \tag{51}$$

In our model, the covariance between productivity and capital, $\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})$, is similar to the covariance between MRPK and capital, $\mathrm{Cov}(\widetilde{q}_{i,t}, \widetilde{a}_{i,t})$, because firms produce homogeneous goods using a constant-returns-to-scale production technology.[16] Intuitively, a higher M_t implies that firms with higher productivity $z_{i,t}$ or MRPK $q_{i,t}$ are associated with less capital $a_{i,t}$, resulting in a lower aggregate TFP according to Proposition 4.

Relation to Existing Empirical Measures of Misallocation. Our model-implied misallocation measure M_t is similar to the capital allocation efficiency measure based on the covariance between size and productivity (e.g., Olley and Pakes, 1996; Bartelsman, Haltiwanger and Scarpetta, 2009, 2013).[17] The state variable M_t constructed in equation

[16] This relationship does not hold in the model of Hsieh and Klenow (2009) because firms produce differentiated goods with different prices in their model.

[17] Olley and Pakes (1996) decompose the total productivity into the unweighted average of plant level productivities and the sample covariance between productivity and output share; they argue that the latter captures capital allocation efficiency because a higher covariance implies that a higher share of output goes to more productive firms. Bartelsman, Haltiwanger and Scarpetta (2009, 2013) generalize this measure by focusing on the covariance between firm-level log productivity and size, where productivity is measured by physical TFP, revenue TFP, or labor productivity and size is measured by the amount of physical output, revenue, or input. They show that the size-productivity relationship carries over to these alternative measures of size and productivity in a class of models.

(51) provides a theoretical justification for using the size-productivity covariance as a measure of capital allocation efficiency. In particular, our model analytically characterizes that a higher M_t (i.e., a lower covariance) reduces aggregate TFP (see equation (48)). Moreover, under the parametric approximation of our model economy, M_t sufficiently summarizes the cross-sectional distribution of firms, $\omega_t(z)$, which highlights the crucial role played by this state variable.

The covariance-type measure for misallocation is essentially similar to the dispersion measures used in the misallocation literature, such as the dispersion of revenue TFP or MRPK (e.g., Foster, Haltiwanger and Syverson, 2008; Hsieh and Klenow, 2009; David and Venkateswaran, 2019) because they all gauge the impacts of capital allocation efficiency on aggregate TFP and rest on the same fundamental presumption that allocation is distorted if the marginal revenue product of a production factor is not equal to its marginal cost. Bartelsman, Haltiwanger and Scarpetta (2013) provide suggestive evidence that the size-productivity relationship is more robust to multiplicative measurement errors than dispersion measures because these classical measurement errors directly increase the standard deviation of measured MRPK but do not change the measured covariance. Relatedly, Eisfeldt and Shi (2018) argue that the productivity dispersion measures are noisy and thus, these measures may not be informative for measuring the business cycle variation in misallocation frictions. These concerns are especially relevant for our paper, given the purpose is to understand the asset pricing implications of misallocation in the context of business cycles. Thus, in Section 4.1, we construct a model-consistent empirical measure for misallocation based on the covariance-type measure M_t in equation (51).

We make two additional remarks on the misallocation measure M_t motivated by our model. First, for tractability, our model builds on Moll (2014) and assumes that firms in the final goods sector use constant-returns-to-scale technology to produce homogeneous goods. Thus, the most efficient allocation is to move all capital to firms with the highest productivity, which is unbounded. Although the first-best allocation is not well defined in our model, M_t is still a valid measure for misallocation due to its monotonic relationship with aggregate TFP and output (see equation (48)). This is enough for our purpose because we focus on the time-variation in M_t and its implications for asset prices. Second, the dispersion measure is not a valid measure for misallocation in our model because firms use constant-returns-to-scale technology to produce homogeneous goods. Under this specification, the dispersion in revenue TFP or MRPK is simply proportional to the dispersion in physical TFP (i.e., the standard deviation of $z_{i,t}$), which is exogenous and constant in the stationary equilibrium due to equation (5).

3.2 Evolution of the Economy

Under the parametric approximation, the economy's transitional dynamics are characterized by the evolution of aggregate capital A_t in the final goods sector, knowledge stock N_t, and misallocation M_t, as summarized in the proposition below.[18]

Proposition 5. *Under our parametric approximation, for all $t \geq 0$, the economy is fully characterized by the evolution of aggregate capital A_t in the final goods sector, knowledge stock N_t, and misallocation M_t, as follows*

$$dA_t = \alpha(1-\varepsilon)Y_t dt - (\delta_k K_t + \delta_a A_t)\, dt - r_{f,t} B_t dt - \rho A_t dt - (\sigma_k K_t - \sigma_{a,t} A_t) dW_t, \tag{52}$$

$$dN_t = \chi\,(\chi v_t)^{\frac{1-h}{h}}\, N_t dt - \delta_b N_t dt, \tag{53}$$

$$dM_t = -\,\theta M_t dt - \frac{Cov(\widetilde{z}_{i,t}, d\widetilde{a}_{i,t})}{\text{var}(\widetilde{z}_{i,t})}, \tag{54}$$

where $Cov(\widetilde{z}_{i,t}, d\widetilde{a}_{i,t})$ is given by equation (76) in Online Appendix 1.6. Define $E_t = N_t/A_t$ as the knowledge stock-capital ratio. Because the economy is homogeneous of degree one in A_t, the three state variables (A_t, N_t, M_t) can be further reduced to two state variables (E_t, M_t).

In equation (52), the accumulation of A_t over $[t, t+\mathrm{d}t)$ is given by the capital share, $\alpha(1-\varepsilon)$, in the production function times the aggregate output, $Y_t\mathrm{d}t$, minus capital depreciation, $(\delta_k K_t + \delta_a A_t)\mathrm{d}t$, interests on bonds, $r_{f,t}B_t\mathrm{d}t$, and dividend payout, $\rho A_t \mathrm{d}t$. The last term $(\sigma_k K_t - \sigma_{a,t} A_t)\mathrm{d}W_t$ captures the variation in A_t due to aggregate shocks. As the purpose of this model is to theoretically demonstrate the asset pricing implications of misallocation, we seek to establish a clean setting without confounding effects from other channels. Thus, we adopt a technical specification, $\sigma_{a,t} = K_t/A_t \sigma_k$. This specification ensures that the evolution of aggregate capital stock is locally deterministic. Thus, economic fluctuations in our model are purely driven by the variation in misallocation M_t in equation (54), allowing us to cleanly analyze the role of M_t. By ruling out other channels, the technical specification will overstate the quantitative effect of misallocation. Having this caveat in mind, we emphasize that the purpose of our quantitative analysis in Section 4 is not to structurally identify the component of misallocation-driven growth fluctuations.

[18]The variables K_t andB_t are not state variables because they are determined by (37) and (42), given A_t.

Instead, our goal is to demonstrate that under a reasonable calibration, the slow-moving misallocation has the capacity to generate low-frequency growth fluctuations, which in turn rationalize the high Sharpe ratio in the capital market and generate large welfare costs.

In equation (53), the accumulation of knowledge stock N_t increases with the value of blueprints v_t because a higher v_t motivates innovators to increase R&D expenditure S_t (equation (46)). Importantly, the misallocation M_t determines the economy's endogenous growth rate over $[t, t + \mathrm{d}t)$. This is because v_t equals the present value of profit flow π_t (equation (8)), and thus v_t is higher when π_t is higher. A lower misallocation M_t increases the economy's TFP Z_t (equation (48)), leading to a higher aggregate output Y_t (equation (40)) and thus a higher profit flow π_t (equation (45)), and ultimately, a higher growth rate of the economy. By linking the final goods sector and the innovation sector through the endogenous TFP Z_t, the allocation of capital $a_{i,t}$ among firms of different productivity $z_{i,t}$ plays a crucial role in determining economic growth.

Equation (54) shows that the evolution of M_t depends on two terms. The first term $-\theta M_t \mathrm{d}t$ reflects time-varying productivity $z_{i,t}$ evolving according to equation (5). Intuitively, a higher θ implies a less persistent idiosyncratic productivity $z_{i,t}$, which pushes the misallocation $M_t = -\mathrm{Cov}(\widetilde{z}_{i,t}, \widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$ towards zero, making M_t less persistent. The second term $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})/\mathrm{var}(\widetilde{z}_{i,t})$ captures the impact of capital accumulation, $\mathrm{d}\widetilde{a}_{i,t}$, evolving according to equation (3). A higher $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ implies that more productive firms accumulate their capital at a higher rate, which reduces misallocation M_t.

Importantly, the variable $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ negatively depends on the aggregate shock $\mathrm{d}W_t$ (see equation (76) in Online Appendix 1.6). Intuitively, a positive shock ($\mathrm{d}W_t > 0$) increases the depreciation rate of capital $k_{i,t}$, which reduces the capital accumulation of more productive firms (i.e., $z_{i,t} \geq \underline{z}_t$) but not those less productive firms (i.e., $z_{i,t} < \underline{z}_t$), which do not produce (see equation (23)). As a result, a positive shock leads to a lower $\mathrm{Cov}(\widetilde{z}_{i,t}, \mathrm{d}\widetilde{a}_{i,t})$ and increases the misallocation M_t, which in turn reduces aggregate output and consumption, indicating that M_t is countercyclical.

3.3 Balanced Growth Path

To clearly illustrate the equilibrium relationship between misallocation and long-run growth, we characterize the economy's balanced growth path in the absence of aggregate shocks (i.e., $\mathrm{d}W_t \equiv 0$).

Proposition 6. *There is a balanced growth path in which $E_t \equiv E$, $M_t \equiv M$, and $H_t \equiv H$ are constant. The aggregate capital A_t, knowledge stock N_t, output Y_t, TFP Z_t, and consumption C_t grow at the same constant rate g, and their ratios are constant.*

The values of these variables and the long-run growth rate g are determined by a system of equations presented in Online Appendix 1.7. The next proposition clearly shows that, in the balanced growth path, there is a negative relationship between the R&D-capital ratio $\overline{S/A}$ and misallocation M.

Proposition 7. *Under our parametric approximation, the R&D-capital ratio, $\overline{S/A}$, is negatively related to misallocation M in the balanced growth path,*

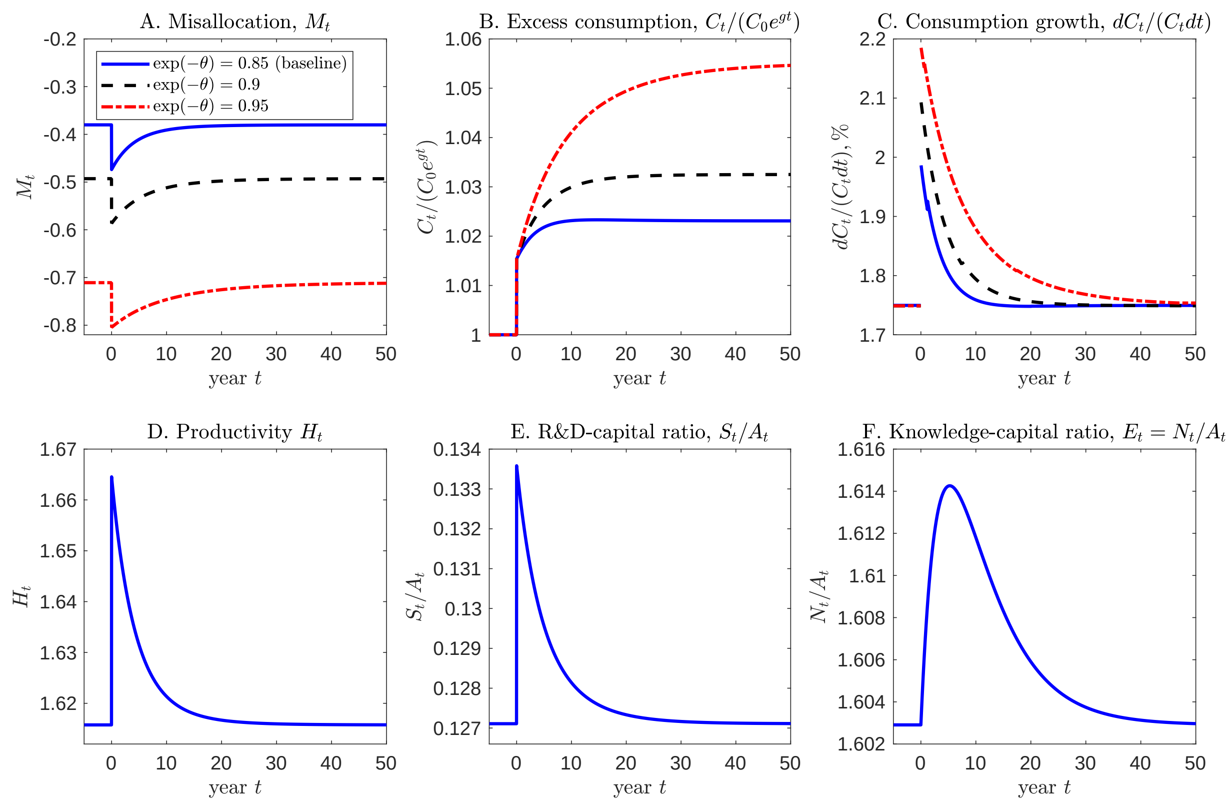
$$\begin{aligned} \ln\left(\overline{S/A}\right) = &-\frac{\alpha\sigma^2}{2h}M + \frac{\alpha\sigma^2}{4h} + \frac{1}{h}\ln(\chi) + \frac{\alpha}{h}\ln(1+\lambda) + \left(1-\frac{\alpha}{h}\right)\ln E \\ &+ \frac{1}{h}\ln\left(\frac{(1-\nu)\varepsilon(\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}}}{r_f+\delta_b}\right) + \frac{\alpha}{h}\ln\left(\Phi\left(\Phi^{-1}\left(\frac{\overline{K/A}}{1+\lambda}\right)+\frac{\sigma}{\sqrt{2}}\right)\right). \end{aligned} \tag{55}$$

In the presence of aggregate shocks, the negative relationship between R&D-capital ratio and misallocation characterized in Proposition 7 still holds. Because R&D is the driving force of economy growth in our model, a higher misallocation leads to a lower growth rate through its negative effect on R&D. Moreover, because aggregate shocks $\mathrm{d}W_t$ affect misallocation M_t (equation (54)), the economy's growth rate is time varying, depending on its misallocation M_t.

3.4 Key Mechanism: Persistence of Misallocation and Growth

In this section, we focus on the balanced growth path characterized in Section 3.3 to clearly illustrate the key theoretical mechanism of the model. We show that a one-time shock has a persistent effect on misallocation M_t, which in turn generates a persistent effect on aggregate growth through its impact on R&D-to-capital ratio (Proposition 7), the driving force of economic growth. Further, we show that the persistence of aggregate growth depends on the persistence of misallocation, which largely depends on the persistence of idiosyncratic productivity.

Impulse Response Function. Consider a one-time unexpected shock, which reduces the misallocation M_t exogenously at $t = 0$. The blue solid lines in Figure 1 plot the transitional dynamics of several major variables for $t \geq 0$ under our benchmark calibration



Note: Consider an unexpected shock that reduces misallocation M_t by $\sigma[M_t] = 0.09$ at $t = 0$. Panels A, B, and C plot the transitional dynamics of misallocation M_t, excess consumption $C_t/(C_0e^{gt})$, and contemporaneous consumption growth rate $\mathrm{d}C_t/(C_t\mathrm{d}t)$ when θ is calibrated at different values. For each choice of θ, we recalibrate the parameter χ so that the consumption growth rate in the balanced growth path is the same as our baseline calibration. All other parameters are set according to our calibration in Table 1. Panels D, E, and F plot the transitional dynamics of final goods sector's productivity H_t, R&D-capital ratio S_t/A_t, and knowledge stock-capital ratio $E_t = N_t/A_t$ for the baseline calibration with $e^{-\theta} = 0.85$.

Figure 1: Transitional dynamics after a one-time shock in misallocation M_t.

(see Table 1). To make the quantitative effects informative, the size of the shock is set at 0.09, corresponding to the standard deviation of M_t under our calibration. Panel A shows that misallocation M_t declines immediately at $t = 0$ and slowly recovers and reaches the steady-state level after about 20 years.

In the absence of aggregate shocks, aggregate consumption would be C_0e^{gt}, growing at a constant annualized rate $g = 1.75\%$ for all $t \geq 0$. We take out the trend effect in C_t by focusing on excess consumption, defined by $C_t/(C_0e^{gt})$. The blue solid line in panel B shows that excess consumption $C_t/(C_0e^{gt})$ stays at one before the shock, and it immediately jumps to about 1.015 when the shock hits at $t = 0$, and continues to increase until reaching the balanced growth path. Even though the shock is transitory, the economy converges to a steady state with permanently higher consumption due to the endogenous accumulation of capital A_t and knowledge stock N_t.

Panel C illustrates a similar idea by plotting the contemporaneous consumption growth rate over $[t, t + \mathrm{d}t)$, defined by $\mathrm{d}C_t/(C_t\mathrm{d}t)$. The blue solid line shows that the consumption growth rate increases dramatically to about 1.98% when the shock hits at $t = 0$. This is because the reduction in misallocation M_t immediately increases the productivity H_t of the final goods sector (panel D). A higher H_t increases the profits of innovators, motivating them to spend more on R&D (panel E), which consequently leads to a higher growth rate of the economy. Crucially, it is the persistence in misallocation M_t (panel A), through its impact on R&D, that results in persistent excess consumption growth relative to the balanced growth path (panels B and C). Panel F plots the evolution of the knowledge stock-capital ratio $E_t = N_t/A_t$, which has hump-shaped dynamics because we only introduce a one-time shock in M_t at $t = 0$.

Role of the Persistence of Idiosyncratic Productivity. As shown in equation (54), the persistence of misallocation depends on the parameter θ, which governs the persistence of idiosyncratic productivity $z_{i,t}$. To further illustrate that the relationship between the persistence of misallocation and the persistence of aggregate consumption growth, we study the transitional dynamics under different values of θ. Specifically, according to equation (5), the yearly autocorrelation in $\ln z_{i,t}$ is $e^{-\theta}$. In panels A, B and C of Figure 1, we compare our baseline calibration with $e^{-\theta} = 0.85$ to two alternative calibrations in which the yearly autocorrelation in $\ln z_{i,t}$ is 0.9 (black dashed line) and 0.95 (red dash-dotted line), respectively.

Panel A shows that the calibration with a higher persistence of $z_{i,t}$ is associated with lower misallocation M_t in the balanced growth path, which follows the insight of Buera and Shin (2011) and Moll (2014). Importantly, the convergence speed of M_t decreases with the persistence of $z_{i,t}$. We compute the half-life of transitions, defined as the time required for M_t to recover to half of its long-run value after the shock. The half life of M_t is 3.0, 4.2, and 6.9 years for $e^{-\theta} = 0.85$, 0.9, and 0.95, respectively, indicating that misallocation becomes more persistent when idiosyncratic productivity is more persistent. Comparing the three curves in panels B and C, it is clear that the economy with a higher persistence of $z_{i,t}$ has more persistent consumption growth after the shock in M_t.

Thus, our model suggests that the persistence of idiosyncratic productivity, $z_{i,t}$, plays an important role in determining the persistence of the growth rate of aggregate consumption, $\mathrm{d}C_t/(C_t\mathrm{d}t)$. The persistence of these two variables is connected with each other via the persistent endogenous misallocation M_t. This result generalizes the key insight of Moll (2014) to an economy with stochastic growth. In a model without long-run growth or aggregate shocks, Moll (2014) shows that transitions to steady states are slower when idiosyncratic productivity shocks become more persistent. Building on this insight, we further show that in a model with endogenous stochastic growth, the persistence of idiosyncratic productivity determines the persistence of aggregate growth through its effect on the persistence of endogenous misallocation. With a proper calibration of θ, this theoretical mechanism allows our model to quantitatively rationalize endogenous low-frequency growth fluctuations through slow-moving misallocation, thereby generating a high Sharpe ratio in the capital market (see Section 4.4) and significant welfare losses of misallocation fluctuations (see Section 4.5).

4 Quantitative Analysis

In this section, we study the quantitative implications of misallocation for economic growth, asset prices, and welfare. Section 4.1 constructs an empirical measure of misallocation motivated by our model. Section 4.2 calibrates and validates the model based on macroeconomic and asset pricing moments. Section 4.3 shows that in both the data and model, misallocation significantly predicts R&D expenditure and future economic growth. Section 4.4 sheds light on the asset pricing implications of misallocation. Finally, Section 4.5 quantifies the costs of misallocation-driven growth fluctuations.

4.1 Data and Empirical Measures

We obtain annual consumption and GDP data from the U.S. Bureau of Economic Analysis (BEA) and stock return data from the Center for Research in Security Prices (CRSP). Consumption and output growth are measured by the log growth rate of per-capita real personal consumption expenditures on nondurable goods and services and the log growth rate of per-capita real GDP. The nominal variables are converted to real terms using the consumer price index (CPI), obtained from CRSP. We obtain data on private business R&D investment from the National Science Foundation (NSF) and the R&D stock from the Bureau of Labor Statistics (BLS). These two time series are considered as empirical counterparts for S_t and N_t, respectively. The ratio of the two (i.e., S_t/N_t) is our empirical measure for R&D intensity. The risk-free rate is constructed using the yield of 3-month Treasury Bills, obtained from CRSP. Firms' dividend yield is computed as the ratio of total dividend over market capitalization, obtained from Compustat.

Empirical Measure of Misallocation. We construct a model-consistent empirical measure of misallocation according to equation (51), $M_t = -\text{Cov}(\widetilde{q}_{i,t}, \widetilde{a}_{i,t})/\text{var}(\widetilde{q}_{i,t})$. Specifically, we regress the empirical measure of log capital $\widetilde{a}_{i,t}$ on log MRPK $\widetilde{q}_{i,t}$ using the cross section of firms in each year t in U.S. Compustat data from 1965 to 2016:[19]

$$\widetilde{a}_{i,t} = \alpha_t + \beta_t \widetilde{q}_{i,t} + \varepsilon_{i,t}, \tag{56}$$

where the estimated coefficient $\widehat{\beta}_t$ directly captures $\text{Cov}(\widetilde{q}_{i,t}, \widetilde{a}_{i,t})/\text{var}(\widetilde{q}_{i,t})$. The empirical measure of M_t is constructed using the HP-filtered time-series of $-\widehat{\beta}_t$ from 1965 to 2016 with a smoothing parameter of 100 (Backus and Kehoe, 1992). The HP filter allows us to extract the cyclical component, following the literature (e.g., Eisfeldt and Rampini, 2006). In the regression specification (56), the empirical measure for log capital $\widetilde{a}_{i,t}$ is constructed using the average log capital of firm i over the past T years, i.e., $\widetilde{a}_{i,t} \equiv T^{-1}\sum_{\tau=1}^{T}\ln(capital_{i,t+1-\tau})$, with $T = 3$. The empirical results are robust to alternative choices of T. Firm i's capital is measured by its net property, plant and equipment, i.e., $capital_{i,t} = ppent_{i,t}$.[20] We construct the empirical measure for $\widetilde{q}_{i,t}$ using the average log $MRPK_{i,t}$ of firm i over the past T years, i.e., $\widetilde{q}_{i,t} \equiv T^{-1}\sum_{\tau=1}^{T}\ln(MRPK_{i,t+1-\tau})$. According to equation (50), we measure $MRPK_{i,t}$ using $MRPK_{i,t} = sale_{i,t}/production\_capital_{i,t}$. Following the model, $production\_capital_{i,t}$ is measured by the sum of the firm's own capital ($ppent_{i,t}$) and rented capital. We measure the amount of rented capital by capitalizing rental expenses, following standard accounting practice and the literature (e.g., Rauh and Sufi, 2011; Rampini and Viswanathan, 2013). Specifically, firm i's rented capital in year t is its total rental expenses in the year multiplied by a factor 10 and capped by a fraction,

[19] Because our theory mainly applies to manufacturing firms, we exclude firms from financial, utility, public administration, and non-tradable industries, where non-tradable industries are defined according to Mian and Sufi (2014). The empirical results are robust if non-tradable industries are included in the sample.

[20] All the empirical results are robust if we use a firm's tangible net worth to construct its capital, i.e., $capital_{i,t} = tangible\_net\_worth_{i,t}$. A firm's tangible net worth is constructed as $tangible\_net\_worth_{i,t} = ppent_{i,t} + current\_assets_{i,t} + other\_assets_{i,t} - total\_liabilities_{i,t}$. As emphasized by Chava and Roberts (2008), lenders commonly use a firm's tangible net worth to assess its ability to support and pay back loans. Naturally, tangible net worth, as a measure for firms' borrowing capacity, is widely reflected in loan covenants (e.g., DeAngelo, DeAngelo and Wruck, 2002; Roberts and Sufi, 2009; Sufi, 2009; Prilmeier, 2017).

0.25, of $ppent_{i,t}$.[21]

Panel A of Figure 2 plots the time series of year-on-year changes in the empirical measure of misallocation. The shaded areas represent periods of economic downturns, including economic recessions and three financial crises.[22] The value of our empirical measure of misallocation increases sharply, in 7 of the 9 economic downturns. This stylized pattern is consistent with the model's prediction that misallocation generally increases during a period of time with macroeconomic recessions or financial turmoil.

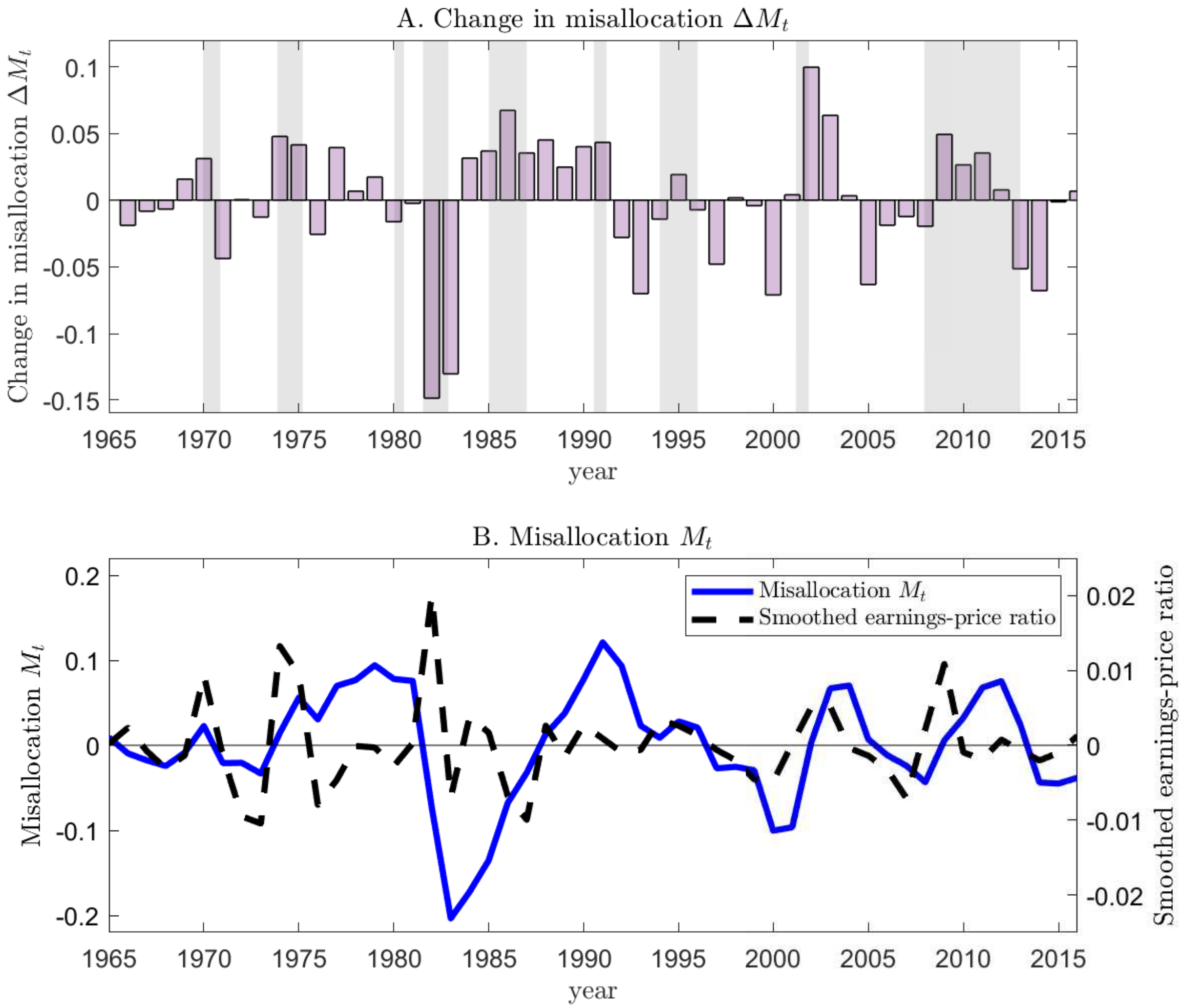
In panel B of Figure 2, we compare the empirical measure of misallocation M_t (the blue solid line) with the smoothed earnings-price ratio (the black dashed line) proposed by Campbell and Shiller (1988*a*). The smoothed earnings-price ratio is commonly used as an empirical proxy for the aggregate discount rate (e.g., Dou, Ji and Wu, 2021, 2022) and its time-series variation roughly has business cycle frequency. Clearly, the empirical measure of misallocation M_t is much more persistent than the smoothed earnings-price ratio. The yearly autocorrelation of M_t is 0.75, which is close to the calibrated persistence of 0.77 by Bansal and Yaron (2004) for the predictable component of consumption growth. If misallocation M_t affects economic growth, as suggested by our model, the highly persistent and volatile M_t seems to capture the low-frequency growth fluctuations, referred to as the medium-term business cycle by Comin and Gertler (2006) or the growth cycle by Kung and Schmid (2015).

4.2 Calibration and Validation of the Model

Panel A of Table 1 presents externally calibrated parameters. Following the standard practice, we set the capital share in the production technology at $\alpha = 0.33$. We set the yearly capital depreciation rates at $\delta_k = \delta_a = 3\%$. We set the share of intermediate inputs at $\varepsilon = 0.5$ according to the estimates of Jones (2011, 2013). The inverse markup is set at $\nu = \epsilon/(\epsilon + (1-\alpha)(1-\epsilon)) = 0.6$ to guarantee the existence of a balanced growth path. Following the standard practice in asset pricing literature, we set the risk aversion at $\gamma = 8$. Following Kung and Schmid (2015), we set the EIS at $\psi = 1.85$, the patent obsolescence rate at $\delta_b = 15\%$, and $h = 0.17$ so that the elasticity of new blueprints with respect to R&D is 0.83. We set the volatility of idiosyncratic productivity at $\sigma = 1.39$

[21] All empirical results are robust if we use a factor 5, 6, or 8 to capitalize rental expenses or cap the amount by a fraction, 0, 0.5, 1, or 2, of $ppent_{i,t}$.

[22] These are the savings and loan (S&L) crisis from January 1986 to December 1987, the Mexican peso crisis from January 1994 to December 1995, and the European sovereign debt crisis from September 2008 to December 2012.



Note: Panel A plots the year-on-year changes in the empirical measure of misallocation, i.e., ΔM_t. The shaded areas represent recessions or severe financial crises. Panel B plots the time series of M_t (left y-axis) and smoothed earnings-price ratio (right y-axis) proposed by Campbell and Shiller (1988*a*).

Figure 2: Time series plot of the empirical measure of misallocation M_t.

according to the calibration of Moll (2014). We set the persistence of idiosyncratic productivity at $\theta = 0.1625$, which implies that the log idiosyncratic productivity has a yearly autocorrelation of $e^{-\theta} = 0.85$, consistent with the estimate of Asker, Collard-Wexler and Loecker (2014*b*) based on U.S. census data as well as the calibration in the macroeconomics literature (e.g., Khan and Thomas, 2008; Moll, 2014; Winberry, 2018, 2021). We set the collateral constraint parameter at $\lambda = 1.1$, which is within the range of

Table 1: Parameter calibration and targeted moments.

Panel A: Externally determined parameters					
Parameter	Symbol	Value	Parameter	Symbol	Value
Capital share	α	0.33	Capital depreciation rate	δ_k, δ_a	0.03
Share of intermediate inputs	ε	0.5	1− R&D elasticity	h	0.17
EIS	ψ	1.85	Risk aversion	γ	8
Patent obsolescence rate	δ_b	0.15	Vol. of idio. productivity	σ	1.39
Inverse markup	ν	0.6	Rent extraction rate	τ	0.01
Collateral constraint	λ	1.1	Persistence of idio. productivity	θ	0.1625

Panel B: Internally calibrated parameters and targeted moments					
Parameter	Symbol	Value	Moments	Data	Model
Subjective discount rate	δ	0.01	Real risk-free rate (%)	1.11	1.58
R&D productivity	χ	1.35	Consumption growth rate (%)	1.76	1.75
Capital depreciation shock	σ_k	0.19	Consumption growth vol. (%)	1.50	1.67
Dividend payout rate	ρ	0.037	Dividend yield (%)	2.35	2.14

the calibration in the macroeconomics literature (e.g., Jermann and Quadrini, 2012; Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014; Dabla-Norris et al., 2021). The rent extraction rate τ is a scaling parameter whose value does not affect firm decisions. We normalize it at $\tau = 1\%$.

The remaining parameters are calibrated by matching the relevant moments summarized in Panel B of Table 1. When constructing the model moments, we simulate a sample for $1,000$ years with a 100-year burn-in period, which is long enough to guarantee the stability of these moments. The discount rate is set at $\delta = 0.01$ to generate a real risk-free rate of about 1.58%. The R&D productivity is set at $\chi = 1.35$ to generate an average consumption growth rate of about 1.75%. Following Storesletten, Telmer and Yaron (2007), we calibrate $\sigma_k = 0.19$ so that the model-implied volatility of consumption growth is about 1.66%. We set the payout rate at $\rho = 3.7\%$ so that the dividend yield is 2.14%.

Table 2 presents untargeted moments as a validation test of the model. Panel A shows that the moments reflecting the persistence of consumption growth implied by the model are roughly consistent with those in the data even though these moments are not directly targeted in our calibration. Panel B shows that the yearly autocorrelation of R&D expenditure growth $\Delta \ln S_t$ and misallocation M_t also have comparable values in the model and data. The model implies a smooth risk-free rate and a high Sharpe ratio of

Table 2: Untargeted moments in the data and model.

Moments	Data	Model	Moments	Data	Model
		Panel A: Consumption moments			
$AC1(\Delta \ln C_t)$ (%)	0.44	0.46	$AC2(\Delta \ln C_t)$ (%)	0.08	0.28
$AC5(\Delta \ln C_t)$ (%)	−0.01	0.00	$AC10(\Delta \ln C_t)$ (%)	0.06	−0.06
$VR2(\Delta \ln C_t)$ (%)	1.52	1.46	$VR5(\Delta \ln C_t)$ (%)	2.02	2.21
		Panel B: Other moments			
$AC1(\Delta \ln S_t)$ (%)	0.30	0.42	$AC1(M_t)$ (%)	0.75	0.73
$SR[R_{w,t}]$	0.36	0.39	$\sigma[r_{f,t}]$ (%)	2.06	0.47

Note: With slight abuse of notations, $\Delta \ln X_t = \ln X_t - \ln X_{t-1}$ represents difference in $\ln X_t$ between year t and year $t-1$, where the yearly value of X_t is computed by integrating $X_t \mathrm{d}t$ in continuous time. $ACk(\Delta \ln C_t)$ refers to the autocorrelation of log consumption growth with a k-year lag. $VRk(\Delta \ln C_t)$ refers to the variance ratio of log consumption growth with a k-year horizon. $AC1(\Delta \ln S_t)$ is the yearly autocorrelation of log private business R&D investment growth. $AC1(M_t)$ is the yearly autocorrelation of misallocation M_t. $SR[R_{w,t}] = \mathbb{E}[R_{w,t} - r_{f,t}]/\sigma[R_{w,t} - r_{f,t}]$ is the Sharpe ratio of the consumption claim.

the consumption claim, consistent with Sharpe ratio of the market portfolio in our data sample. We discuss the mechanisms that generate asset pricing implications in Section 4.4.

4.3 Misallocation, R&D and Growth

Our model implies that misallocation affects R&D intensity, which determines growth. Thus, the highly persistent empirical measure of misallocation M_t can be covarying with the slow-moving component of expected growth. In this section, we study the relationship between misallocation, R&D and growth. Rather than emphasizing M_t as a predictor for growth, our goal is to provide evidence that M_t captures the low-frequency growth fluctuations based on predicative regressions over long horizons.

In panel A of Table 3, we study the relationship between misallocation M_t and R&D intensity. In both the data and model (i.e., simulated data), we regress R&D intensity in the current year (t) and the next year ($t+1$) on misallocation M_t, as follows:

$$\frac{S_{t+h}}{N_{t+h}} = \alpha + \beta M_t + \nu_{t+h}, \tag{57}$$

where $h = 0, 1$. The results indicate that a higher misallocation is associated with a decline in contemporaneous R&D intensity and predicts a lower R&D intensity in the next year.

Table 3: Misallocation, R&D, and growth in the data and model.

Panel A: R&D intensity (S_t/N_t)				
	t		$t+1$	
	Data	Model	Data	Model
β	-0.106^{***}	-0.039^{***}	-0.094^{***}	-0.042^{***}
	$[-3.793]$	$[-9.065]$	$[-3.172]$	$[-11.821]$

Panel B: Consumption growth ($\Delta \ln C_t$)										
	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	-0.083^{***}	-0.140^{***}	-0.138^{***}	-0.201^{***}	-0.178^{***}	-0.246^{***}	-0.207^{***}	-0.275^{***}	-0.227^{***}	-0.276^{***}
	$[-3.027]$	$[-8.353]$	$[-3.232]$	$[-6.114]$	$[-3.363]$	$[-5.239]$	$[-3.700]$	$[-4.321]$	$[-3.781]$	$[-3.436]$

Panel C: Output growth ($\Delta \ln Y_t$)										
	$t \to t+1$		$t \to t+2$		$t \to t+3$		$t \to t+4$		$t \to t+5$	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
β	-0.094^{**}	-0.109^{***}	-0.139^{**}	-0.243^{***}	-0.163^{*}	-0.218^{***}	-0.193^{**}	-0.225^{***}	-0.218^{**}	-0.233^{***}
	$[-2.044]$	$[-3.357]$	$[-2.145]$	$[-6.606]$	$[-1.957]$	$[-4.053]$	$[-2.296]$	$[-3.525]$	$[-2.492]$	$[-3.123]$

Note: The data sample is yearly and spans the period from 1965 to 2016. In the model, we simulate a sample of 52 years as in the data. t-statistics are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

Next, we examine whether misallocation M_t covaries with the slow-moving component of expected growth by testing whether misallocation negatively predicts future consumption growth in the data and model. We run the following regression:

$$\Delta \ln C_{t,t+1} + \cdots + \Delta \ln C_{t+h-1,t+h} = \alpha + \beta M_t + \nu_{t,t+h}, \tag{58}$$

where $h = 1, \cdots, 5$ and $\Delta \ln C_{t+h-1,t+h}$ is the one-year log consumption growth from year $t+h-1$ to $t+h$. Panel B of Table 3 presents the results of projecting future consumption growth over horizons of one to five years on misallocation M_t. In both the data and model, the slope coefficients are negative and statistically significant. The coefficients are more negative for longer horizons as consumption growth is persistent. Our estimates indicate that misallocation M_t seems to comove with the slow-moving component of expected consumption growth. We further run regressions similar to (58) using future log output growth as the dependent variable. Panel C of Table 3 presents the results of projecting future output growth over horizons of one to five years on misallocation M_t. The patterns are similar to those of consumption growth in panel B.

Taken together, we find empirical evidence that the aggregate growth rates of consumption and output can be predicted by our empirical measure of misallocation M_t, especially over long horizons. Our findings lend empirical support to the notion of misallocation-driven low-frequency growth fluctuations. In the simulated data of our model, similar patterns are observed due to the key mechanism elaborated in Section 3.4. Our model thus helps rationalize and identify misallocation as an economic source of low-frequency growth fluctuations in the data.

4.4 Asset Pricing Implications of Misallocation

Table 4 studies the asset pricing implications of misallocation. Column (1) presents the implications in the baseline model. The claim to aggregate consumption has a high Sharpe ratio of 0.39, which is similar to that of the market portfolio in the data. Because the model is calibrated to match an annualized volatility of consumption growth of 1.5%, the excess return of the consumption claim has an annualized volatility of only 1.39%. Thus, the average excess return is low due to the small volatility. The risk-free rate has an average value of 1.58% and a low volatility, as in the data. We also compute the ratio of the one-year SDF's volatility to its mean, $\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$, which determines the maximal Sharpe ratio in the model. The baseline calibration implies a high value, 0.61.

Next, we study different model specifications. In column (2), we exogenously fix the misallocation state variable M_t at its long-run mean $\mathbb{E}[M_t]$. The volatility of the consumption claim's excess returns drops to zero and the Sharpe ratio is not defined. This is because in our model, the aggregate shock $\mathrm{d}W_t$ drives economic fluctuations purely through its effect on M_t, while the evolutions of A_t and E_t are locally deterministic (see Proposition 5). This property differentiates our theoretical mechanism from those of Kaltenbrunner and Lochstoer (2010) and Kung and Schmid (2015), whose models generate low-frequency growth fluctuations through time-varying aggregate capital stock or R&D expenditure, rather than the covariance between capital and productivity across firms (i.e., M_t).

In column (3), we quantitatively investigate the role of economic growth, determined by the growth rate of knowledge stock N_t. We consider an alternative specification with no economic growth in column (3), by setting $\mathrm{d}N_t \equiv 0$.[23] Compared with the

[23]Under this specification, the economy's aggregate output and consumption still fluctuate due to

Table 4: Asset pricing implications under different model specifications.

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$	(3) $\mathrm{d}N_t \equiv 0$	(4) $e^{-\theta}$ $= 0.2$	(5) $e^{-\theta}$ $= 0.45$	(6) CRRA ($\gamma = 1/\psi$) $= 1.5$	(7) CRRA ($\gamma = 1/\psi$) $= 3$
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54	0	0.02	0.01	0.08	0.02	0.02
$\sigma[R^e_{w,t}]$ (%)	1.39	0	0.72	1.17	1.09	1.01	0.57
$SR[R_{w,t}]$	0.39	–	0.02	0.01	0.08	0.02	0.04
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87	0.98	1.93	1.88	3.60	6.17
$\sigma[r_{f,t}]$ (%)	0.47	0	0.34	0.33	0.41	0.47	0.57
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0	0.03	0.06	0.10	0.03	0.05

Note: In the table, $R^e_{w,t} = R_{w,t} - r_{f,t}$ is the consumption claim's return $R_{w,t}$ in excess of the risk-free rate $r_{f,t}$; $SR[R_{w,t}] = \mathbb{E}[R^e_{w,t}]/\sigma[R^e_{w,t}]$ is the Sharpe ratio of the consumption claim; and $\sigma[\Lambda_{t+1}/\Lambda_t]/\mathbb{E}[\Lambda_{t+1}/\Lambda_t]$ is the ratio of the one-year SDF's volatility to its mean. Column (1) presents the results under the baseline calibration. In column (2), we adopt the same baseline calibration but eliminate fluctuations in misallocation by imposing $M_t \equiv \mathbb{E}[M_t]$ exogenously. In column (3), we adopt the same baseline calibration but eliminate the growth of knowledge stock N_t by imposing $\mathrm{d}N_t \equiv 0$ exogenously. In columns (4) to (5), we set different values of parameter θ. In columns (6) to (7), we set different values of parameter γ while imposing $\gamma = 1/\psi$. For columns (4) to (7), we calibrate χ and σ_k to generate the same model-implied average consumption growth rate and volatility as those reported in panel B of Table 1. Other parameters are set at the same values as the baseline calibration.

baseline model in column (1), the volatility of the consumption claim's excess returns drops by about half, from 1.39% to 0.72%. The average excess return declines even more significantly, resulting in a Sharpe ratio of merely 0.02.

In columns (4) and (5), we further show that merely having fluctuations in economic growth is not sufficient to rationalize a high Sharpe ratio; it is important for misallocation fluctuations to generate low-frequency growth fluctuations. Specifically, following the insight illustrated in panels A to C of Figure 1, the persistence of idiosyncratic productivity determines the persistence of growth. In columns (4) and (5), we set $e^{-\theta}$ at 0.2 and 0.45, respectively, which in turn results in a lower yearly autocorrelation of consumption growth than that in the baseline calibration with $e^{-\theta} = 0.85$. Compared with column (1), the Sharpe ratio of the consumption claim and the maximal Sharpe ratio determined by the SDF drop significantly when idiosyncratic shocks are not persistent. These results highlight the importance of low-frequency growth fluctuations in amplifying the impacts of misallocation fluctuations on risk premia. Our findings complement the main insights of Buera and Shin (2011) and Moll (2014) who analyze the impacts of the persistence

aggregate shocks. However, there is no long-run growth as the average growth rates of Y_t and C_t are zero.

of idiosyncratic productivity on TFP, welfare, and the speed of transition through the self-financing channel.

In columns (6) and (7), we consider the specification where the representative agent has a standard CRRA preference by setting $\gamma = 1/\psi$. In this setting, the Sharpe ratio implied by the model is very low, but the risk-free rate is much higher due to the low EIS.

Taken together, the results in columns (3) to (7) indicate that it is the combination of low-frequency growth fluctuations and the recursive preference of the representative agent that generates the high Sharpe ratio in the capital market. The intuition is as follows. On the one hand, the recursive preference ensures that the representative agent's marginal utility today is affected not only by news about contemptuous consumption growth, but also, crucially, by news about future consumption growth. Thus, innovations in future consumption growth can generate valuation effects through the SDF. On the other hand, because consumption growth is persistent, a temporary innovation in consumption growth would generate a persistent effect in future, which significantly amplifies the impacts of future consumption growth on the marginal utility today. If we instead consider non-recursive preference, such as the standard CRRA utility function, the low-frequency consumption growth fluctuations would not have significant valuation effects because the representative agent's marginal utility only depends on the contemporaneous consumption growth.

4.5 The Costs of Misallocation-Driven Growth Fluctuations

We evaluate the costs of misallocation-driven growth fluctuations in the stochastic stationary equilibrium through the lens of our model. In Section 4.5.1, we show that the misallocation-driven growth fluctuations can generate significant welfare costs due to the combination of two key properties of the model, endogenous low-frequency growth fluctuations and the recursive preference of the representative agent. In Section 4.5.2, we quantify the costs of misallocation fluctuations in an alternative way, measuring the potential output gain if the amount of capital reallocation observed in booms could be achieved in recessions.

4.5.1 Welfare Costs of Misallocation-Driven Growth Fluctuations

In our model, consumption fluctuations are almost entirely driven by fluctuations in misallocation. By measuring the welfare costs of consumption fluctuations, we can thus

Table 5: Welfare gains from removing consumption fluctuations.

	(1) Baseline	(2) $\mathrm{d}N_t \equiv 0$	(3) $e^{-\theta}$	(4)	(5) CRRA ($\gamma = 1/\psi$)	(6)
			$= 0.2$	$= 0.45$	$= 1.5$	$= 3$
Welfare gains (%)	10.34	0.33	0.24	0.98	0.58	0.65

Note: The welfare gains from removing consumption fluctuations are computed using $\overline{U}_0/U_0 - 1$, where U_0 and $\overline{U}_0$ represent the representative agent's utility at $t = 0$, with and without (i.e., setting $\sigma_k = 0$) consumption fluctuations, respectively. When computing $\overline{U}_0$, the parameter χ is recalibrated to have the same average consumption growth rate while all other parameter values remain unchanged. The specification in each column is described in Table 4.

provide a quantitative assessment for the welfare costs of misallocation-driven growth fluctuations within the model. In reality, consumption fluctuations are likely driven by other aggregate variables. Having this concern in mind, our purpose here is not to structurally separate the costs of consumption fluctuations attributed to misallocation fluctuations. Instead, our goal is to show that misallocation fluctuations have the capacity to generate large welfare costs by inducing consumption fluctuations, in a calibrated model that matches aggregate consumption moments (panel A of Table 2).

Specifically, we solve a similarly parameterized model without aggregate shocks (i.e., $\sigma_k = 0$) and compare the representative agent's utility gain relative to the model with aggregate shocks. Column (1) of Table 5 reports that the welfare gain from removing all consumption fluctuations is 10.34% under the baseline calibration.

In columns (2) to (6) of Table 5, we compute the welfare gains from removing consumption fluctuations under different specifications, similar to those in Table 4. Specifically, columns (2) to (4) show that the welfare gains would be small if misallocation cannot affect economic growth (i.e., setting $\mathrm{d}N_t \equiv 0$) or if misallocation is not persistent enough to generate low-frequency growth fluctuations.[24] Columns (5) and (6) show that if preference is non-recursive (i.e., setting $\gamma = 1/\psi$), the welfare gains are also small.

Taken together, these results indicate that our model implies a large welfare cost of misallocation-driven consumption fluctuations due to the combination of two properties.

[24] Columns (3) and (4) show that as idiosyncratic productivity becomes more persistent (i.e., higher $e^{-\theta}$), the welfare gain from removing consumption fluctuations increases. This finding is related to the key insight of Moll (2014), who shows that as idiosyncratic productivity becomes more persistent, the transition speed from a distorted initial state to the steady state is slower, resulting in potentially larger welfare losses during transitions. In our model with stochastic growth, the slow "transition" in response to aggregate shocks generates endogenous low-frequency growth fluctuations, which result in large welfare costs under the recursive preference of the representative agent.

First, following the mechanism discussed in Section 3.4, the model is able to generate low-frequency growth fluctuations through slow-moving misallocation. Second, the representative agent has a recursive preference and thus, news about future consumption growth affects his current marginal utility. When these two properties simultaneously exist, a temporary innovation in consumption growth would have a persistent effect in the future, which in turn significantly affects the representative agent's current marginal utility. Thus, the welfare gain from eliminating consumption fluctuations is quantified to be large.

As shown in Table 4, these two properties also enable the model to rationalize the high Sharpe ratio in the capital market. In our model, there is a tight connection between the welfare cost of consumption fluctuations and the Sharpe ratio in the capital market. Intuitively, both measures are higher when the representative agent's marginal utility exhibits larger variations in response to aggregate shocks. This tight connection is exploited by Alvarez and Jermann (2004) to estimate the welfare gains from eliminating all consumption uncertainty by directly applying the no-arbitrage principles on financial market data without specifying consumer preference. We also implement the method proposed by Alvarez and Jermann (2004) in our 1965-2016 sample and estimate that the welfare gain from eliminating all consumption uncertainty ranges from 6.03% to 23.97%, which nests the value implied by our structural model.[25]

The results in Tables 4 and 5 show that misallocation-driven growth fluctuations can have significant implications for asset prices and welfare. As misallocation arises from firms' financial constraints in our model, these results are related to the literature on the connection between financial frictions and misallocation (e.g., Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014). A direct comparison of our model's quantitative implications with these models could be difficult due to the differences in model setup. For example, our model has stochastic growth driven by misallocation fluctuations whereas these models quantify losses from misallocation in steady states or transitions, without aggregate shocks. Also, our model features both the final goods sector and the intermediate goods sector, but for tractability, we only consider misallocation in former.

Despite the differences in model setup, our findings in Table 5 are broadly consistent with the literature. For example, consistent with the calibration of Buera and Shin (2013) and Moll (2014), our calibration of large idiosyncratic shocks implies that firm-

[25] Alvarez and Jermann (2004) propose different estimation methods to show robustness. We use their first method, which projects consumption growth onto the payoff space spanned by a set of tradable assets. The estimates and implementation details of other methods are reported in Table 4 of Online Appendix 4.1.

level productivity is not very persistent. As a result, purely through the variation in misallocation M_t, the model is able to generate a 2.48% volatility of TFP, as in the data. This result is consistent with the finding of Buera and Shin (2013) that the misallocation resulting from financial frictions can generate sizable TFP losses.

While Buera and Shin (2013) purely focus on quantifying misallocation through the intensive margin (i.e., differences in MRPK among active firms due to financial frictions), other papers highlight the importance of misallocation through the extensive margin (i.e., productive firms may not be active due to financial frictions, e.g., Banerjee and Moll, 2010; Buera, Kaboski and Shin, 2011; Midrigan and Xu, 2014). Depending on the calibration and model setup, Buera, Kaboski and Shin (2011) quantify that both extensive and intensive margins are important whereas Midrigan and Xu (2014) estimate large TFP losses through the extensive margin rather than the intensive margin. In our model, the misallocation due to financial frictions reduces the final goods sector's productivity H_t, which captures the intensive margin effect. A lower H_t, in turn, reduces the profits of innovators. Through the free-entry condition (10), this further leads to a lower growth rate of the variety of intermediate goods, $\mathrm{d}N_t/N_t$ (see equation (9)), which can be thought of as capturing the extensive margin effect.[26] The results in column (3) of Table 4 and column (2) of Table 5 indicate that the extensive margin plays a crucial role in rationalizing the high Sharpe ratio in the capital market and generating a large welfare cost of misallocation-driven growth fluctuations. These findings support the significant role of extensive-margin misallocation quantified by Midrigan and Xu (2014).

4.5.2 Output Gains from Capital Reallocation

In both the data and model, capital reallocation is procyclical as the aggregate reallocation rate is higher in booms than recessions. Eisfeldt and Shi (2018) propose a method to quantify the cost of misallocation fluctuations over business cycles. The key idea of this method is to measure the potential output gain if the amount of capital reallocation observed in booms could be achieved in recessions. This method's main advantage is that it incorporates flow data on capital reallocation to help measure the cost of increased misallocation during recessions. The quantity data on flows are presumably more precisely measured than MRPK, which depends on particular model specifications.

In the model, productive firms ($z_{i,t} \geq \underline{z}_t$) lease capital from unproductive firms

[26] Note that in the intermediate good sector, there is no misallocation through the intensive margin because producers are homogeneous.

($z_{i,t} < \underline{z}_t$) as shown in Lemma 1. Thus, the aggregate reallocation rate over $[t, t + \mathrm{d}t)$ is given by

$$R_t = \frac{1}{A_t} \int_{\underline{z}_t}^{\infty} \int_0^{\infty} \lambda a \varphi_t(a, z) \mathrm{d}a \mathrm{d}z = \lambda(1 - \Omega_t(\underline{z}_t)), \tag{59}$$

where $1 - \Omega_t(\underline{z}_t)$ captures the share of aggregate capital in the final goods sector held by productive firms ($z_{i,t} \geq \underline{z}_t$). Under our calibration, $\underline{z}_t$ is countercyclical, with $\mathrm{corr}(\ln \underline{z}_t, \Delta \ln(C_t)) \approx -0.35$. Thus, the aggregate reallocation rate R_t is procyclical even with a constant λ.

Using the method proposed by Eisfeldt and Shi (2018), we quantify the potential output gain in recessions if the reallocation rate of capital among firms during recessions is assumed to be as high as that during booms.[27] We apply this method to both the actual data in our 1965-2016 sample and the simulated data of our model. The estimated potential gains in recessions from capital reallocation are 3.58% and 3.09%, respectively, indicating that misallocation fluctuations have large effects on output fluctuations. The similarity in the two estimates provide a further validation of the model.

5 Cross-Sectional Evidence

Our theory's main implication is that the volatile and persistent time-series variation in misallocation captures the low-frequency component of the time-series variation in aggregate growth. Although our model does not analyze cross-sectional implications, we provide cross-sectional evidence to further support the theoretical mechanism. In Section 5.1, we estimate the market price of risk for the misallocation factor and study its cross-sectional asset pricing implications. In Section 5.2, we show that firms with higher book-to-market ratios are more negatively exposed to the misallocation factor, which provides further support that the slow-moving misallocation captures low-frequency growth fluctuations. Finally, in Section 5.3, we provide evidence that misallocation drives long-run growth through its impact on R&D by exploiting AJCA as a policy shock to firms' financial constraints.

[27] The implementation details are in Online Appendix 4.2.

5.1 Misallocation as A Macroeconomic Risk Factor

Our model implies that the misallocation M_t plays a significant role in determining the SDF of representative agent through its effects on aggregate consumption growth. To examine the empirical relevance of this mechanism, we test whether the empirical misallocation measure M_t is a risk factor significantly priced in the cross section of assets.

We consider standard test assets, including 25 size-sorted and book-to-market-sorted portfolios, 10 momentum-sorted portfolios, and 6 maturity-sorted Treasury bond portfolios. For each asset i, we estimate the factor loadings using the following time-series regression:

$$R^e_{i,t} = c_i + \sum_k \beta_{i,k} f_{k,t} + \nu_{i,t}, \tag{60}$$

where $R^e_{i,t} = R_{i,t} - r_{f,t}$ is the excess return of asset i over the risk-free rate and $f_{k,t}$ represents risk factor k. We then estimate the cross-sectional price of risk associated with the factors $f_{k,t}$ by running a cross-sectional regression of time-series average excess returns, $\mathbb{E}[R^e_{i,t}]$, on risk factor exposures estimated in equation (60) as follows,

$$\mathbb{E}[R^e_{i,t}] = \alpha + \sum_k \widehat{\beta}_{i,k} \lambda_k + \epsilon_i, \tag{61}$$

where the estimated $\widehat{\lambda}_k$ is the price of risk for factor k and $\widehat{\alpha}$ is the average cross-sectional pricing error or zero-beta rate.

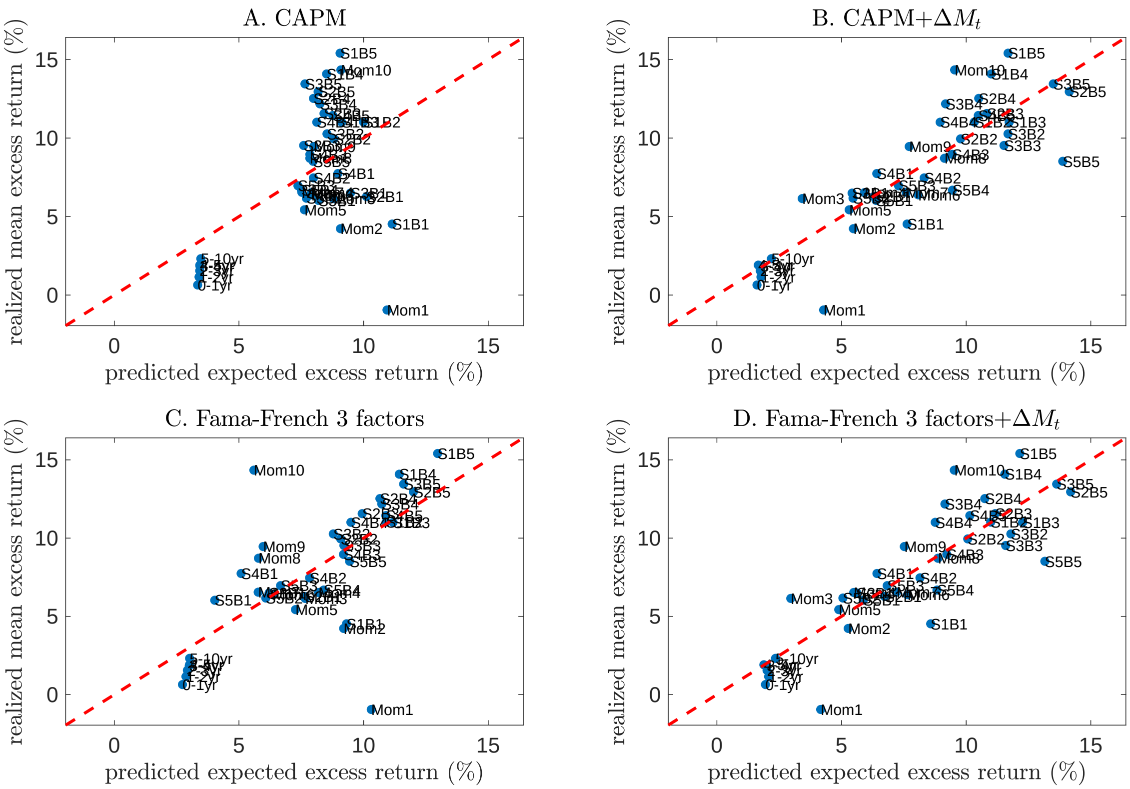
The above estimation procedure is implemented using different linear factor models. The results are presented in Table 6 and visualized in Figures 3 and 4. As a benchmark, column (1) of Table 6 reports the results of CAPM, which includes market excess returns as the single risk factor. It clearly shows that the exposure to market risk cannot explain the spread in average returns across portfolios. The cross-sectional intercept is statistically significant and the factor price of risk is statistically insignificant. The pricing errors are large, with a high total mean absolute pricing error (MAPE) of 2.76% and a low adjusted R-squared of 0.30. Column (2) of Table 6 presents the results based on a two-factor model that includes the year-on-year changes in the empirical misallocation measure, ΔM_t, as an additional risk factor. The price of risk for ΔM_t is -0.08, which is negative and statistically significant as implied by our model.[28] Relative to CAPM, the adjusted R-squared increases significantly to 0.45 and the total MAPE declines significantly from

[28] The magnitude of the price of risk for ΔM_t does not represent the risk premium of ΔM_t because the misallocation factor ΔM_t does not lie in the space of excess returns.

Table 6: Portfolio returns and model fit.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Mkt	$Mkt, \Delta M$	FF	$FF, \Delta M$	Mkt, CG	$Mkt, \Delta M, CG$	FF, CG	$FF, \Delta M, CG$
Panel A: Prices of risk								
Intercept	3.34***	1.42	2.67***	1.78	2.44	1.41	2.34**	1.46
	[3.47]	[0.84]	[3.65]	[1.51]	[1.76]	[0.89]	[2.65]	[1.16]
Mkt	4.92	6.08	3.66	5.35	4.61	5.83	4.53	6.20
	[1.28]	[0.97]	[0.98]	[0.96]	[0.95]	[1.01]	[1.08]	[1.05]
ΔM		−0.08**		−0.08***		−0.07***		−0.08**
		[−2.54]		[−2.61]		[−2.81]		[−2.45]
SMB			3.01	2.18			2.64	1.83
			[1.05]	[0.51]			[0.82]	[0.40]
HML			4.36	4.80			4.40	4.84
			[1.46]	[1.08]			[1.31]	[1.03]
CG					0.02**	0.01	0.02***	0.02*
					[2.13]	[0.83]	[2.66]	[1.84]
Panel B: Test diagnostics								
Total MAPE	2.76	1.46	1.90	1.45	2.03	1.39	1.95	1.47
Size and B/M 25	2.77	1.53	1.30	1.47	1.49	1.40	1.38	1.58
Momentum 10	3.30	1.96	3.72	1.98	3.62	1.97	3.71	1.75
Bond 6	1.85	0.37	1.36	0.48	1.61	0.40	1.43	0.55
Adjusted R-squared	0.30	0.45	0.62	0.71	0.45	0.53	0.63	0.72

Note: This table presents pricing results for 41 test assets, including 25 size-sorted and book-to-market-sorted portfolios, 10 momentum-sorted portfolios, and 6 maturity-sorted Treasury bond portfolios. Each model is estimated using equation (61). Mkt is the market's excess return over the risk-free rate. ΔM is the misallocation factor, which is the year-on-year changes in the empirical misallocation measure M_t. SMB and HML are the two factors in the FF3 model, capturing the excess returns of small caps over big caps and of value stocks over growth stocks, repectively. Panel A reports the prices of risk. Shanken t-statistics are reported in brackets. Panel B reports test diagnostics, including MAPE and the adjusted R-squared. The sample is yearly and spans the period from 1965 to 2016. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

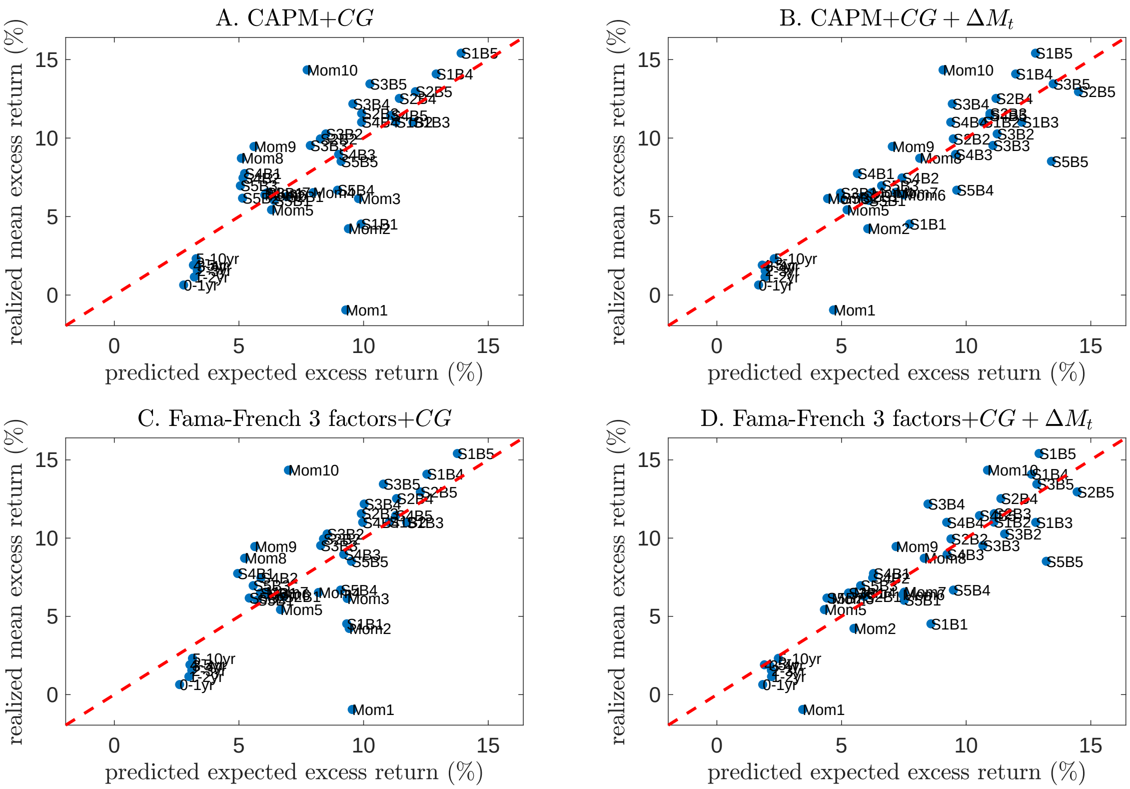


Note: This figure plots the realized mean excess returns of 35 equity portfolios (25 size-sorted and book-to-market-sorted portfolios and 10 momentum-sorted portfolios) and 6 maturity-sorted Treasury bond portfolios against the expected excess returns predicted by various linear factor asset pricing models. The sample is yearly and spans the period from 1965 to 2016.

Figure 3: Realized versus predicted mean excess returns in factor models with M_t.

to 1.46%. The test assets are lined up very close to the 45-degree line in the two factor model (panel B of Figure 3), which is in sharp contrast to the prediction of CAPM (panel A of Figure 3).

As another benchmark, column (3) of Table 6 presents the results of the Fama-French three-factor (FF3) model. Comparing columns (2) and (3) of Table 6, the FF3 model achieves a higher adjusted R-squared of 0.62. However, the two-factor model with market returns and the misallocation factor ΔM_t has a lower total MAPE. The two-factor model outperforms the FF3 model especially for the 10 momentum-sorted portfolios (3.72% compared to 1.96%). It is well known that the FF3 model has a poor explanatory power for momentum-sorted portfolio returns. The cross-sectional fit is clearly displayed in panels B and C of Figure 3, which shows that the two-factor model outperforms the



Note: This figure plots the realized mean excess returns of 35 equity portfolios (25 size-sorted and book-to-market-sorted portfolios and 10 momentum-sorted portfolios) and 6 maturity-sorted Treasury bond portfolios against the expected excess returns predicted by various linear factor asset pricing models. The sample is yearly and spans the period from 1965 to 2016.

Figure 4: Realized versus predicted mean excess returns in factor models with M_t and accumulated future consumption growth.

FF3 model mainly due to the improved fit for momentum-sorted portfolios. In column (4) of Table 6, we further include the misallocation factor ΔM_t to the FF3 model to construct a four-factor model. Compared with the FF3 model, the cross-sectional fit further improves as shown by the lower total MAPE and higher adjusted R-squared in the four-factor model. The improvement is mainly due to improved explanatory power for momentum-sorted portfolio.

Our model suggests that the low-frequency component of aggregate consumption growth is generated by the slow-moving misallocation. If this mechanism is empirically relevant, we expect the long-run expected consumption growth to have little explanatory power for portfolio returns after including the misallocation factor ΔM_t in linear factor models. Following Parker and Julliard (2005), we use accumulated future consumption growth to approximate long-run expected consumption growth. Column (5) of Table 6 and panel A of Figure 4 show that the two-factor model with market returns and accumulated future consumption growth can fit the returns of our test portfolios well, with an adjusted R-squared of 0.45. In column (6) of Table 6 and panel B of Figure 4, we augment this two-factor model with the misallocation factor ΔM_t to construct a three-factor model. We find that the relation between realized mean excess returns and predicted mean excess returns across our test portfolios stays almost unchanged, implying that expected consumption growth and misallocation are indeed similarly priced in the cross section of test assets. However, the coefficient on accumulated future consumption growth becomes statistically insignificant after including ΔM_t as a factor whereas the coefficient on ΔM_t is statistically significant. Similar patterns are shown in columns (7) and (8) of Table 6 and panels C and D of Figure 4, when we include the misallocation factor ΔM_t in a four-factor model that contains the Fama-French three factors and accumulated future consumption growth.

The strong pricing power of misallocation factor, as a (macro) nontradable asset pricing factor, is an important, nontrivial empirical finding. As emphasized by Cochrane (2017), it is the sole job of macro-finance to understand what are the primitive sources of systematic risk, by suggesting (macro) nontradable factors, and explain why they earn a premium.[29] However, not many studies find that (macro) nontradable factors motivated by macro-finance models empirically outperform or drive out (ad hoc) tradable factors such as Fama-French factors in explaining the cross section of expected asset returns,[30] partly because the measurement error in nontradable factors causes attenuation bias in the estimates of factor exposures.

5.2 Cash Flow Exposure to the Misallocation Factor

To support the key theoretical mechanism that the slow-moving misallocation drives low-frequency growth fluctuations, we provide further cross-sectional evidence on firms' cash flow exposure to the misallocation factor. Our starting point is the robust evidence found in the asset pricing literature (Bansal, Dittmar and Lundblad, 2005; Parker and

[29] Other recent reviews on macro-finance models also highlight this point (e.g., Brunnermeier, Eisenbach and Sannikov, 2012; Dou et al., 2020*a*).

[30] A few exceptions include durable consumption growth (Yogo, 2006; Gomes, Kogan and Yogo, 2009), expenditure shares of housing (Piazzesi, Schneider and Tuzel, 2007), market liquidity (Pástor and Stambaugh, 2003), intermediary leverages (Adrian, Etula and Muir, 2014; He, Kelly and Manela, 2017), and common fund flows (Dou, Kogan and Wu, 2023), among others.

Table 7: Exposure to misallocation M_t across firms sorted on the book-to-market ratio.

$BE_{i,t-1}/ME_{i,t-1}$	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
β_1^p	0.032	-0.160^{*}	-0.348^{***}	-0.425^{**}	-0.505^{***}	-0.538^{**}
	[0.217]	[−1.769]	[−3.015]	[−2.437]	[−2.756]	[−2.281]

Note: In each year t, we sort firms into quintiles on their book-to-market ratios $BE_{i,t-1}/ME_{i,t-1}$ in year $t-1$. For each quintile portfolio, we estimate β_1^p according to specification (62). The sample spans the period from 1965 to 2016. t-statistics are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

Julliard, 2005; Hansen, Heaton and Li, 2008; Santos and Veronesi, 2010): the cash flows of value firms load more positively on accumulated consumption growth than those of growth firms. Given that a higher misallocation predicts a lower consumption growth over long-horizons in both the data and model (panel B of Table 3), if the theoretical mechanism has empirical relevance, we should find the cash flows of value firms load more negatively on misallocation in the data.

To test this prediction, we follow the empirical strategy of Santos and Veronesi (2010). In each year t, we sort firms into quintiles based on their book-to-market ratios $BE_{i,t-1}/ME_{i,t-1}$ in year $t-1$, where $BE_{i,t-1}$ is the book equity from Compustat and $ME_{i,t-1}$ is the market equity from CRSP. For each quintile portfolio, we compute the value-weighted return on equity (ROE) across all firms within the portfolio, where a firm's ROE is its income before extraordinary items divided by its common equity. Let $ROE^p_{t+j,j+1}$ denote the value-weighted ROE at year $t+j$ of the portfolio p, which was formed $j+1$ years earlier, i.e., in year $t-1$. We run a regression similar to the specification adopted by Santos and Veronesi (2010), except for including accumulated misallocation shocks as an additional independent variable:

$$\sum_{j=0}^{4} \rho^j ROE^p_{t+j,j+1} = \beta_0^p + \beta_1^p \sum_{j=0}^{4} \rho^j \Delta M_{t+j} + \beta_2^p \sum_{j=0}^{4} \rho^j ROE^{Mkt}_{t+j} + \nu_t, \tag{62}$$

where $\rho = 0.95$ is a constant as in Santos and Veronesi (2010). The variable ΔM_t is the year-on-year changes in M_t and the variable ROE^{Mkt}_t is the ROE of the market portfolio. The coefficient of interest is β_1^p, which captures the loadings of accumulated ROE on accumulated misallocation shocks.

Table 7 presents the results. The accumulated ROE of firms with high book-to-market ratios (i.e., value firms in the quintile group 5 labeled as Q5) is significantly more negatively exposed to accumulated year-on-year changes in misallocation than that of firms with low book-to-market ratios (i.e., growth firms in the quintile group 1 labeled as Q1). The loadings monotonically decrease from 0.032 to -0.505 as the book-to-market ratio increases from Q1 to Q5. The difference in the loadings between Q1 and Q5 (Q5$-$Q1) is -0.538, which is statistically significant.

5.3 Impacts of AJCA on Misallocation and R&D

In the model, misallocation drives long-run growth through its impact on R&D. In this section, we provide evidence for this mechanism by examining industry-level responses to a policy shock that alleviates firms' financial constraints.

The AJCA passed in 2004 allows domestic firms in the U.S. to repatriate their foreign profits at a tax rate of 5.25%, whereas the tax rate is 35% under the prior law. This policy change effectively relaxes the financial constraints of treated firms, significantly boosting the investments of financially constrained firms (Faulkender and Petersen, 2012). According to our model, relaxed financial constraints would lead to lower misallocation, providing firms more incentive to conduct R&D. To test this prediction, we estimate the impact of AJCA on industry-level misallocation and R&D expenditure by exploiting industries' differential exposure to AJCA using a DID method.

Specifically, we construct industry-level measures for misallocation, R&D-capital ratio, and AJCA exposure using U.S. Compustat data. We use three-digit SIC codes (SIC3) to define industries. We exclude industries whose median number of firms is smaller than 10 to ensure accurate estimation of industry-level misallocation, which is constructed following the procedures described in Section 4.1, except for running regression (56) based on firms within each industry. The industry-level R&D-capital ratio is constructed as the ratio of the total R&D expenditure to total capital of firms within the industry. To capture an industry's exposure to AJCA, we construct an industry-level measure for foreign business intensity, which is the the proportion of firms in the industry whose pre-tax income from abroad during the 3-year period prior to AJCA (i.e., from 2001 to 2003) exceeds 5%. We consider industries with foreign business intensity above 33% as treated industries and the other industries as untreated industries. Treated industries are matched with untreated industries using the nearest neighbor matching method (see Online Appendix 4.3) based on six industry-level characteristics.[31] All industry-level

[31] The six industry-level characteristics are mean and standard deviation of firms' sales, mean and standard deviation of firms' profit margin, mean and standard deviation of firms' Tobin's Q. We construct a firm's (net) profit margin using its income before extraordinary items divided by its sales as in Dou, Ji and Wu (2021),

Table 8: Impacts of AJCA on misallocation and R&D.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Industry-level misallocation							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
-0.470^{***}	-0.211	-0.250	-0.046	-0.404^{**}	-0.584^{***}	-0.773^{***}	-0.627^{**}
[−2.700]	[−0.663]	[−1.295]	[−0.266]	[−2.107]	[−2.644]	[−2.729]	[−1.968]
Panel B: Industry-level R&D-capital ratio							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
0.018^{***}	-0.013	-0.005	-0.002	0.010^{**}	0.013^{**}	0.015^{**}	0.014
[2.856]	[−1.508]	[−0.924]	[−0.685]	[2.001]	[2.144]	[2.062]	[1.511]
Panel C: Industry-level R&D-capital ratio controlling for misallocation							
α	α_{-4}	α_{-3}	α_{-2}	α_0	α_1	α_2	α_3
0.013^{**}	-0.013	-0.005	-0.002	0.007	0.009	0.009	0.010
[2.017]	[−1.259]	[−0.896]	[−0.558]	[1.227]	[1.220]	[1.031]	[0.943]

Note: Panel A estimates the impacts of AJCA on industry-level misallocation. Column (1) reports the estimated $\widehat{\alpha}$ in specification (63). Columns (2) to (8) report the estimated α_τ in specification (64) for $\tau = -4, -3, -2, 0, 1, 2, 3$. All coefficients are normalized relative to $\tau = -1$. Panel B estimates the impacts of AJCA on industry-level R&D-capital ratio. Panel C estimates the impacts of AJCA on industry-level R&D-capital ratio, controlling for industry-level misallocation. t-statistics are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

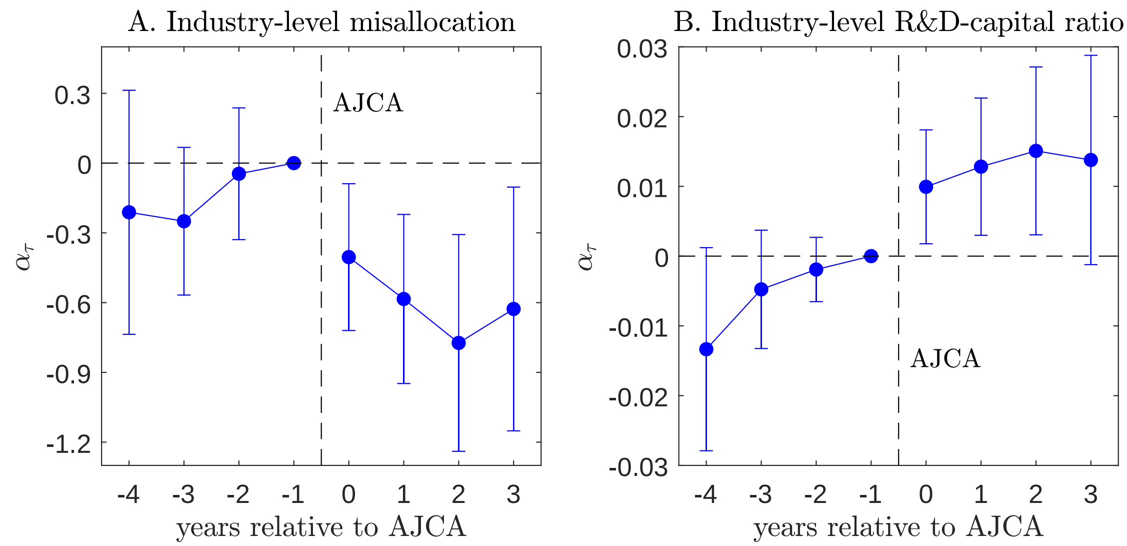
characteristics are averaged over the 3-year period prior to AJCA.

We run the following regression using industry-year observations for the period from 2000 to 2007:

$$Y_{s,t} = \alpha Treat_s \times Post_{t \geq 2004} + \beta_1 Treat_s + \beta_2 Post_t + \epsilon_{s,t}, \tag{63}$$

where $Treat_s = 1$ if industry s is a treated industry and $Treat_s = 0$ otherwise. The variable $Post_{t \geq 2004}$ is a time indicator that equals one for years after 2004. The coefficient of interest is α, which estimates the average effect of AJCA on the outcome variable $Y_{s,t}$ of treated industries. Our interested outcome variables are industry-level misallocation ($M_{s,t}$) and R&D-capital ratio ($RD_{s,t}$). The estimated coefficients are presented in column (1) of panels A and B in Table 8. Our results indicate that AJCA results in significantly lower misallocation and higher R&D-capital ratio in treated industries.

and a firm's Tobin's Q as $Tobin\_Q_{i,t} = (total\_assets_{i,t} + market\_equity_{i,t} - book\_equity_{i,t})/total\_asset_{i,t}$, following Gompers, Ishii and Metrick (2003).



Note: The solid lines visualize the empirical estimates in columns (2) to (8) of panels A and B in Table 8, respectively. All coefficients are normalized relative to $\tau = -1$. The vertical bars represent the corresponding 90% confidence intervals.

Figure 5: Impacts of AJCA on misallocation and R&D.

Next, we estimate the effect of AJCA in each year by running the following regression:

$$Y_{s,t} = \sum_{\tau=-4}^{3} \alpha_\tau Treat_s \times Year_t^\tau + \beta_1 Treat_s + \sum_{\tau=-4}^{3} \beta_{2,\tau} Year_t^\tau + \epsilon_{s,t}, \tag{64}$$

where $Year_t^\tau$ is an indicator variable that captures the time difference relative to year 2004, and it equals 1 if $t = 2004 + \tau$ and 0 otherwise. The coefficient α_τ estimates the impact of AJCA on the outcome variable $Y_{s,t}$ of treated industries τ year after (or before if $\tau < 0$) the year (i.e., 2004) in which this policy was implemented. The estimated impacts on industry-level misallocation and R&D-capital ratio are presented in columns (2) to (8) of panels A and B in Table 8, respectively, and visualized in Figure 5. The leading terms of the estimated treatment effects are close to 0 and statistically insignificant, suggesting that the parallel trend assumption is satisfied in the years before 2004. Regarding the three years after 2004, we estimate that AJCA has significant negative effects on industry-level misallocation and significant positive effects on industry-level R&D-capital ratio.

Furthermore, we provide evidence that the positive impact of AJCA on the industry-level R&D-capital ratio is mostly achieved through the change in industry-level misallocation. Specifically, we modify specification (63) as follows:

$$RD_{s,t} = \alpha Treat_s \times Post_{t\geq 2004} + \beta_1 Treat_s + \beta_2 Post_t + \beta_3 M_{s,t} + \beta_4 Treat_s \times M_{s,t} + \epsilon_{s,t}, \quad (65)$$

which controls for industry-level misallocation $M_{s,t}$ and its interaction term with $Treat_s$. The estimated coefficient is presented in column (1) of panel C in Table 8, which indicates that AJCA has a much less significant effect on R&D-capital ratio after controlling for industry-level misallocation.

We further estimate the impact of AJCA on industry-level R&D-capital ratio in each year, controlling for industry-level misallocation, by running the following regression:

$$RD_{s,t} = \sum_{\tau=-4}^{3} \alpha_\tau Treat_s \times Year_t^\tau + \beta_1 Treat_s + \sum_{\tau=-4}^{3} \beta_{2,\tau} Year_t^\tau + \beta_3 M_{s,t} + \beta_4 Treat_s \times M_{s,t} + \epsilon_{s,t}. \quad (66)$$

Columns (2) to (8) of panel C in Table 8 report the estimates in each year, which indicate that the impacts of AJCA on industry-level R&D-capital ratio are statistically insignificant after controlling for industry-level misallocation. We show that these results are robust if we estimate the impacts of AJCA using alternative empirical specifications (see Online Appendix 4.4).

6 Conclusion

This paper sheds light on the connection between misallocation and the systematic risk that shapes asset prices in capital markets. Through the lens of an analytically tractable model with endogenous growth and misallocation, we show that macroeconomic shocks can have persistent impacts on capital misallocation. The slow-moving misallocation, in turn, generates low-frequency growth fluctuations. When agents have recursive preferences, the misallocation-driven low-frequency growth fluctuations can have first-order asset pricing implications in capital markets and lead to substantial welfare losses.

In the data, we construct a misallocation measure motivated by the model and find evidence that the aggregate growth rates of consumption and output can be predicted by misallocation over long horizons. Moreover, as a macroeconomic risk factor, misallocation explains the cross-sectional returns of standard test portfolios, suggesting that it also captures low-frequency fluctuations in stock returns.

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